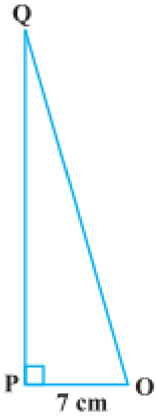
TRIGONOMETRY

Class 10 - Mathematics

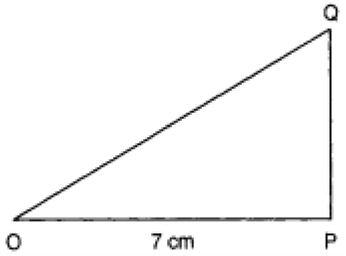
Time Allowed: 59 minutes **Maximum Marks: 25**

1. In $\triangle OPQ$ right angled at P, OP = 7 cm, OQ - PQ = 1 cm. Determine the values of sin Q and cos Q. **[3]**



2. Prove that: $\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{cosecA-1}{cosecA+1}$ **[2]**
3. Evaluate: $\frac{5\cos^2 60^\circ+4\sec^2 30^\circ-\tan^2 45^\circ}{\sin^2 30^\circ+\cos^2 30^\circ}$ **[2]**
4. Prove $\sqrt{\frac{1+\sin A}{1-\sin A}}$ = sec A + tan A, where the angles involved are acute angles for which the expressions are defined. **[2]**
5. Prove $\frac{\sin\theta-2\sin^3\theta}{2\cos^2\theta-\cos\theta}=\tan\theta$, where the angles involved are acute angles for which the expressions are defined. **[2]**
6. Prove (cosec A - sin A) (sec A - cos A) = $\frac{1}{\tan A+\cot A}$, where the angles involved are acute angles for which the expressions are defined. **[2]**
7. Prove $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$, where the angles involved are acute angles for which the expressions are defined. **[2]**
8. Prove $(cosec\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$, where the angles involved are acute angles for which the expressions are defined. **[2]**
9. Prove $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$ = cosec A + cot A, using the identity cosec^2 A = 1 + cot^2 A. where the angles involved are acute angles for which the expressions are defined. **[2]**
10. Prove $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$ = 2 sec A, where the angles involved are acute angles for which the expressions are defined. **[2]**
11. Prove (sin A + cosec A)^2 + (cos A + sec A)^2 = 7 + tan^{2}A + cot^{2}A, where the angles involved are acute angles for which the expressions are defined. **[2]**
12. Prove $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$ = 1 + sec A cos ecA, where the angles involved are acute angles for which the expressions are defined. **[2]**

Solution
TRIGONOMETRY
Class 10 - Mathematics

1. 

In $\triangle OPQ$, by Pythagoras theorem

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow (PQ+1)^2 = OP^2 + PQ^2\ [\because OQ - PQ = 1 \Rightarrow OQ = 1 + PQ]$

$\Rightarrow PQ^2 + 2PQ + 1 = 7^2 + PQ^2$

$\Rightarrow 2PQ + 1 = 49$

$\Rightarrow 2PQ = 48$

$\Rightarrow PQ = 24 \text{ cm}$

$\therefore OQ - PQ = 1\text{cm} \Rightarrow$ OQ - 24 = 1 $\Rightarrow$ OQ = 25 cm

Now, $\sin Q = \frac{OP}{OQ} = \frac{7}{25}$

and, $\cos Q = \frac{PQ}{OQ} = \frac{24}{25}$

2. LHS = $\frac{\cot A - \cos A}{\cot A + \cos A}$

$= \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$

$= \frac{\frac{\cos A - \sin A \cos A}{\sin A}}{\frac{\cos A + \sin A \cos A}{\sin A}}$

$= \frac{\cos A(1 - \sin A)}{\cos A(1 + \sin A)}$

$= \frac{1 - \sin A}{1 + \sin A}$

$= \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$

$= \frac{cosec A - 1}{cosec A + 1}$ = RHS

3. Given: $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$

$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1+3}{4}}$

$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}}$

$= \frac{\frac{67}{12}}{1}$

$= \frac{67}{12}$

4. L.H.S. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$

$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} \left[\because (a+b)(a-b) = a^2 - b^2\right]$

$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \left[\because 1 - \sin^2\theta = \cos^2\theta\right]$

$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A = R \cdot H. S.$

5. LHS

$= \frac{\sin\theta - 2\sin^3\theta}{2\cos^2\theta - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(\cos^2\theta + \sin^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \cos^2\theta - \sin^2\theta)} \quad \because \cos^2\theta + \sin^2\theta = 1$

$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)} = \tan\theta$

= RHS

6. LHS

$= (cos A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right) = \frac{1-\sin^2 A}{\sin A}\frac{1-\cos^2 A}{\cos A}$

$= \frac{\cos^2 A}{\sin A}\frac{\sin^2 A}{\cos A}, \ldots\ldots \because \sin^2 A + \cos^2 A = 1 = \frac{\frac{\sin A\cos A}{\sin A\cos A}}{\frac{\sin^2 A}{\sin A\cos A} + \frac{\cos^2 A}{\sin A\cos A}}$

.... Dividing the numerator and denominator by sin A cos A

$= \frac{1}{\tan A + \cot A}$

= RHS

7. $\text{LHS} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = \cos A + 1 = 1 + \cos A$

$= \frac{(1+\cos A)(1-\cos A)}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A}$

$= \frac{\sin^2 A}{1-\cos A} \cdot \because \sin^2 A + \cos^2 A = 1$

= RHS

8. $LHS = (cosec\,\theta - \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta} = RHS$

9. Taking L.H.S

$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing Numerator and Denominator by sin A

$= \frac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}}$

$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$

Using the formula $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\cot A - 1 + cosecA}{\cot A + 1 - cosecA}$

Using the identity cosec^2A = 1 + cot^2A

$= \frac{\cot A - \left(cosec^2 A - \cot^2 A\right) + cosecA}{\cot A + 1 - cosecA}$

$= \frac{(\cot A + cosecA) - \left(cosec^2 A - \cot^2 A\right)}{\cot A + 1 - cosecA}$

$= \frac{(\cot A + cosecA)(1 - cosecA + \cot A)}{\cot A + 1 - cosecA}$

= cot A + cosec A

= R.H.S

10. $LHS = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A} = \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)\cos A}$

$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} \because \sin^2 A + \cos^2 A = 1$

$= \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \frac{2}{\cos A} = 2 \cdot \frac{1}{\cos A} = 2\sec A = RHS$

11. To prove: (sinA + cosecA)^2 + (cosA + secA)^2 = 7 + tan^{2}A + cot^{2}A

taking L.H.S

Using the formula (a+b)^2 = a^2 + b^2 + 2ab to get,

= (sin^{2}A + cosec^{2}A + 2sinA cosecA) + (cos^{2}A + sec^{2}A + 2 cos A sec A)

Since sinθ =$\frac{1}{cosec\theta}$ and $\cos\theta = \frac{1}{\sec\theta}$

$= \left(\sin^2 A + \csc^2 A + 2\sin A \frac{1}{\sin A}\right) + \left(\cos^2 A + \sec^2 A + 2\cos A \frac{1}{\cos A}\right)$

= sin^{2}A + cosec^{2}A + 2 + cos^{2}A + sec^{2}A + 2

= (sin^{2}A + cos^{2}A) + cosec^{2}A + sec^{2}A + 2 + 2

Using the identities sin^{2}A + cos^{2}A = 1, sec^{2}A = 1 + tan^{2}A and cosec^{2}A = 1 + cot^{2}A to get

= 1+ 1 + tan^{2}A + 1 + cot^{2}A + 2 + 2

= 1 + 2 + 2 + 2 + tan^{2}A + cot^{2}A

= 7 + tan^{2}A + cot^{2}A

= R.H.S.

Hence proved

12. LHS-

$\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$= \frac{\tan A}{1-\frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1-\tan A}$

$= \frac{\tan A}{\frac{\tan A - 1}{\tan A}} + \frac{1}{\tan A(1-\tan A)}$

$= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A(1-\tan A)}$

$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$

$=\frac{(\tan A-1)(\tan^2 A+\tan A+1)}{\tan A(\tan A-1)}$ [$a^3-b^3=(a-b)(a^2+ab+b^2)$]

$=\frac{\tan^2 A+\tan A+1}{\tan A}$

$=\tan A+1+\cot A$

$=\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}+1$

$=\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}+1$

$=\frac{1}{\sin A\cos A}+1$

= sec A cosec A+ 1

= R.H.S