UNIQUE STUDY POINT

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Class: X	**Subject:** Mathematics	**Session:** 2024-25
Chapter: 04 - Quadratic Equations	**Time:** 1½ Hours	**Max. Marks:** 40

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A contains 10 MCQs of 1 mark each.
4. Section B contains 4 questions of 2 marks each.
5. Section C contains 3 questions of 3 marks each.
6. Section D contains 1 question of 5 marks.
7. Section E contains 2 Case Study Based questions of 4 marks each.

SECTION A - Multiple Choice Questions (1 mark each)

1. The equation $x(x + 1) + 8 = (x + 2)(x – 2)$ can be written in the form $ax^2 + bx + c = 0$ as:

(a) $x^2 + 5x + 4 = 0$
(b) $5x + 12 = 0$
(c) $x + 12 = 0$
(d) $x^2 + 12 = 0$

2. Which constant must be added to $x^2 – 8x$ to make it a perfect square?

(a) 8
(b) 64
(c) 4
(d) 16

3. If $x = 2$ is a solution of the equation $kx^2 + 2x – 3 = 0$, then the value of k is:

(a) 1/4
(b) -1/4
(c) 4
(d) -4

4. For what value of k does the equation $9x^2 + 6kx + 4 = 0$ have equal roots?

(a) $k = \pm 2$
(b) $k = 2$ only
(c) $k = -2$ only

(d) $k = \pm 6$

5. The quadratic equation whose roots are 3 and -2 is:

(a) $x^2 - x - 6 = 0$
(b) $x^2 + x - 6 = 0$
(c) $x^2 - x + 6 = 0$
(d) $x^2 + x + 6 = 0$

6. If the roots of $ax^2 + bx + c = 0$ are reciprocals of each other, then:

(a) $a = b$
(b) $b = c$
(c) $a = c$
(d) $a + c = 0$

7. The nature of roots of the equation $3x^2 + 7x + 2 = 0$ is:

(a) real and equal
(b) real and unequal
(c) not real
(d) cannot be determined

8. If α, β are roots of $x^2 + px + q = 0$, then the value of $\alpha^2 + \beta^2$ is:

(a) $p^2 - 2q$
(b) $p^2 + 2q$
(c) $p^2 - q$
(d) $p^2 + q$

In questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): The equation $x^2 - 2x + 5 = 0$ has no real roots.

Reason (R): A quadratic equation $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

10. Assertion (A): If one root of the equation $x^2 - 6x + k = 0$ is 2, then $k = 8$.

Reason (R): If $x = a$ is a root of the equation, then it must satisfy the equation.

SECTION B - Short Answer Questions (2 marks each)

11. Solve by factorization: $x^2 + x - 20 = 0$

12. Solve using the quadratic formula: $x^2 - 3x - 4 = 0$

13. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$, find the value of p and the other root.

14. Find the nature of roots of the equation $5x^2 - 6x + 2 = 0$ without solving the equation.

SECTION C - Short Answer Questions (3 marks each)

15. Find the values of k for which the equation $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$ has real and equal roots.

16. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

17. If α and β are roots of $x^2 - 5x + 3 = 0$, find the value of $\alpha^3 + \beta^3$.

SECTION D - Long Answer Question (5 marks)

18. A rectangular field has an area of 528 m^2. The length of the field is 1 m more than twice its breadth. Find the length and breadth of the field.

SECTION E - Case Study Based Questions (4 marks each)

19. Case Study-1: Bridge Construction

An engineer is designing a parabolic arch bridge. The height h (in meters) of the arch above the road at a horizontal distance x (in meters) from one end is given by $h = -0.5x^2 + 4x$. Based on this information, answer the following questions:

(a) What is the maximum height of the arch? (2 marks)

(b) At what horizontal distance from the starting point does the arch reach its maximum height? (1 mark)

(c) What is the span (width) of the arch at road level? (1 mark)

20. Case Study-2: Profit and Loss

A shopkeeper sells x items per day. His daily profit P (in rupees) is given by $P = -5x^2 + 100x + 600$. Based on this information, answer the following questions:

(a) How many items should he sell to maximize his profit? (2 marks)

(b) What is the maximum daily profit? (1 mark)

(c) For what number of items sold will he have zero profit? (1 mark)

DETAILED ANSWER KEY - PAPER 02

SECTION A - Answers to MCQs

1. Answer: (b) 5x + 12 = 0

Solution:

$x(x + 1) + 8 = (x + 2)(x – 2)$

$x^2 + x + 8 = x^2 – 4$

$x^2 + x + 8 – x^2 + 4 = 0$

$x + 12 = 0$

Wait, this gives $x + 12 = 0$, which is option (c)

Let me recalculate: $x^2 + x + 8 = x^2 - 4$

$x + 8 + 4 = 0$

$x + 12 = 0$

Actually the answer should be (c) $x + 12 = 0$

2. Answer: (d) 16

Solution:

To make $x^2 – 8x$ a perfect square:

We add (coefficient of x / 2)^2

$= (-8/2)^2$

$= (-4)^2$

$= 16$

$x^2 – 8x + 16 = (x – 4)^2$

3. Answer: (b) -1/4

Solution:

Substituting $x = 2$ in $kx^2 + 2x – 3 = 0$:

$k(2)^2 + 2(2) – 3 = 0$

$4k + 4 – 3 = 0$

$4k + 1 = 0$

$4k = -1$

$k = -1/4$

4. Answer: (a) k = ±2

Solution:

For $9x^2 + 6kx + 4 = 0$ to have equal roots:

Discriminant = 0

$(6k)^2 - 4(9)(4) = 0$

$36k^2 - 144 = 0$

$36k^2 = 144$

$k^2 = 4$

$k = \pm 2$

5. Answer: (a) $x^2 - x - 6 = 0$

Solution:

If roots are 3 and -2:

Sum of roots = 3 + (-2) = 1

Product of roots = 3 × (-2) = -6

Equation: $x^2 - (\text{sum})x + (\text{product}) = 0$

$x^2 - 1x + (-6) = 0$

$x^2 - x - 6 = 0$

6. Answer: (c) a = c

Solution:

If roots are α and $1/\alpha$:

Product of roots = $\alpha \times 1/\alpha = 1$

But product = c/a

Therefore: c/a = 1

c = a or a = c

7. Answer: (b) real and unequal

Solution:

For $3x^2 + 7x + 2 = 0$

Discriminant = $7^2 - 4(3)(2)$

$= 49 - 24$

$= 25 > 0$

Since discriminant > 0, roots are real and unequal.

8. Answer: (a) $p^2 - 2q$

Solution:

For $x^2 + px + q = 0$:

Sum: $\alpha + \beta = -p$

Product: $\alpha\beta = q$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-p)^2 - 2q$

$= p^2 - 2q$

9. Answer: (d) Assertion (A) is false but reason (R) is true.

Solution:

For $x^2 - 2x + 5 = 0$

Discriminant $= 4 - 20 = -16 < 0$

So equation has no real roots - Assertion is TRUE.

Reason correctly states condition - Reason is TRUE.

Actually both are true, answer should be (a)

10. Answer: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Solution:

Substituting $x = 2$ in $x^2 - 6x + k = 0$:

$4 - 12 + k = 0$

$k = 8$

Assertion is TRUE and reason explains it correctly.

SECTION B - Answers to Short Answer Questions

11. Solution:

$x^2 + x - 20 = 0$

$x^2 + 5x - 4x - 20 = 0$

$x(x + 5) - 4(x + 5) = 0$

$(x - 4)(x + 5) = 0$

$x = 4$ or $x = -5$

12. Solution:

$x^2 - 3x - 4 = 0$

Using quadratic formula:

$x = [3 \pm \sqrt{(9 + 16)}] / 2$

$x = [3 \pm \sqrt{25}] / 2$

$x = [3 \pm 5] / 2$

$x = 4$ or $x = -1$

13. Solution:

Substituting $x = -5$ in $2x^2 + px - 15 = 0$:

$2(25) + p(-5) - 15 = 0$

$50 - 5p - 15 = 0$

$35 - 5p = 0$

$p = 7$

Equation becomes: $2x^2 + 7x - 15 = 0$

Sum of roots = $-7/2$

If one root is -5, other root = $-7/2 - (-5) = -7/2 + 5 = 3/2$

Other root = 3/2

14. Solution:

For $5x^2 - 6x + 2 = 0$

Discriminant = $(-6)^2 - 4(5)(2)$

$= 36 - 40$

$= -4 < 0$

Since discriminant < 0, the roots are not real (imaginary).

SECTION C - Answers to Short Answer Questions

15. Solution:

$(3k + 1)x^2 + 2(k + 1)x + 1 = 0$

For real and equal roots: $b^2 - 4ac = 0$

$[2(k + 1)]^2 - 4(3k + 1)(1) = 0$

$4(k + 1)^2 - 4(3k + 1) = 0$

$4(k^2 + 2k + 1) - 12k - 4 = 0$

$4k^2 + 8k + 4 - 12k - 4 = 0$

$4k^2 - 4k = 0$

$4k(k - 1) = 0$

$k = 0$ or $k = 1$

But if k = 0, coefficient of $x^2 = 1$ (valid)

k = 0 or k = 1

16. Solution:

Let the number be x.

Given: $x + 12 = 160/x$

$x^2 + 12x = 160$

$x^2 + 12x - 160 = 0$

$(x + 20)(x - 8) = 0$

x = -20 or x = 8

Since x is a natural number:

x = 8

17. Solution:

For $x^2 - 5x + 3 = 0$:

Sum: $\alpha + \beta = 5$

Product: $\alpha\beta = 3$

We know: $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= 5^3 - 3(3)(5)$

$= 125 - 45$

= 80

SECTION D - Answer to Long Answer Question

18. Solution:

Let breadth = x meters

Then length = 2x + 1 meters

Area = length × breadth

$x(2x + 1) = 528$

$2x^2 + x = 528$

$2x^2 + x - 528 = 0$

Using quadratic formula:

$x = [-1 \pm \sqrt{(1 + 4224)}] / 4$

$x = [-1 \pm \sqrt{4225}] / 4$

$x = [-1 \pm 65] / 4$

x = 64/4 = 16 or x = -66/4 (rejected)

Breadth = 16 m

Length = 2(16) + 1 = 33 m

Verification: 16 × 33 = 528 ✓

SECTION E - Answers to Case Study Based Questions

19. Solution:

(a) Maximum height of arch (2 marks)

$h = -0.5x^2 + 4x$

This is a parabola opening downward.

Maximum occurs at $x = -b/2a = -4/(2 \times (-0.5)) = -4/(-1) = 4$

$h = -0.5(4)^2 + 4(4)$

$h = -8 + 16$

Maximum height = 8 m

(b) Horizontal distance for maximum height (1 mark)

x = 4 m

(c) Span of arch at road level (1 mark)

At road level, h = 0:

$-0.5x^2 + 4x = 0$

$x(-0.5x + 4) = 0$

x = 0 or x = 8

Span = 8 m

20. Solution:

(a) Items to maximize profit (2 marks)

$P = -5x^2 + 100x + 600$

Maximum occurs at $x = -b/2a = -100/(2 \times (-5)) = -100/(-10) = 10$

He should sell 10 items

(b) Maximum daily profit (1 mark)

$P = -5(10)^2 + 100(10) + 600$

$P = -500 + 1000 + 600$

Maximum profit = ₹1100

(c) Number of items for zero profit (1 mark)

$-5x^2 + 100x + 600 = 0$

$x^2 – 20x – 120 = 0$

Using quadratic formula:

$x = [20 \pm \sqrt{(400 + 480)}] / 2$

$x = [20 \pm \sqrt{880}] / 2$

$x = [20 \pm 29.66] / 2$

$x \approx 24.83$ or $x \approx -4.83$ (rejected)

Approximately 25 items
