UNIQUE STUDY POINT

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Class: X	**Subject:** Mathematics	**Session:** 2024-25
Chapter: 05 - Arithmetic Progression	**Time:** 1½ Hours	**Max. Marks:** 40

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A contains 10 MCQs of 1 mark each.
4. Section B contains 4 questions of 2 marks each.
5. Section C contains 3 questions of 3 marks each.
6. Section D contains 1 question of 5 marks.
7. Section E contains 2 Case Study Based questions of 4 marks each.
8. Use of calculators is not permitted.

SECTION A - Multiple Choice Questions (1 mark each)

1. The 21st term of the AP whose first two terms are –3 and 4 is:

(a) 17
(b) 137
(c) 143
(d) –143

2. If k, 2k – 1 and 2k + 1 are three consecutive terms of an A.P., then the value of k is:

(a) 2
(b) 3
(c) –3
(d) 5

3. The sum of the first 16 terms of the A.P.: 10, 6, 2, ... is:

(a) –320
(b) 320
(c) –352
(d) –400

4. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?

(a) 30
(b) 33
(c) 37
(d) 38

5. Which term of the AP: 21, 42, 63, 84... is 210?

(a) 9th
(b) 10th
(c) 11th
(d) 12th

6. The common difference of an A.P., whose nth term is $a_n = (3n + 7)$, is:

(a) 3
(b) 7
(c) 10
(d) 6

7. The value of p for which (2p + 1), 10 and (5p + 5) are three consecutive terms of an A.P., is:

(a) –1
(b) –2
(c) 1
(d) 2

8. The number of terms of an A.P. 5, 9, 13, ... 185 is:

(a) 31
(b) 51
(c) 41
(d) 46

9. Assertion (A): Common difference of an A.P. in which $a_{21} - a_7 = 84$ is 14.

Reason (R): nth term of AP is given by $a_n = a + (n - 1)d$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

10. Assertion (A): If the second term of an A.P., is 13 and the fifth term is 25, then its 7th term is 33.

Reason (R): If the common difference of an A.P. is 5, then $a_{18} - a_{13}$ is 25.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

SECTION B - Short Answer Questions (2 marks each)

11. Determine the A.P. whose third term is 16 and 7th term exceeds the 5th term by 12.

12. Which term of the A.P. 3, 15, 27, 39, ... will be 120 more than its 21st term?

13. If seven times the 7th term of an A.P. is equal to eleven times the 11th term, then what will be its 18th term?

14. Find how many integers between 200 and 500 are divisible by 8.

SECTION C - Short Answer Questions (3 marks each)

15. Solve for x : 1 + 5 + 9 + 13 + ... + x = 1326

16. Find the middle terms of the A.P. 7, 13, 19 .... 241.

17. The first term of an A.P. is –5 and the last term is 45. If the sum of the terms of the A.P. is 120, then find the number of terms and the common difference.

SECTION D - Long Answer Question (5 marks)

18. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers.

SECTION E - Case Study Based Questions (4 marks each)

19. A school auditorium has to be constructed with a capacity of 2000 people. The chairs in the auditorium are arranged in a concave shape facing towards the stage in such a way that each succeeding row has 5 seats more than the previous one.

(a) If the first row has 15 seats, then how many seats will be there in 12th row? **(1 mark)**

(b) If there are 15 rows in the auditorium, then how many seats will be there in the middle row? **(1 mark)**

(c) If total 1875 guests were there in the auditorium for a particular event, then how many rows will be needed to make all of them sit? **(2 marks)**

OR

(c) If total 1250 guests were there in the auditorium for a particular event, then how many rows will be left blank out of total 30 rows? **(2 marks)**

20. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86 m at the Asian Grand Prix in 2017 is the maximum distance for an Indian female athlete. Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day. Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9 cm every week. During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

(a) How many throws Sanjitha practiced on 11th day of the camp? **(1 mark)**

(b) What would be Sanjitha's throw distance at the end of 6 weeks? **(1 mark)**

OR

(b) When will she be able to achieve a throw of 11.16 m? **(1 mark)**

(c) How many throws did she do during the entire camp of 15 days? **(2 marks)**

DETAILED ANSWER KEY - PAPER 05

SECTION A - Answers to MCQs

1. (b) 137

Solution:

Given: $a_1 = -3$, $a_2 = 4$

Common difference $d = a_2 - a_1 = 4 - (-3) = 7$

For 21st term: $a_{21} = a + 20d = -3 + 20(7) = -3 + 140 = 137$

2. (b) 3

Solution:

For three consecutive terms in AP: 2(middle term) = first term + third term

$2(2k - 1) = k + (2k + 1)$

$4k - 2 = 3k + 1$

$k = 3$

3. (a) -320

Solution:

Given: $a = 10$, $d = 6 - 10 = -4$, $n = 16$

$S_n = n/2[2a + (n - 1)d]$

$S_{16} = 16/2[2(10) + 15(-4)]$

$S_{16} = 8[20 - 60] = 8(-40) = -320$

4. (b) 33

Solution:

Given: $a_2 = 13$, $a_5 = 25$

$a + d = 13$ ... (i)

$a + 4d = 25$ ... (ii)

Subtracting (i) from (ii): $3d = 12$, $d = 4$

From (i): $a = 13 - 4 = 9$

$a_7 = a + 6d = 9 + 6(4) = 9 + 24 = 33$

5. (b) 10th

Solution:

Given: $a = 21$, $d = 42 - 21 = 21$, $a_n = 210$

$a_n = a + (n - 1)d$

$210 = 21 + (n - 1)21$

$189 = (n - 1)21$

$n - 1 = 9$

$n = 10$

6. (a) 3

Solution:

Given: $a_n = 3n + 7$

$a_1 = 3(1) + 7 = 10$

$a_2 = 3(2) + 7 = 13$

Common difference $d = a_2 - a_1 = 13 - 10 = 3$

7. (c) 1

Solution:

For three consecutive terms in AP: 2(middle term) = first term + third term

$2(10) = (2p + 1) + (5p + 5)$

$20 = 7p + 6$

$14 = 7p$

p = 2... Wait, let me recalculate:

$2(10) = 2p + 1 + 5p + 5$

$20 = 7p + 6$

$7p = 14$

$p = 2$

Note: Answer should be (d) 2, not (c) 1

8. (d) 46

Solution:

Given: $a = 5$, $d = 9 - 5 = 4$, $a_n = 185$

$a_n = a + (n - 1)d$

$185 = 5 + (n - 1)4$

$180 = 4(n - 1)$

$n - 1 = 45$

$n = 46$

9. (a)

Solution:

Using $a_n = a + (n - 1)d$:

$a_{21} = a + 20d$ and $a_7 = a + 6d$

$a_{21} - a_7 = (a + 20d) - (a + 6d) = 14d$

Given: $a_{21} - a_7 = 84$

Therefore: $14d = 84$, $d = 6$

Assertion is false (d = 6, not 14), but Reason is true.

Correct answer: (d)

10. (b)

Solution:

For Assertion:

$a_2 = 13$, $a_5 = 25$

$a + d = 13$, $a + 4d = 25$

Solving: $d = 4$, $a = 9$

$a_7 = 9 + 6(4) = 33$ ✓ (Assertion is true)

For Reason:

If d = 5, then $a_{18} - a_{13} = (a + 17d) - (a + 12d) = 5d = 25$ ✓ (Reason is true)

Both are true but Reason doesn't explain Assertion.

SECTION B - Answers to Short Answer Questions

11.

Solution:

Given: $a_3 = 16$ and $a_7 - a_5 = 12$

We know: $a_3 = a + 2d = 16$ ... (i)
And: $a_7 - a_5 = (a + 6d) - (a + 4d) = 2d = 12$
Therefore: d = 6

From (i): a + 2(6) = 16
a = 16 – 12 = 4

Answer: The A.P. is 4, 10, 16, 22, 28, ...

12.

Solution:

Given A.P.: 3, 15, 27, 39, ...
Here: a = 3, d = 15 – 3 = 12

$a_{21} = a + 20d = 3 + 20(12) = 3 + 240 = 243$

Required term = $a_{21} + 120 = 243 + 120 = 363$

Let this be nth term:
$a_n = a + (n - 1)d$
$363 = 3 + (n - 1)12$
$360 = 12(n - 1)$
$n - 1 = 30$
$n = 31$

Answer: 31st term

13.

Solution:

Given: $7 \times a_7 = 11 \times a_{11}$
$7(a + 6d) = 11(a + 10d)$
$7a + 42d = 11a + 110d$
$-4a = 68d$
$a = -17d$

$a_{18} = a + 17d = -17d + 17d = 0$

Answer: 18th term = 0

14.

Solution:

First integer divisible by 8 after 200 = 200 ÷ 8 = 25, so 25 × 8 = 200
Next one = 208 (first term, a = 208)
Last integer divisible by 8 before 500 = 496 (last term)

This forms an A.P.: 208, 216, 224, ..., 496
Here: a = 208, d = 8, a_n = 496

$a_n = a + (n - 1)d$
$496 = 208 + (n - 1)8$
$288 = 8(n - 1)$
$n - 1 = 36$
$n = 37$

Answer: 37 integers

SECTION C - Answers to Short Answer Questions

15.

Solution:

The series is: 1 + 5 + 9 + 13 + ... + x = 1326
This is an A.P. with a = 1, d = 4

Let x be the nth term, so $a_n = x$
$x = 1 + (n - 1)4 = 1 + 4n - 4 = 4n - 3$

Sum formula: $S_n = n/2(\text{first term} + \text{last term})$
$1326 = n/2(1 + x)$
$2652 = n(1 + x)$
$2652 = n(1 + 4n - 3)$
$2652 = n(4n - 2)$
$2652 = 4n^2 - 2n$
$4n^2 - 2n - 2652 = 0$
$2n^2 - n - 1326 = 0$

Using quadratic formula:
$n = [1 \pm \sqrt{(1 + 10608)}]/4 = [1 \pm \sqrt{10609}]/4 = [1 \pm 103]/4$
$n = 104/4 = 26$ (taking positive value)

$x = 4n - 3 = 4(26) - 3 = 104 - 3 = 101$

Answer: x = 101

16.

Solution:

Given A.P.: 7, 13, 19, ..., 241
Here: a = 7, d = 6, last term = 241

First, find number of terms:
$a_n = a + (n - 1)d$
$241 = 7 + (n - 1)6$
$234 = 6(n - 1)$
$n - 1 = 39$
$n = 40$

Since n = 40 (even), there are two middle terms:
20th term and 21st term

$a_{20} = 7 + 19(6) = 7 + 114 = 121$
$a_{21} = 7 + 20(6) = 7 + 120 = 127$

Answer: Middle terms are 121 and 127

17.

Solution:

Given: a = –5, l = 45, $S_n = 120$

Using $S_n = n/2(a + l)$:
$120 = n/2(-5 + 45)$
$120 = n/2(40)$
$120 = 20n$
$n = 6$

Now, $l = a + (n - 1)d$
$45 = -5 + (6 - 1)d$
$45 = -5 + 5d$
$50 = 5d$
$d = 10$

Answer: Number of terms = 6, Common difference = 10

SECTION D - Answer to Long Answer Question

18.

Solution:

Let the four consecutive terms in A.P. be: (a – 3d), (a – d), (a + d), (a + 3d)

Given condition 1: Sum = 32
$(a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$
$4a = 32$
$a = 8$

Given condition 2: Ratio of products = 7 : 15

$[(a - 3d)(a + 3d)] / [(a - d)(a + d)] = 7/15$

$(a^2 - 9d^2) / (a^2 - d^2) = 7/15$

Cross-multiplying:

$15(a^2 - 9d^2) = 7(a^2 - d^2)$

$15a^2 - 135d^2 = 7a^2 - 7d^2$

$8a^2 = 128d^2$

$a^2 = 16d^2$

Since a = 8:

$64 = 16d^2$

$d^2 = 4$

$d = \pm 2$

Case 1: When d = 2:

Numbers are: 8 – 6 = 2, 8 – 2 = 6, 8 + 2 = 10, 8 + 6 = 14

Case 2: When d = –2:

Numbers are: 8 + 6 = 14, 8 + 2 = 10, 8 – 2 = 6, 8 – 6 = 2

Answer: The numbers are 2, 6, 10, 14

SECTION E - Answers to Case Study Based Questions

19.

(a) Seats in 12th row:

Given: First row has 15 seats, d = 5

$a_{12} = a + 11d = 15 + 11(5) = 15 + 55 = 70$

Answer: 70 seats

(b) Middle row seats (15 rows):

Middle row = 8th row

$a_8 = 15 + 7(5) = 15 + 35 = 50$

Answer: 50 seats

(c) Rows needed for 1875 guests:

$S_n = n/2[2a + (n - 1)d]$

$1875 = n/2[2(15) + (n - 1)5]$

$1875 = n/2[30 + 5n - 5]$

$1875 = n/2[25 + 5n]$

$3750 = n(25 + 5n)$

$3750 = 25n + 5n^2$

$5n^2 + 25n - 3750 = 0$

$n^2 + 5n - 750 = 0$

Using quadratic formula:

$n = [-5 \pm \sqrt{(25 + 3000)}]/2 = [-5 \pm \sqrt{3025}]/2 = [-5 \pm 55]/2$

n = 50/2 = 25 (taking positive value)

Answer: 25 rows

(c) OR - Blank rows for 1250 guests (30 total rows):

$S_n = n/2[2a + (n – 1)d]$

$1250 = n/2[30 + 5n – 5]$

$2500 = n(25 + 5n)$

$5n^2 + 25n – 2500 = 0$

$n^2 + 5n – 500 = 0$

Using quadratic formula:

$n = [–5 ± \sqrt{(25 + 2000)}]/2 = [–5 ± \sqrt{2025}]/2 = [–5 ± 45]/2$

$n = 40/2 = 20$

Blank rows = 30 – 20 = 10

Answer: 10 rows will be left blank

20.

(a) Throws on 11th day:

First day: a = 40, d = 12

$a_{11} = 40 + 10(12) = 40 + 120 = 160$

Answer: 160 throws

(b) Throw distance at end of 6 weeks:

Initial distance = 7.56 m = 756 cm

Improvement = 9 cm per week

After 6 weeks: 756 + 6(9) = 756 + 54 = 810 cm = 8.10 m

Answer: 8.10 m

(b) OR - When will throw reach 11.16 m:

Target = 11.16 m = 1116 cm

Initial = 756 cm

Improvement needed = 1116 – 756 = 360 cm

Number of weeks = 360 ÷ 9 = 40 weeks

Answer: After 40 weeks

(c) Total throws in 15 days camp:

a = 40, d = 12, n = 15

$S_n = n/2[2a + (n – 1)d]$

$S_{15} = 15/2[2(40) + 14(12)]$

$S_{15} = 15/2[80 + 168]$

$S_{15} = 15/2 \times 248$

$S_{15} = 15 \times 124 = 1860$

Answer: 1860 throws

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