UNIQUE STUDY POINT

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion- Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1,1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take $\pi = 22/7$ wherever required if not stated.
11. Use of calculators is not allowed.

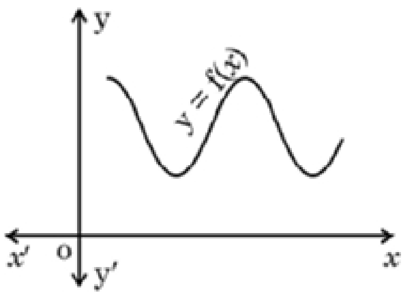
Section A

1. If a is a non-zero rational and $\sqrt{b}$ is irrational, then $a\sqrt{b}$ is: **[1]**

a) a natural number b) a rational number

c) an integer d) an irrational number

2. The graph of y = f(x) is shown in the figure for some polynomial f(x). **[1]**

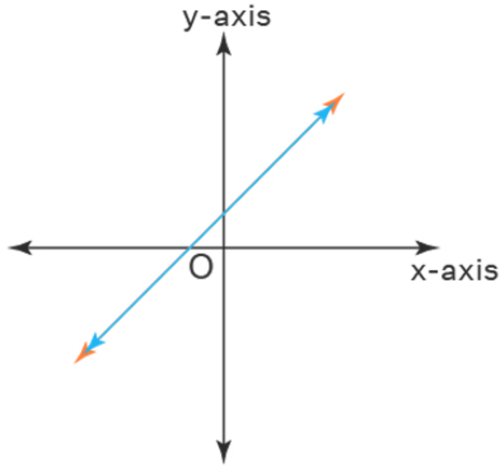


The number of zeroes of f(x) is

a) 2 b) 0

c) 3 d) 4

3. A system of linear equations is said to be consistent, if it has **[1]**



a) exactly one solution

b) one or many solutions

c) no solution

d) two solutions

4. $(x + 1)^2 - x^2 = 0$ has **[1]**

a) 1 real root

b) 4 real roots

c) 2 real roots

d) no real roots

5. The sum of first 16 terms of the AP 10, 6, 2, ... is **[1]**

a) -400

b) 320

c) -320

d) 352

6. The distance between the points A(5, -4) and B(4, -5) is **[1]**

a) $9\sqrt{2}$ units

b) 1 unit

c) 2 units

d) $\sqrt{2}$ units

7. If the line segment joining the points A (x_1, y_1) and B(x_2, y_2) is divided by a point P in the ratio 1 : k internally, then the coordinates of the point P are **[1]**

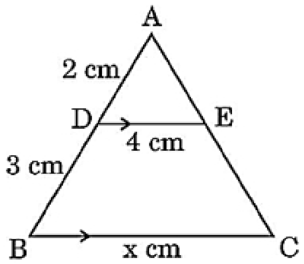
a) $\left(\frac{x_1+kx_2}{1+k}, \frac{y_1+ky_2}{1+k}\right)$

b) $\left(\frac{x_2+kx_1}{1-k}, \frac{y_2+ky_1}{1-k}\right)$

c) $\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

d) $\left(\frac{x_2-kx_1}{1+k}, \frac{y_2-ky_1}{1+k}\right)$

8. In the given figure, DE || BC. The value of x is: **[1]**



a) 8

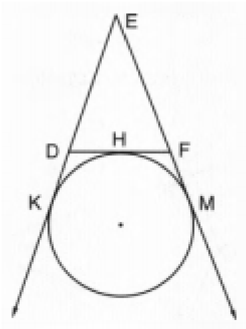
b) 6

c) 12.5

d) 10

9. In Figure, a circle touches the side DF of $\triangle$EDF at H and touches ED and EF produced at K and M **[1]**

respectively. If EK = 9 cm, then the perimeter of △EDF is



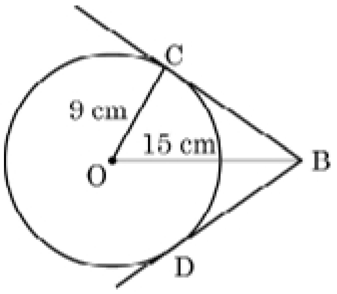
a) 12 cm

b) 13.5 cm

c) 9 cm

d) 18 cm

10. In the given figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OB = 15 cm, then the length (BC + BD) is: **[1]**



a) 36 cm

b) 24 cm

c) 18 cm

d) 12 cm

11. If x = a sec θ and y = b tan θ, then $b^2x^2 - a^2y^2 =$ **[1]**

a) $a^2 - b^2$

b) $a^2 b^2$

c) $a^2 + b^2$

d) ab

12. $\left(\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}\right)$ is equal to: **[1]**

a) sin 30°

b) cos 60^o

c) sin 60^o

d) tan 60^o

13. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower (in metres) is **[1]**

a) 75

b) $150\sqrt{3}$

c) $150\sqrt{2}$

d) $50\sqrt{3}$

14. The area of a sector whose perimeter is four times its radius r units, is **[1]**

a) $2r^2$ sq. units

b) $\frac{r^2}{2}$sq units

c) r^2 sq. units

d) $\frac{r^2}{4}$sq units

15. A pendulum swings through an angle of 30^o and describes an arc 8.8 cm in length. Find the length of the pendulum. **[1]**

a) 17 cm

b) 8.8 cm

c) 15.8 cm | d) 16.8 cm

16. Two dice are thrown together. The probability that they show different numbers is: **[1]**

a) $\frac{1}{3}$ | b) $\frac{2}{3}$
c) $\frac{1}{6}$ | d) $\frac{5}{6}$

17. The probability that it will rain on a particular day is 0.76. The probability that it will not rain on that day is **[1]**

a) 0 | b) 0.24
c) 0.76 | d) 1

18. In a data, if l = 60, h = 15, f_1 = 16, f_0 = 6, f_2 = 6, then the mode is **[1]**

a) 60 | b) 67.5
c) 62 | d) 72

19. **Assertion (A):** In a solid hemisphere of radius 10 cm, a right cone of same radius is removed out. The volume of the remaining solid is 523.33 cm^3 [Take π = 3.14 and $\sqrt{2}$ = 1.4] **[1]**
Reason (R): Expression used here to calculate volume of remaining solid = Volume of hemisphere - Volume of cone

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** Common difference of an AP in which a_{21} - a_7 = 84 is 14 **[1]**
Reason (R): nth term of AP is given by a_n = a + (n - 1)d

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

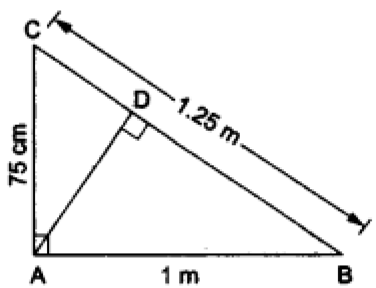
Section B

21. Given that HCF (306,1314) = 18. Find LCM (306,1314). **[2]**

OR

Show that 5 + 3$\sqrt{2}$ is an irrational number.

22. In the given figure, $\angle CAB = 90^\circ$ and $AD \perp BC$. Show that $\triangle BDA \sim \triangle BAC$. If AC = 75 cm, AB = 1 m and BC = 1.25m, Find AD. **[2]**



23. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral. **[2]**

24. If sin A = $\frac{\sqrt{3}}{2}$, find the value of $2\cot^2 A$ -1. **[2]**

OR

Prove that: $(\sqrt{3}+1)$ (3 – cot 30°) = $\tan^3$ 60° – 2 tan 60°

25. In a circle of radius 10.5 cm, the minor arc is one-fifth of the major arc. Find the area of the sector corresponding to the major arc. **[2]**

Section C

26. 105 goats, 140 donkeys and 175 cows have to be taken across a river. There is only one boat which will have to make many trips in order to do so. The lazy boatman has his own conditions for transporting them. He insists that he will take the same number of animals in every trip and they have to be of the same kind. He will naturally like to take the largest possible number each time. Can you tell how many animals went in each trip? **[3]**

27. If one root of the quadratic polynomial $2x^2 - 3x + p$ is 3, find the other root. Also, find the value of p. **[3]**

28. Five coins were simultaneously tossed 1000 times and at each toss the number of heads were observed. The number of tosses during which 0,1,2,3,4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss. **[3]**

No. of heads per toss	**No. of tosses**
0	38
1	144
2	342
3	287
4	164
5	25
Total	**1000**

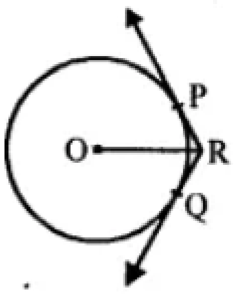
29. The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and breadth is increased by 5 metres. Find the dimensions of the rectangle. **[3]**

OR

If 2 is added to the numerator of a fraction, it reduces to $\frac{1}{2}$ and if 1 is subtracted from the denominator, it reduces to $\frac{1}{3}$. Find the fraction.

30. In an acute angled triangle ABC, if tan (A+B-C) = 1 and, sec (B+C-A)=2, find the values of A, B and C. **[3]**

31. In the given figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle$PRQ = 120°, then prove that OR = PR + RQ. **[3]**



OR

A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Section D

32. Calculate the mode of the following frequency distribution table : **[5]**

Marks	Number of students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

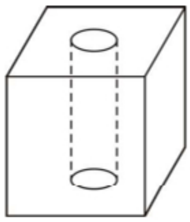
33. PQRS is a trapezium with PQ $\|$ SR Diagonals PR and SQ intersect at M and $\Delta PMS \sim \Delta QMR$. **[5]**
Prove that PS = QR.

34. A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day. Represent the situations mathematically (quadratic equation). **[5]**

OR

In a flight of 600 km, the speed of the aircraft was slowed down due to bad weather. The average speed of the trip was decreased by 200 km/hr and thus the time of flight increased by 30 minutes. Find the average speed of the aircraft originally.

35. In Figure, from a solid cube of side 7 cm, a cylinder of radius 2.1 cm and height 7 cm is scooped out. Find the total surface area of the remaining solid. **[5]**



OR

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**
Akshat's father is planning some construction work in his terrace area. He ordered 360 bricks and instructed the supplier to keep the bricks in such as way that the bottom row has 30 bricks and next is one less than that and so on.

The supplier stacked these 360 bricks in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on.

i. In how many rows, 360 bricks are placed? (1)

ii. How many bricks are there in the top row? (1)

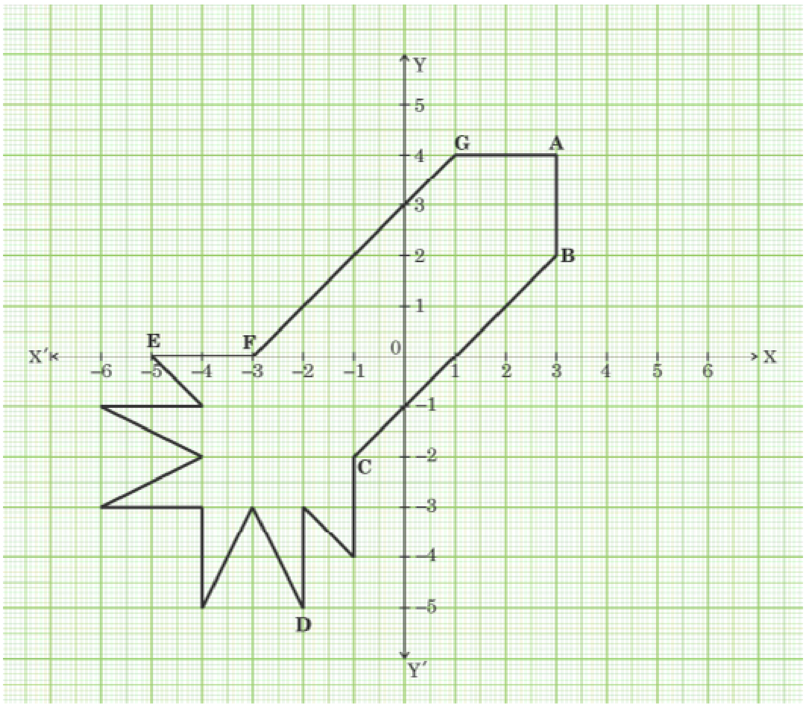
iii. How many bricks are there in 10^{th} row? (2)

OR

If which row 26 bricks are there? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Ryan, from a very young age, was fascinated by the twinkling of stars and the vastness of space. He always dreamt of becoming an astronaut one day. So he started to sketch his own rocket designs on the graph sheet. One such design is given below:



Based on the above, answer the following questions:

i. Find the mid-point of the segment joining F and G. **(1)**

ii. a. What is the distance between the points A and C? **(2)**

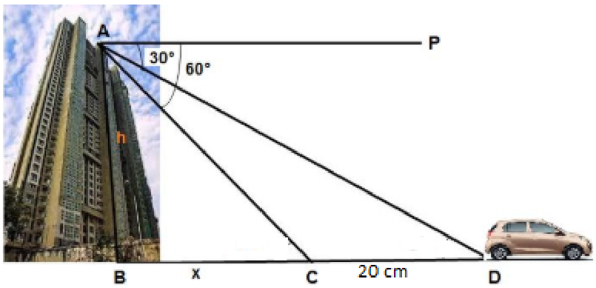
OR

b. Find the coordinates of the point which divides the line segment joining the points A and B in the ratio 1 : 3 internally. **(2)**

iii. What are the coordinates of the point D? **(1)**

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^{o}. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^o.



i. Find the value of x. (1)

ii. Find the height of the building AB. (1)

iii. Find the distance between top of the building and a car at position D? (2)

OR

Find the distance between top of the building and a car at position C? (2)

Solution
UNIQUE STUDY POINT
Class 10 - Mathematics
Section A

1.
(d) an irrational number
Explanation:
If possible let $a\sqrt{b}$ be rational.
Then $a\sqrt{b} = \frac{p}{q}$, where p and q are non-zero integers, having no common factor other than 1.
Now, $a\sqrt{b} = \frac{p}{q}$
$\Rightarrow \sqrt{b} = \frac{p}{aq}$ ... (i)
But, p and aq are both rational and aq $\neq$ 0
$\because \frac{p}{aq}$ is rational.
Therefore, from eq. (i), it follows that $\sqrt{b}$ is rational.
The contradiction arises by assuming that $a\sqrt{b}$ is rational.
Hence, $a\sqrt{b}$ is irrational.

2.
(b) 0
Explanation:
Here y = f(x) is not intersecting or touching the X-axis.
$\therefore$ Number of zeroes of f(x) = 0

3.
(b) one or many solutions
Explanation:
A system of linear equations is said to be consistent if it has at least one solution or can have many solutions. If a consistent system has an infinite number of solutions, it is dependent. When you graph the equations, both equations represent the same line. If a system has no solution, it is said to be inconsistent. The graphs of the lines do not intersect, so the graphs are parallel and there is no solution.

4. **(a)** 1 real root
Explanation:
Given: $(x + 1)^2 - x^2 = 0$
$\Rightarrow x^2 + 1 + 2x - x^2 = 0$
$\Rightarrow 2x + 1 = 0$
$\Rightarrow x = \frac{-1}{2}$
Therefore, $(x + 1)^2 - x^2 = 0$ is a linear polynomial and has one real root.

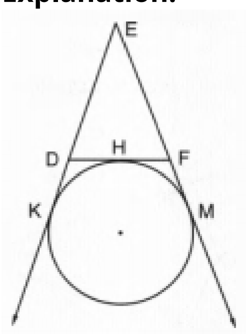
5.
(c) -320
Explanation:
$l = T_{16} = 10 + 15 \times (-4) = -50$.
$\therefore$ sum = $\frac{n}{2}(a+l)$ = $\frac{16}{2}$ (10 - 50) = 8 $\times$ (-40) = -320

6.
(d) $\sqrt{2}$ units
Explanation:

A (5, -4) B (4, -5)
AB = $\sqrt{(5-4)^2+(-4+5)^2}$
= $\sqrt{1^2+1^2}$
AB = $\sqrt{2}$ units

7.
(c) $\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$
Explanation:
Let coordinates of P be (x, y) which divides the line joining A(x_1, y_1) and B(x_2, y_2) in the ratio 1 : k
$m_1 : m_2 = 1 : k$
$\therefore$ x = $\frac{m_1x_2+m_2x_1}{m_1+m_2}$
= $\frac{1\times x_2+k\times x_1}{1+k}$ = $\frac{x_2+kx_1}{1+k}$
And y = $\frac{m_1y_2+m_2y_1}{m_1+m_2}$
= $\frac{1\times y_2+k\times y_1}{1+k}$
= $\frac{y_2+ky_1}{1+k}$
$\therefore$ P$\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

8.
(d) 10
Explanation:
In $\triangle$ADE and $\triangle$ABC
$\angle$D = $\angle$B {Corresponding angle}
$\angle$E = $\angle$C {Corresponding angle}
$\therefore$ $\triangle$ADE and $\triangle$ABC (by A A Similarity)
$\frac{AD}{AB}$ = $\frac{DE}{BC}$
$\frac{2}{5}$ = $\frac{4}{X}$
X = $\frac{5\times4}{2}$ = 10
= 10 cm

9.
(d) 18 cm
Explanation:


In $\triangle \mathrm{DEF}$
DF touches the circle at H
and circle touches ED and EF Produced at K and M respectively
EK = 9 cm
EK and EM are the tangents to the circle
EM = EK = 9 cm
Similarly DH and DK are the tangent

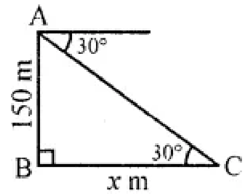
DH = DK and FH and FM are tangents
FH = FM
Now, perimeter of $\Delta\mathrm{DEF}$
= ED + DF + EF
= ED + DH + FH + EF
= ED + DK + FM + EF
= EK + EM
= 9 + 9
= 18 cm

10.
(b) 24 cm
Explanation:
We know that, a tangent to a circle is perpendicular to the radius at the point of contact.
So, $\triangle$OCB is right a triangle, right angled at C.
Hence, by Pythagoras' theorem, we have:
$BC^2 = OB^2 - OC^2$
$\Rightarrow BC^2 = 225 - 81 = 144$
BC = 12 cm
We also know that, the tangents drawn from the same external point to a circle are equal.
Since BC and BD are tangents drawn from the same external point, B, we have:
BC = BD = 12 cm.
So, BC + BD = 24 cm.
Hence, BC + BD = 24 cm.

11.
(b) $a^2 b^2$
Explanation:
Here we have x = a sec θ and y = b tan θ
Therefore, $b^2x^2 - a^2y^2 = b^2 (a \sec\theta)^2 - a^2 (b \tan\theta)^2$
$= a^2b^2 \sec^2\theta - a^2b^2 \tan^2\theta$
$= a^2b^2 (\sec^2\theta - \tan^2\theta)$
$= a^2b^2 \times 1$
$= a^2b^2$

12.
(c) sin 60°
Explanation:
$\frac{2tan30^o}{1+tan^2 30^o} = \frac{2\times\frac{1}{\sqrt{3}}}{1+(\frac{1}{\sqrt{3}})^2}$
$\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$
= sin60°

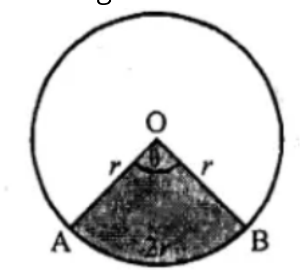
13.
(b) $150\sqrt{3}$
Explanation:
Let AB be the tower of height 150 m
C is car and angle of depression is 30°



Therefore, $\angle$ACB = 30^{o} (alternate angle)
In right-angled triangle ABC,
$\frac{BC}{AB}$ = cot 30^{o}
$\Rightarrow \frac{BC}{150} = \sqrt{3} \Rightarrow$ BC = 150 $\sqrt{3}$ m
That is, the distance of the car from the tower is 150$\sqrt{3}$ m.

14.

(c) r^2 sq. units
Explanation:
Radius of sector = r
Perimeter = 4r
and length of arc = 4r - 2r = 2r



$\therefore$ Let angle at the centre = θ
Then, $2\pi r = \frac{\theta}{360^\circ}$ and $2r = \frac{\theta}{360^\circ}$
$\Rightarrow \pi \times \frac{\theta}{360^\circ}$ = 1 ...(i)
Now area = $\pi r^2 \times \frac{\theta}{360^\circ} = r^2\left(\pi \times \frac{\theta}{360^\circ}\right)$
= $r^2 \times 1$ [from (i)]
= r^2

15.

(d) 16.8 cm
Explanation:
Length of the pendulum = Radius of a sector of the circle
Arc length = 8.8
{tex}\frac{θ}{360} (2πr){/tex} = 8.8
$\frac{30}{360} \times 2 \times \frac{22}{7} \times r$ = 8.8
r = 16.8 cm

16.

(d) $\frac{5}{6}$
Explanation:
E = dice show different no. favourable case for same no.
= (1, 1) (2, 2) (3, 3) (4, 4) (5, 5) (6, 6)
= 6
P(same no.) $= \frac{6}{36} = \frac{1}{6}$
p(not same no.) $= 1 - \frac{1}{6}$
$= \frac{5}{6}$

17.

(b) 0.24

Explanation:

Given: P (It will rain on a particular day) = 0.76

$\therefore$ P (It will not rain on a particular day) = 1 - P (It will rain particular day)

= 1 - 0.76 = 0.24

18.

(b) 67.5

Explanation:

Mode = $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

= 60 + $\frac{16-6}{2\times16-6-6}$ $\times$ 15

= 60 + $\frac{10}{32-12}$ $\times$ 15

= 60 + $\frac{10}{20}$ $\times$ 15

= 60 + 7.5

= 67.5

19.

(d) A is false but R is true.

Explanation:

A is false but R is true.

20.

(d) A is false but R is true.

Explanation:

We have,

a_n = a + (n - 1)d

a_{21} - a_7 = {a + (21 - 1)d} - {a + (7 - 1)d} = 84

a + 20d - a - 6d = 84

14d = 84

d = $\frac{18}{14}$ = 6

d = 6

So, A is false but R is true.

Section B

21. $HCF(306,1314)=18$

$LCM(306,1314)=?$

Let, a = 306

b = 1314

LCM (a, b) $\times$ HCF (a,b) = a $\times$b

or, LCM (a, b) $\times$ 18 = 306 $\times 1314$

or LCM(a,b) = $\frac{306\times1314}{18}$ =22338

Therefore, $LCM(306,1314)=22338$

OR

Let us assume that 5 + 3$\sqrt{2}$, is a rational number.

Then there exist co primes a and b such that

5 + 3$\sqrt{2}$=$\frac{a}{b}$

3$\sqrt{2}$=$\frac{a}{b}$-5

$= \frac{a-5b}{b}$

So $\sqrt{2} = \frac{a-5b}{3b}$ -------(i)

$\frac{a-5b}{3b}$ is rational so this shows that $\sqrt{2}$ is rational

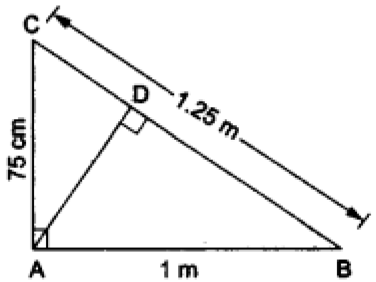
But $\sqrt{2}$ is irrational.

∴(i) presents a contradiction.,

Hence 5 + 3$\sqrt{2}$ is an irrational number.

22. Given, $\angle CAB = 90^\circ$ and $AD \perp BC$.

Also given, AC = 75 cm = 0.75 m , AB = 1 m and BC = 1.25 m.



In $\triangle \mathrm{BDA}$ and $\triangle \mathrm{BAC}$, we have:

$\angle BDA = \angle BAC = 90^0$

$\angle DBA = \angle CBA$ (common)

$\therefore \triangle \mathrm{BDA} \sim \triangle \mathrm{BAC}$ [By AA similarity theorem]

$\Rightarrow \frac{AD}{AC} = \frac{AB}{BC}$ [By proportionality theorem]

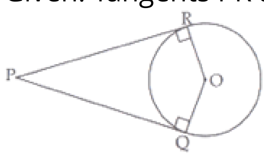
$\Rightarrow \frac{AD}{0.75} = \frac{1}{1.25}$

$\Rightarrow \mathrm{AD} = \frac{0.75}{1.25}$

= 0.6 m or 60 cm

∴AD = 60 cm.

23. Given: Tangents PR and PQ from an external point P to a circle with centre O.



To prove: Quadrilateral QORP is cyclic.

Proof: RO and RP are the radius and tangent respectively at contact point R.

Therefore, ∠PRO = 90°

Similarly ∠PQO = 90°

In quadrilateral QOPR, we have

∠P + ∠R + ∠O + ∠Q = 360°

⇒ ∠P + ∠90° + ∠O + ∠90° = 360°

⇒ ∠P + ∠O = 360° - 180° = 180°

These are opposite angles of quadrilateral QORP and are supplementary.

Therefore, Quadrilateral QORP is cyclic. hence, proved.

24. $2cot^2 A - 1 = 2(cosec^2 A - 1) - 1 \left(\because \cot^2\theta = -1 + \text{cosec}^2\theta\right)$

$= 2cosec^2 A - 2 - 1$

$= \frac{2}{\sin^2 A} - 3 \ (\because cosec\theta = \frac{1}{sin\theta})$

$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3$

$2cot^2 A - 1 = \frac{8}{3} - 3 = \frac{8-9}{3} = \frac{-1}{3}$

OR

We have to prove that: $(\sqrt{3}+1)$ (3 – cot 30°) = tan^3 60° – 2 tan 60°

Here, LHS = $(\sqrt{3}+1)$ (3 – cot 30°)

$= (\sqrt{3}+1)(3-\sqrt{3})$
$= \sqrt{3}(3-\sqrt{3})+1(3-\sqrt{3})$
$= 3\sqrt{3}-3+3-\sqrt{3}$
$= 2\sqrt{3}$

RHS = $\tan^3 60° - 2\sin 60°$

$= (\sqrt{3})^3 - 2\times\frac{\sqrt{3}}{2}$
$= 3\sqrt{3}-\sqrt{3}$
$= 2\sqrt{3}$

⇒ LHS = RHS

Hence, proved.

25. Let the major arc be x cm long

Then, length of the minor arc $= \frac{1}{5}x\ cm$

Circumference$= \left(x+\frac{1}{5}x\right)\mathrm{cm}$

$= \frac{6x}{5}\mathrm{cm}$

$\frac{6x}{5} = 2\times\frac{22}{7}\times\frac{21}{2}$

$\Rightarrow x = 55\mathrm{cm}$

Required area $= \left(\frac{1}{2}\times 55\times\frac{21}{2}\right)\mathrm{cm}^2 \quad [Area = \frac{1}{2}r-1]$

= 288.75 cm^2

Section C

26. **Given:** Number of goats for trip = 105

Number of donkey for trip = 140

Number of cows for trip = 175

Therefore, The largest number of animals in one trip = HCF of 105. 140 and 175.

First consider 105 and 140

By applying Euclid’s division lemma, we get

140 = 105 $\times$ 1 + 35

105 = 35 $\times$ 3 + 0

Therefore, HCF of 105 and 140 = 35

Now consider 35 and 175

Again applying Euclid’s division lemma, we get

175 = 35 $\times$ 5 +0

HCF of 105, 140 and 175 is 35.

So 35 animals of same kind can go for trip in a single trip and number of trip is 105/35 +140/35 + 175/35 = 12

27. The given quadratic polynomial is p(x) = $2x^2$ - 3x + p

Since, 3 is a root (zero) of p(x)

$\Rightarrow$ $2(3)^2$ - 3 $\times$ 3 + p = 0

$\Rightarrow$18 - 9 + p = 0

$\Rightarrow$9 + p = 0

$\Rightarrow$ p = - 9

Now p(x) = $2x^2$ - 3x - 9

= $2x^2$- 6x + 3x - 9

= 2x (x - 3) + 3 (x - 3)

= (x - 3) (2x + 3)

For roots of polynomial, p(x) = 0

$\Rightarrow$(x - 3) (2x + 3) = 0

$\Rightarrow$ x = 3 or x = -$\frac{3}{2}$

Hence the other root is -$\frac{3}{2}$.

No of heads per toss	**No of tosses**	

28.

		$f_i x_i$
0	38	0
1	144	144
2	342	684
3	287	861
4	164	656
5	25	125
	$\sum f_i$ = 1000	$\sum f_i x_i$ = 2470

Mean number of heads per toss = $\frac{\sum f_i x_i}{\sum f_i} = \frac{2470}{1000} = 2.47$

Therefore, Mean = 2.47

29. Let us suppose that the length and breadth of the rectangle be x m and y m respectively.

Then, Area of rectangle = xy meter^2

Now, according to question if length is increased by 7m and the breadth is decreased by 3m, the area remains same

$\therefore$ xy = (x + 7)(y - 3)

$\Rightarrow$ xy = xy - 3x + 7y - 21

$\Rightarrow$ 3x - 7y = -21 ....(i)

Again, according to question when length is decreased by 7m and breadth is increased by 5m, then area remains unaffected

$\therefore$ xy = (x - 7)(y + 5)

$\Rightarrow$ xy = xy + 5x - 7y - 35

$\Rightarrow$ 35 = 5x - 7y

$\Rightarrow$ 5x - 7y = 35 ....(ii)

Subtracting equation (i) from (ii), we get

5x - 7y -(3x- 7y) = 35 - (-21)

or, 5x - 7y -3- + 7y = 35 + 21

$\Rightarrow$ 2x = 56

$\Rightarrow x = \frac{56}{2} = 28$

Put the value of x = 28 in equation (ii), we get

5 $\times$ 28 - 7y = 35

$\Rightarrow$ 140 - 7y = 35

$\Rightarrow$ -7y = 35 - 140

$\Rightarrow$ -7y = -105

$\Rightarrow y = \frac{105}{7} = 15$

Therefore, dimensions of the rectangle are 28m and 15m respectively.

OR

Let the fraction be $\frac{x}{y}$

According to question,

$\frac{x+2}{y} = \frac{1}{2}$

or 2x - y = -4 ....(i)

and $\frac{x}{y-1} = \frac{1}{3}$

or 3x - y = -1 ...(ii)

on solving eq (i) and (ii), we get,

x = 3, y = 10

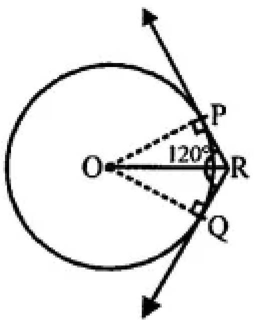
$\therefore$ fraction is $\frac{3}{10}$

30. According to the question,

tan (A+B-C) = 1

$\Rightarrow$ tan (A+B-C) = $\tan 45^\circ$
$\Rightarrow$ A + B - C = 45° ....(1)
Also given, sec (B+C-A) = 2
$\Rightarrow$ sec (B + C - A) = sec 60°
$\therefore B + C - A = 60^\circ$ ....(2)
Adding equation (1) & (2);
$(A+B-C)+(B+C-A)=45^\circ+60^\circ$
$\Rightarrow \quad 2B=105^\circ$
$\Rightarrow \quad B=52\frac{1}{2}^\circ$
Putting $B=52\frac{1}{2}^\circ$ in equation (2); we get :-
$52\frac{1}{2}^\circ+C-A=60^\circ$
$\Rightarrow C-A=7\frac{1}{2}^\circ$ ......(3)
Also, in ΔABC, we have
$A+B+C=180^\circ$
$\Rightarrow \quad A+52\frac{1}{2}^\circ+C=180^\circ \quad \left[\because B=52\frac{1}{2}^\circ\right]$
$\Rightarrow \quad C+A=127\frac{1}{2}^\circ$ ......(4)
Adding and subtracting (3) and (4), we get
2C = 135° and 2A = 120°
$\Rightarrow C=67\frac{1}{2}^\circ$ and A = 60°
Hence, we get the values of A = 60°,
$B=52\frac{1}{2}^\circ$
and $C=67\frac{1}{2}^\circ$.

31. In the given figure, two tangents RQ and RP are drawn from the external point R to the circle with centre O.
∠PRQ = 120°
To prove: OR = PR + RQ
Construction: Join OP and OQ.
Also join OR.

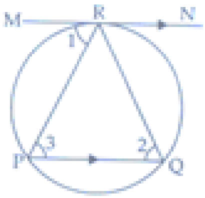


Proof: OR bisects the ∠PRQ
$\therefore \angle \mathrm{PRO}=\angle \mathrm{QRO}=\frac{120^\circ}{2}=60^\circ$
∵ OP and OQ are radii and RP and RQ are tangents.
∴ OP ⊥ PR and $\mathrm{OQ} \perp \mathrm{QR}$
In right △ OPR
$\angle \mathrm{POR}=180^\circ-(90^\circ+60^\circ)$
$=180^\circ-150^\circ=30^\circ$
Similarly,
$\angle \mathrm{QOR}=30^\circ$
and $\cos\theta=\frac{\mathrm{PR}}{\mathrm{OR}}$
$\Rightarrow \cos 60^\circ=\frac{\mathrm{PR}}{\mathrm{OR}} \Rightarrow \frac{1}{2}=\frac{\mathrm{PR}}{\mathrm{OR}}$
$\Rightarrow$ 2PR=OR ......(i)

Similarly, in right △OQR
⇒ 2QR=OR ....(ii)
Adding (i) and (ii)
⇒ 2PR + 2QR = 2OR
⇒ OR = PR + RQ
Hence Proved.

OR

Given: In a circle a chord PQ and a tangent MRN at R such that QP || MRN



To prove: R bisects the arc PRQ.
Construction**:** Join RP and RQ.
Proof: Chord RP subtends ∠1 with tangent MN and ∠2 in alternates segment of circle so ∠1 = ∠2.
MRN || PQ
∴ ∠1 = ∠3 [Alternate interior angles]
⇒ ∠2 = ∠3
⇒ PR = RQ [Sides opp. to equal ∠s in Δ RPQ]
∵ Equal chords subtend equal arcs in a circle so
arc PR = arc RQ
or R bisect the arc PRQ. Hence, proved.

Section D

32. The given data are:

Marks	**Number of students**
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

From above data we can calculate range data as following:

Marks	**Number of students(f)**
25 - 35	52 - 47 = 5
35 - 45	47 - 37 = 10
45 - 55	37 - 17 = 20
55 - 65	17 - 8 = 9
65 - 75	8 - 2 = 6
75 - 85	2 - 0 = 2
85 - 95	0

From table it is clear that maximum class frequency is 20 belonging to class interval 45 - 55
Modal class = 45 - 55
Lower limit (l) of modal class = 45
Class size (h) = 10

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 10

Frequency (f_2) of class succeeding the modal class = 9

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 45 + \left(\frac{20-10}{2\times20-10-9}\right) \times 10$

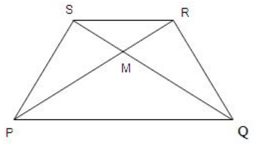
$= 45 + \frac{10}{21} \times 10$

= 45 + 4.76

= 49.76

Therefore mode of data is 49.76

33. Given : $\Delta PMS \sim \Delta QMR$ and $PQ||SR$.



To show PS = QR

$\because \triangle PMS \sim \triangle QMR$

$\therefore \frac{PS}{QR} = \frac{PM}{QM} = \frac{MS}{MR}$ ...(i)

[corresponding sides of similar triangles are proportional]

Now, consider $\triangle PMQ$ and $\triangle RMS$

In these triangles, we have

$\angle PMQ = \angle RMS$ [vertically opposite angles]

$\angle MPQ = \angle MRS$ [alternate angles]

$\because \triangle PMQ \sim \triangle RMS$ [AA criteria]

$\therefore \frac{PM}{RM} = \frac{MQ}{MS}$

[corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{PM}{QM} = \frac{MR}{MS}$ ...(ii)

From Eq (i) and Eq (ii), we get

$\Rightarrow \frac{MS}{MR} = \frac{MR}{MS}$

$\Rightarrow MS^2 = MR^2$

$\Rightarrow MS = MR$

From Eq(i) , we get

$\therefore \frac{PS}{QR} = \frac{MS}{MR}$

$\frac{PS}{QR} = 1$

$\Rightarrow PS = QR$ Hence proved.

34. Let the number of toys produced be *x*.

$\therefore$ Cost of production of each toy = Rs (55 − *x*)

It is given that, total production of the toys = Rs 750

$\therefore$ x(55 – x) = 750

⇒ x^2 – 55x + 750 = 0

Now to factorize this equation we have to find two numbers such that their product is 750 and sum is 55

⇒ x^2 – 25x – 30x + 750 = 0

⇒ x(x – 25) – 30(x – 25) = 0

⇒ (x – 25)(x – 30) = 0

Either x – 25 = 0 or *x* − 30 = 0

i.e., *x* = 25 or *x* = 30

Hence, the number of toys will be either 25 or 30.

OR

Let average speed of aircraft be x km/h

$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$

x^2 - 200x – 240000 = 0

(x - 600) (x + 400) = 0

x = 600 km/h

$\therefore$ Original speed = 600 km/h

35. We have;

A Cube,

Cube's $\frac{length}{Edge}$, a = 7 cm

A Cylinder:

Cylinder's Radius, r = 2.1 cm or r = $\frac{21}{10}$cm

Cylinder's Height, h = 7 cm

$\because$ A cylinder is scooped out from a cube,

$\therefore$ TSA of the resulting cuboid:

= TSA of whole Cube - 2 $\times$ (Area of upper circle or Area of lower circle) + CSA of the scooped out Cylinder

= $6a^2 + 2\pi rh - 2 \times (\pi r^2)$

= $6 \times (7)^2 + 2 \times (22 \div 7 \times 2.1 \times 7) - 2 \times [22 \div 7 \times (2.1)^2]$

= $6 \times 49 + (44 \div 7 \times 14.7) - (44 \div 7 \times 4.41)$

= 294 + 92.4 - 27.72

= 294 + 64.68

= 358.68 cm^2

Hence, the total surface area of the remaining solid is 358.68 cm^2

OR

According to the question,a hemispherical depression is cut from one face of the cubical block such that the diameter l of the hemisphere is equal to the edge of the cube.

Let the radius of hemisphere = r

$\therefore r = \frac{l}{2}$

Now, the required surface area = Surface area of cubical block - Area of base of hemisphere + Curved surface area of hemisphere.

$= 6(\text{ side })^2 - \pi r^2 + 2\pi r^2$

$= 6l^2 - \pi\left(\frac{l}{2}\right)^2 + 2\pi\left(\frac{l}{2}\right)^2$

$= 6l^2 - \frac{\pi l^2}{4} + \frac{\pi}{2}l^2$

$= 6l^2 + \frac{\pi l^2}{4}$

Surface area $= \frac{1}{4}(24+\pi)l^2$ units.

$= \frac{1}{4}\left(24 + \frac{22}{7}\right)l^2$

Section E

36. i. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

i.e. $S_n = 360$

$\Rightarrow \frac{n}{2}[2 \times 30 + (n-1)(-1)] = 360$ $\{S_n = \frac{n}{2}(2a + (n-1)d)$ }

$\Rightarrow 720 = n(60 - n + 1)$

$\Rightarrow$ 720 = 60n - $n^2 + n$

$\Rightarrow n^2 - 61n + 720 = 0$

$\Rightarrow n^2 - 16n - 45n + 720 = 0$ [by factorization]

$\Rightarrow n(n-16) - 45(n-16) = 0$

$\Rightarrow (n-16)(n-45) = 0$

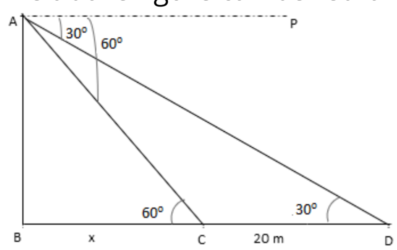
$\Rightarrow (n-16)=0$ or $(n-45)=0$
$\Rightarrow n=16$ or $n=45$
Hence, number of rows is either 45 or 16.
n = 45 not possible so n = 16
$a_{45}=30+(45-1)(-1)$ $\{a_n=a+(n-1)d\}$
$=30-44=-14$ [$\because$ The number of logs cannot be negative]
Hence the number of rows is 16.

ii. Number of bricks in the bottom row = 30. in the next row = 29, and so on.
Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1
Suppose number of rows is n, then sum of number of bricks in n rows should be 360.
Number of bricks on top row are $n=16$,
a_{16} = 30 + (16 - 1) (-1) $\{a_n = a + (n - 1)d\}$
= 30 - 15 = 15
Hence, and number of bricks in the top row is 15.

iii. Number of bricks in the bottom row = 30. in the next row = 29, and so on.
therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.
Suppose number of rows is n, then sum of number of bricks in n rows should be 360
Number of bricks in 10th row a = 30, d = -1, n = 10
$a_n = a + (n - 1)d$
$\Rightarrow a_{10} = 30 + 9 \times -1$
$\Rightarrow a_{10} = 30 - 9 = 21$
Therefore, number of bricks in 10th row are 21.
OR
Number of bricks in the bottom row = 30. in the next row = 29, and so on.
Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.
Suppose number of rows is n, then sum of number of bricks in n rows should be 360.
a_n = 26, a = 30, d = -1
$a_n = a + (n - 1)d$
$\Rightarrow 26 = 30 + (n - 1) \times -1$
$\Rightarrow 26 - 30 = -n + 1$
$\Rightarrow n = 5$
Hence 26 bricks are in 5th row.

37. i. Mid point of FG is $\left(\frac{-3+1}{2}, \frac{0+4}{2}\right)$ = (-1, 2)
ii. a. $\mathrm{AC}=\sqrt{(-1-3)^2+(-2-4)^2}$
$=\sqrt{52}$ or $2\sqrt{13}$
OR
b. The coordinates of required point are $\left(\frac{1\times3+3\times3}{1+3}, \frac{1\times2+3\times4}{1+3}\right)$ i.e. $\left(3, \frac{7}{2}\right)$
iii. D(-2, -5)

38. i. The above figure can be redrawn as shown below:



From the figure,

let AB = h and BC = x

In $\triangle$ABC,

tan 60 = $\frac{\mathrm{AB}}{\mathrm{BC}} = \frac{\mathrm{h}}{x}$

$\sqrt{3}$ = $\frac{\mathrm{h}}{x}$

h = $\sqrt{3}$x ...(i)

In $\triangle$ABD,

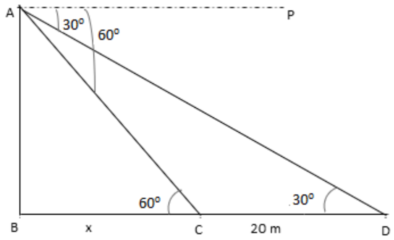
tan 30 = $\frac{\mathrm{AB}}{\mathrm{BD}} = \frac{\mathrm{h}}{x+20}$

$\frac{1}{\sqrt{3}}$ = $\frac{\sqrt{3}x}{x+20}$ [using (i)]

x + 20 = 3x

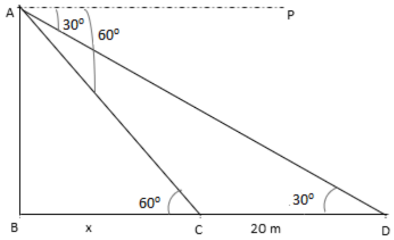
x = 10 m

ii. The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = 10$\sqrt{3}$ = 17.32 m

iii. The above figure can be redrawn as shown below:



Distance from top of the building to point D.

In $\triangle$ABD

sin 30° = $\frac{AB}{AD}$

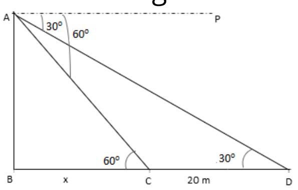
$\Rightarrow AD = \frac{AB}{\sin 30^0}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}m$

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point C is

In $\triangle$ABC

sin 60° = $\frac{AB}{AC}$

$\Rightarrow AC = \frac{AB}{\sin 60^0}$

$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AD = 20 m