

# UNIQUE STUDY POINT

By Sumeet Sahu

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Unique Study Point, Amitesh Nagar, Indore, MP | Contact: 8103405051

<b>Class:</b> X	<b>Subject:</b> Mathematics	<b>Session:</b> 2024-25
<b>Chapter:</b> 05 - Arithmetic Progression	<b>Time:</b> 1½ Hours	<b>Max. Marks:</b> 40

## General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A contains 10 MCQs of 1 mark each.
4. Section B contains 4 questions of 2 marks each.
5. Section C contains 3 questions of 3 marks each.
6. Section D contains 1 question of 5 marks.
7. Section E contains 2 Case Study Based questions of 4 marks each.
8. Use of calculators is not permitted.

## SECTION A - Multiple Choice Questions (1 mark each)

1. In an AP, if  $d = -2$  and  $a_7 = 4$ , then  $a$  is:  
(a) 16  
(b) 14  
(c) 12  
(d) 10
2. The sum of all numbers between 100 and 200 divisible by 4 is:  
(a) 3600  
(b) 3700  
(c) 3750  
(d) 3800
3. If 7th term of an AP is  $\frac{1}{9}$  and its 9th term is  $\frac{1}{7}$ , then its 63rd term is:  
(a) 0  
(b) 1  
(c)  $\frac{1}{63}$   
(d) 63
4. The sum of first 25 terms of an AP whose  $n$ th term is given by  $a_n = 7 - 3n$  is:  
(a) -800  
(b) -775  
(c) -750  
(d) -725
5. Two APs have the same common difference. The first term of one is 2 and that of the other is 7. The difference between their 10th terms is:

- (a) 2
- (b) 5
- (c) 7
- (d) 9

6. If the sum of first  $n$  terms is  $(3n^2 + 2n)$ , then the 20th term is:

- (a) 117
- (b) 118
- (c) 119
- (d) 120

7. The 15th term from the end of the AP 4, 9, 14, ..., 254 is:

- (a) 174
- (b) 179
- (c) 184
- (d) 189

8. If six times the sixth term of an AP is equal to nine times the ninth term, then the 15th term is:

- (a) -15
- (b) 0
- (c) 15
- (d) 30

9. **Assertion (A):** In the AP 2, 7, 12, 17, ..., the 50th term is 247.

**Reason (R):**  $a_n = a + (n-1)d$

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

10. **Assertion (A):** The sum of first  $n$  even natural numbers is  $n(n+1)$ .

**Reason (R):** Even numbers form an AP with  $a=2$ ,  $d=2$ .

- (a) Both A and R are true and R is the correct explanation of A.
- (b) Both A and R are true but R is not the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

### SECTION B - Short Answer Questions (2 marks each)

11. For what value of  $k$  will  $k+9$ ,  $2k-1$  and  $2k+7$  be consecutive terms of an AP?

12. The sum of first 15 terms of an AP is 750 and its first term is 15. Find the 20th term.

13. If the  $m$ th term of an AP is  $1/n$  and  $n$ th term is  $1/m$ , prove that the sum of  $mn$  terms is  $(mn + 1)/2$ .

14. Find the sum of first 24 terms of the AP  $a_1, a_2, a_3, \dots$  if it is known that  $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ .

### SECTION C - Short Answer Questions (3 marks each)

15. The sum of three numbers in AP is 30 and the ratio of first to third is 3:7. Find the numbers.

16. If the sum of  $n$  terms of an AP is  $2n^2 + 5n$ , then find its  $n$ th term. Hence, find its 16th term.

17. How many terms of the AP 24, 21, 18, 15, ... must be taken so that their sum is 78? Explain the double answer.

#### SECTION D - Long Answer Question (5 marks)

18. The sum of first  $n$  terms of an AP is given by  $S_n = (2n^2 + 3n)$ . Find:

- (i) The  $n$ th term
- (ii) The common difference
- (iii) The 10th term
- (iv) Verify your answer by calculating  $S_{10}$  using both formulas

#### SECTION E - Case Study Based Questions (4 marks each)

19. A spiral is made up of successive semicircles with centers alternately at points A and B, starting with center at A. The diameters of the semicircles are 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown.

- (a) What is the diameter of the 10th semicircle? **(1 mark)**
- (b) What is the total length of the spiral made up of 10 semicircles? (Take  $\pi = 22/7$ ) **(1 mark)**
- (c) How many semicircles are needed to make a spiral of total length 1650 cm? **(2 marks)**

**OR**

(c) If the pattern continues, what is the total length of first 20 semicircles? **(2 marks)**

20. A merchant borrows ₹50,000 and agrees to repay with a total interest of ₹12,000 in 20 monthly installments. Each installment being less than the preceding one by ₹250.

- (a) What is the first installment? **(1 mark)**
- (b) What is the last installment? **(1 mark)**
- (c) If instead the installments increased by ₹150 per month, what would be the first and last installments? **(2 marks)**

**OR**

(c) What is the value of the 10th installment in the original scheme? **(2 marks)**

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SECTION A - Answers to MCQs

1. (a) 16

$$a_7 = a + 6d = 4$$

$$a + 6(-2) = 4$$

$$a - 12 = 4$$

$$a = 16$$

2. (c) 3750

Numbers: 104, 108, ..., 196 (25 terms)

$$S = 25/2(104 + 196) = 25 \times 150 = 3750$$

3. (b) 1

$$a_7 = a + 6d = 1/9$$

$$a_9 = a + 8d = 1/7$$

$$\text{Solving: } 2d = 1/7 - 1/9 = 2/63, d = 1/63$$

$$a = 1/9 - 6/63 = 1/9 - 2/21 = 1/63$$

$$a_{63} = 1/63 + 62/63 = 1$$

4. (a) -800

$$a_n = 7 - 3n$$

$$a_1 = 4, a_{25} = 7 - 75 = -68$$

$$S_{25} = 25/2(4 - 68) = 25 \times (-32) = -800$$

5. (b) 5

$$\text{AP1: } a_{10} = 2 + 9d$$

$$\text{AP2: } A_{10} = 7 + 9d$$

$$\text{Difference} = 7 + 9d - 2 - 9d = 5$$

6. (c) 119

$$S_n = 3n^2 + 2n$$

$$a_{20} = S_{20} - S_{19}$$

$$= 3(400) + 40 - 3(361) - 38$$

$$= 1240 - 1121 = 119$$

7. (b) 179

AP: 4, 9, 14, ..., 254 (d = 5)

15th from end =  $254 - 14(5) = 254 - 70 = 184$ ... Wait: 15th from end =  $254 - (15-1)(5) = 254 - 70 = 184$

Actually, should be:  $254 - 14 \times 5 = 184$ , but answer is 179. Let me recalculate: term from end at position k = last - (k-1)d  
 15th from end =  $254 - 14(5) = 254 - 70 = 184$  *Should be (c) 184, not (b) 179 - possible error in options*

8. (b) 0

$$6a_6 = 9a_9$$

$$6(a + 5d) = 9(a + 8d)$$

$$6a + 30d = 9a + 72d$$

$$3a = -42d$$

$$a = -14d$$

$$a_{15} = a + 14d = -14d + 14d = 0$$

9. (a)

$$a = 2, d = 5$$

$$a_{50} = 2 + 49(5) = 2 + 245 = 247 \checkmark$$

Both true, R explains A.

10. (a)

Even numbers: 2, 4, 6, ..., 2n

$$S = n/2(2 + 2n) = n(n+1) \checkmark$$

Both true, R explains A.

## SECTION B - Answers to Short Answer Questions

11.

For AP: 2(middle) = first + third

$$2(2k-1) = (k+9) + (2k+7)$$

$$4k - 2 = 3k + 16$$

$$k = 18$$

**Answer:**  $k = 18$

12.

$$S_{15} = 750, a = 15$$

$$15/2[2(15) + 14d] = 750$$

$$30 + 14d = 100$$

$$d = 5$$

$$a_{20} = 15 + 19(5) = 110$$

**Answer:** 110

13.

$$a_m = a + (m-1)d = 1/n \dots (i)$$

$$a_n = a + (n-1)d = 1/m \dots (ii)$$

$$\text{Subtracting: } (m-n)d = 1/n - 1/m = (m-n)/mn$$

$$d = 1/mn$$

$$\text{From (i): } a = 1/n - (m-1)/mn = (m - m + 1)/mn = 1/mn$$

$$S_{mn} = mn/2[2(1/mn) + (mn-1)(1/mn)]$$

$$= mn/2[(2 + mn - 1)/mn]$$

$$= (mn + 1)/2$$

**Hence proved.**

14.

$$\text{Given: } a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$$

$$\text{Note: } a_1 + a_{24} = a_5 + a_{20} = a_{10} + a_{15}$$

$$\text{So: } 3(a_1 + a_{24}) = 225$$

$$a_1 + a_{24} = 75$$

$$S_{24} = 24/2(a_1 + a_{24}) = 12 \times 75 = 900$$

**Answer:** 900

## SECTION C - Answers to Short Answer Questions

15.

Let numbers be (a-d), a, (a+d)

$$\text{Sum: } 3a = 30, a = 10$$

$$\text{Ratio: } (a-d)/(a+d) = 3/7$$

$$7(10-d) = 3(10+d)$$

$$70 - 7d = 30 + 3d$$

$$40 = 10d, d = 4$$

Numbers: 6, 10, 14

**Answer:** 6, 10, 14

16.

$$S_n = 2n^2 + 5n$$

$$a_n = S_n - S_{n-1}$$

$$= 2n^2 + 5n - 2(n-1)^2 - 5(n-1)$$

$$= 2n^2 + 5n - 2n^2 + 4n - 2 - 5n + 5$$

$$= 4n + 3$$

$$a_{16} = 4(16) + 3 = 67$$

**Answer:**  $a_n = 4n + 3$ ,  $a_{16} = 67$

17.

AP: 24, 21, 18, 15, ... (a = 24, d = -3)

$$S_n = n/2[2(24) + (n-1)(-3)] = 78$$

$$n[48 - 3n + 3] = 156$$

$$51n - 3n^2 = 156$$

$$3n^2 - 51n + 156 = 0$$

$$n^2 - 17n + 52 = 0$$

$$(n - 4)(n - 13) = 0$$

$$n = 4 \text{ or } n = 13$$

**Explanation:** After 4 terms (sum = 78), the terms become negative. The sum again becomes 78 when we add terms up to the 13th term (as negative terms reduce the sum back to 78).

**Answer:**  $n = 4$  or  $n = 13$

## SECTION D - Answer to Long Answer Question

18.

$$\text{Given: } S_n = 2n^2 + 3n$$

**(i) nth term:**

$$a_n = S_n - S_{n-1}$$

$$= 2n^2 + 3n - 2(n-1)^2 - 3(n-1)$$

$$= 2n^2 + 3n - 2(n^2 - 2n + 1) - 3n + 3$$

$$= 2n^2 + 3n - 2n^2 + 4n - 2 - 3n + 3$$

$$= 4n + 1$$

**(ii) Common difference:**

$$a_1 = 4(1) + 1 = 5$$

$$a_2 = 4(2) + 1 = 9$$

$$d = a_2 - a_1 = 9 - 5 = 4$$

**(iii) 10th term:**

$$a_{10} = 4(10) + 1 = 41$$

**(iv) Verification:**

Using  $S_n = 2n^2 + 3n$ :

$$S_{10} = 2(100) + 30 = 230$$

Using  $S_n = n/2[2a + (n-1)d]$ :

$$S_{10} = 10/2[2(5) + 9(4)] = 5[10 + 36] = 5(46) = 230 \checkmark$$

**Both match!**

## SECTION E - Answers to Case Study Based Questions

19.

**(a):** Diameters: 0.5, 1.0, 1.5, 2.0, ... ( $d = 0.5$ )

$$d_{10} = 0.5 + 9(0.5) = 5 \text{ cm}$$

**(b):** Length =  $\pi/2 \times$  (sum of diameters)

$$\text{Sum} = 10/2(0.5 + 5) = 5 \times 5.5 = 27.5$$

$$\text{Length} = 22/7 \times 1/2 \times 27.5 = 11 \times 2.5 = 27.5 \dots \text{Actually: } \pi/2 \times 27.5 = 22/14 \times 27.5 = 43.21 \text{ cm}$$

$$\text{More precisely: } (22/7) \times (27.5/2) = 11 \times 27.5/7 = 302.5/7 \approx 43.2 \text{ cm}$$

**(c):** If total length = 1650 cm, solving gives  $n \approx 60$  semicircles

**(c) OR:** For 20 semicircles, sum of diameters =  $20/2(0.5 + 10) = 105$

$$\text{Length} = \pi/2 \times 105 = 22/14 \times 105 = 165 \text{ cm}$$

20.

**(a):** Total to repay =  $50000 + 12000 = ₹62,000$

Installments form AP with  $d = -250$ ,  $n = 20$

$$S_{20} = 20/2[2a + 19(-250)] = 62000$$

$$2a - 4750 = 6200$$

$$a = 5475$$

$$\text{First installment} = ₹5475$$

**(b):** Last =  $a + 19d = 5475 - 4750 = ₹725$

**(c):** If  $d = 150$ :

$$10[2a + 19(150)] = 62000$$

$$2a + 2850 = 6200$$

$$a = 1675$$

$$\text{First} = ₹1675, \text{ Last} = 1675 + 2850 = ₹4525$$

**(c) OR:** 10th installment =  $5475 - 9(250) = 5475 - 2250 = ₹3225$

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