UNIQUE STUDY POINT

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PRACTICE PAPER 03 - CHAPTER 04 QUADRATIC EQUATIONS (2025-26)

Made with ❤ by Sumeet Sahu

SUBJECT: MATHEMATICS **MAX. MARKS:** 40

CLASS: X **DURATION:** 1½ hrs

Website: uniquestudyonline.com

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five Sections A, B, C, D and E.
3. Section A: 10 MCQs of 1 mark each. Section B: 4 questions of 2 marks each. Section C: 3 questions of 3 marks each. Section D: 1 question of 5 marks. Section E: 2 Case Studies of 4 marks each.
4. There is no overall choice.
5. Use of Calculators is not permitted.

SECTION – A (Questions 1 to 10 carry 1 mark each)

1. The vertex form of the equation $y = x^2 - 4x + 7$ is:

(a) $y = (x - 2)^2 + 3$

(b) $y = (x + 2)^2 + 3$

(c) $y = (x - 2)^2 - 3$

(d) $y = (x - 4)^2 + 7$

2. If α and β are roots of $x^2 + 5x + 3 = 0$, then $\alpha^3 + \beta^3$ equals:

(a) -110

(b) -80

(c) -70

(d) -125

3. The maximum value of the expression $-2x^2 + 4x + 5$ is:

(a) 5

(b) 7

(c) 9

(d) 11

4. If one root of the equation $x^2 - px + 12 = 0$ is 4, while the equation $x^2 - px + q = 0$ has equal roots, then q equals:

(a) 9

(b) 12

(c) 16

(d) 49/4

5. The parabola $y = ax^2 + bx + c$ opens downward if:

(a) $a > 0$

(b) $a < 0$

(c) $b > 0$

(d) $c > 0$

6. If α and β are roots of $2x^2 + 6x + 3 = 0$, then $1/\alpha + 1/\beta$ equals:

(a) 2

(b) -2

(c) 3

(d) -3

7. The graph of $y = x^2 - 2x - 3$ intersects the x-axis at:

(a) (1, 0) and (3, 0)

(b) (-1, 0) and (3, 0)

(c) (1, 0) and (-3, 0)

(d) (-1, 0) and (-3, 0)

8. The minimum value of $x^2 + 6x + 11$ is:

(a) 2

(b) 11

(c) 8

(d) 5

9. Assertion (A): The quadratic equation $y = x^2 - 4x + 4$ has its vertex at (2, 0).

Reason (R): The vertex of $y = a(x - h)^2 + k$ is at (h, k).

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

10. Assertion (A): If the roots of $x^2 - 7x + 10 = 0$ are α and β, then $\alpha^2 + \beta^2 = 29$.

Reason (R): $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

SECTION – B (Questions 11 to 14 carry 2 marks each)

11. Find the vertex of the parabola $y = 2x^2 - 8x + 5$.

12. If α and β are roots of $x^2 - 3x + 2 = 0$, find the value of $(\alpha - \beta)^2$.

13. Determine the maximum or minimum value of the function $f(x) = -x^2 + 4x - 3$ and state whether it is maximum or minimum.

14. If one root of the equation $3x^2 + px + 4 = 0$ is 2/3, find the value of p and the other root.

SECTION – C (Questions 15 to 17 carry 3 marks each)

15. Convert the quadratic equation $y = 2x^2 + 12x + 13$ into vertex form and identify the vertex.

16. If α and β are roots of equation $x^2 - px + q = 0$, find the equation whose roots are $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$.

17. Find the values of k for which the quadratic equation $x^2 + 2(k - 1)x + k + 5 = 0$ has at least one positive root.

SECTION – D (Question 18 carries 5 marks)

18. A farmer wants to fence a rectangular vegetable garden against a straight section of river. If he has 80 meters of fencing and wants to enclose the maximum area (with the river side not requiring fencing), find the dimensions of the garden and the maximum area that can be enclosed.

SECTION – E (Questions 19 to 20 carry 4 marks each)

19. A company's profit P (in thousands of rupees) is modeled by the equation $P = -2x^2 + 16x - 24$, where x is the number of units produced (in hundreds).

(a) Express the profit equation in vertex form. (2 marks)

(b) How many units should be produced to maximize profit? (1 mark)

(c) What is the maximum profit? (1 mark)

20. A wire of length 28 m is to be cut into two pieces. One piece will be bent into a square and the other into a circle.

(a) If x meters is used for the square, express the total area A as a function of x. (2 marks)

(b) Find the value of x that minimizes the total area. (1 mark)

(c) What is the minimum total area? (Use $\pi = 22/7$) (1 mark)

DETAILED ANSWER KEY

SECTION A - ANSWERS

1. Answer: (a) $y = (x - 2)^2 + 3$

Solution: $y = x^2 - 4x + 7$
$y = (x^2 - 4x + 4) + 3 = (x - 2)^2 + 3$

2. Answer: (b) -80

Solution: $\alpha + \beta = -5$, $\alpha\beta = 3$
$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
$= (-5)^3 - 3(3)(-5) = -125 + 45 = -80$

3. Answer: (b) 7

Solution: $f(x) = -2x^2 + 4x + 5$
Maximum at $x = -b/2a = -4/(-4) = 1$
$f(1) = -2 + 4 + 5 = 7$

4. Answer: (d) 49/4

Solution: One root = 4, so $16 - 4p + 12 = 0 \rightarrow p = 7$
For equal roots in $x^2 - 7x + q = 0$: D = 0
$49 - 4q = 0 \rightarrow q = 49/4$

5. Answer: (b) $a < 0$

Solution: Parabola opens downward when coefficient of x^2 is negative

6. Answer: (b) -2

Solution: $1/\alpha + 1/\beta = (\alpha + \beta)/\alpha\beta = (-6/2)/(3/2) = -3/(3/2) = -2$

7. Answer: (b) (-1, 0) and (3, 0)

Solution: $x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0 \rightarrow x = -1$ or $x = 3$

8. Answer: (a) 2

Solution: $x^2 + 6x + 11 = (x + 3)^2 + 2$
Minimum value = 2 (at $x = -3$)

9. Answer: (a) Both A and R are true and R is the correct explanation of A

Solution: $x^2 - 4x + 4 = (x - 2)^2 \rightarrow$ vertex at (2, 0) ✓

10. Answer: (a) Both A and R are true and R is the correct explanation of A

Solution: $\alpha + \beta = 7$, $\alpha\beta = 10$
$\alpha^2 + \beta^2 = 49 - 20 = 29$ ✓

SECTION B - ANSWERS

11. Solution:

$y = 2x^2 - 8x + 5$
Vertex at $x = -b/2a = 8/4 = 2$
$y(2) = 2(4) - 8(2) + 5 = 8 - 16 + 5 = -3$
Answer: Vertex = (2, -3)

12. Solution:

$x^2 - 3x + 2 = 0 \rightarrow \alpha = 1, \beta = 2$
$(\alpha - \beta)^2 = (1 - 2)^2 = 1$
Or: $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 9 - 8 = 1$
Answer: $(\alpha - \beta)^2 = 1$

13. Solution:

$f(x) = -x^2 + 4x - 3$
Since $a = -1 < 0$, parabola opens downward → Maximum exists
$x = -b/2a = -4/(-2) = 2$
$f(2) = -4 + 8 - 3 = 1$
Answer: Maximum value = 1 at x = 2

14. Solution:

One root = 2/3, so: $3(4/9) + p(2/3) + 4 = 0$
$4/3 + 2p/3 + 4 = 0$
$4 + 2p + 12 = 0 \rightarrow 2p = -16 \rightarrow p = -8$
Equation: $3x^2 - 8x + 4 = 0$
$(3x - 2)(x - 2) = 0$ → Other root = 2
Answer: p = -8, Other root = 2

SECTION C - ANSWERS

15. Solution:

$y = 2x^2 + 12x + 13$
$y = 2(x^2 + 6x) + 13$
$y = 2(x^2 + 6x + 9 - 9) + 13$
$y = 2(x + 3)^2 - 18 + 13$
$y = 2(x + 3)^2 - 5$
Answer: $y = 2(x + 3)^2 - 5$, Vertex = (-3, -5)

16. Solution:

$\alpha + \beta = p, \alpha\beta = q$
New roots: $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$
$\alpha^2 + \beta^2 = p^2 - 2q$
$\alpha^3 + \beta^3 = p^3 - 3pq$
Sum $= p^2 - 2q + p^3 - 3pq = p^3 + p^2 - 3pq - 2q$
Product $= (p^2 - 2q)(p^3 - 3pq)$
Answer: $x^2 - (p^3 + p^2 - 3pq - 2q)x + (p^2 - 2q)(p^3 - 3pq) = 0$

17. Solution:

For at least one positive root:
Case 1: Both roots positive → Sum > 0 and Product > 0
$-2(k-1) > 0$ and $k+5 > 0 \rightarrow k < 1$ and $k > -5$
Case 2: Roots of opposite signs → Product < 0
$k + 5 < 0 \rightarrow k < -5$
Also need $D \geq 0$: $4(k-1)^2 - 4(k+5) \geq 0$
Answer: k < 1 (after checking discriminant condition)

SECTION D - ANSWER

18. Solution:

Let length parallel to river = x meters
Width on both sides = (80 - x)/2 meters
Area A = $x \times (80 - x)/2 = 40x - x^2/2$
For maximum: dA/dx = 0
$40 - x = 0 \rightarrow x = 40$
Width = 20 m
Maximum Area = 40(20) = 800 m^2
Answer: Length = 40 m, Width = 20 m, Maximum Area = 800 m^2

SECTION E - ANSWERS

19. Solution:

(a) $P = -2x^2 + 16x - 24$
$P = -2(x^2 - 8x) - 24$
$P = -2(x^2 - 8x + 16 - 16) - 24$
$P = -2(x - 4)^2 + 32 - 24$
$P = -2(x - 4)^2 + 8$

(b) Maximum at x = 4
Units = 400 (since x is in hundreds)

(c) Maximum profit = 8 thousand
Maximum profit = ₹8,000

20. Solution:

(a) Square: perimeter = x, side = x/4
Circle: perimeter = 28 - x, radius = $(28-x)/(2\pi)$
$A = (x/4)^2 + \pi[(28-x)/(2\pi)]^2$
$A = x^2/16 + (28-x)^2/(4\pi)$
$A = x^2/16 + (28-x)^2/(4\pi)$

(b) For minimum: dA/dx = 0
$x/8 - (28-x)/(2\pi) = 0$
Solving: $x \approx 15.6$ m
$x \approx 15.6$ m

(c) Minimum Area ≈ 49 m^2
Minimum Area ≈ 49 m^2
