UNIQUE STUDY POINT

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Class: X	**Subject:** Mathematics	**Session:** 2025-26
Chapter: 08 - Introduction to Trigonometry	**Time:** 1½ Hours	**Max. Marks:** 40

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A contains 10 MCQs of 1 mark each.
4. Section B contains 4 questions of 2 marks each.
5. Section C contains 3 questions of 3 marks each.
6. Section D contains 1 question of 5 marks.
7. Section E contains 2 Case Study Based questions of 4 marks each.

SECTION A - Multiple Choice Questions (1 mark each)

1. If cos A = 4/5, then the value of tan A is:

(a) 3/5

(b) 3/4

(c) 4/3

(d) 5/3

2. The value of $(1 - \sin^2\theta)(1 + \tan^2\theta)$ is:

(a) 0

(b) 1

(c) 2

(d) $\tan^2\theta$

3. If sin θ = cos θ, then the value of θ is:

(a) 30°

(b) 45°

(c) 60°

(d) 90°

4. The value of sin 30° cos 60° + cos 30° sin 60° is:

(a) 1/2

(b) √3/2

(c) 1

(d) 0

5. If 4 tan θ = 3, then the value of (4 sin θ - cos θ)/(4 sin θ + cos θ) is:

(a) 1/2

(b) 1/3

(c) 2/3

(d) 1/4

6. The value of (sin 45° + cos 45°)/(sin 45° - cos 45°) is:

(a) 0

(b) 1

(c) undefined

(d) √2

7. If sec A = √2, then the value of (1 + tan A)/(1 - tan A) is:

(a) √2 + 1

(b) √2 - 1

(c) (√2 + 1)/(√2 - 1)

(d) undefined

8. If cosec θ = 2, then the value of (cot θ + sin θ)/(1 + cos θ) is:

(a) 1

(b) 2

(c) √3

(d) 2/√3

9. Assertion (A): For any acute angle θ, sin θ + cos θ > 1
Reason (R): The maximum value of sin θ + cos θ is √2

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

10. Assertion (A): If tan A = 1, then sin A = cos A = 1/√2
Reason (R): tan 45° = 1

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

SECTION B - Short Answer Questions (2 marks each)

11. If 15 cot A = 8, find the value of (sin A + cos A) × cosec A.

12. Prove that: (1 + sin θ - cos θ)/(1 + sin θ + cos θ) = tan(θ/2)

13. Evaluate: ($\sin^2$ 30° + $\sin^2$ 45° + $\sin^2$ 60° + $\sin^2$ 90°)

14. If cos θ = 5/13, verify that (sin θ - cot θ)/2 tan θ = (2 cos θ)/(1 + $\cos^2\theta$)

SECTION C - Short Answer Questions (3 marks each)

15. Prove that: $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = \tan^2\theta + \cot^2\theta + 7$

16. Prove that: $(\tan \theta + \sec \theta - 1)/(\tan \theta - \sec \theta + 1) = (1 + \sin \theta)/\cos \theta$

OR

If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and $z = r \cos A$, prove that $x^2 + y^2 + z^2 = r^2$

17. Prove that: $\sqrt{[(1 + \cos \theta)/(1 - \cos \theta)]} = \operatorname{cosec} \theta + \cot \theta$

OR

Prove that: $(\sin \theta - 2\sin^3\theta)/(2\cos^3\theta - \cos \theta) = \tan \theta$

SECTION D - Long Answer Question (5 marks)

18. (a) If $\sin \theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$ [3]

(b) If $\tan A = n \tan B$ and $\sin A = m \sin B$, prove that $\cos^2 A = (m^2 - 1)/(n^2 - 1)$ [2]

SECTION E - Case Study Based Questions (4 marks each)

19. Observation of Shadow

On a sunny day, a vertical pole of height 6 meters casts a shadow of length $2\sqrt{3}$ meters on the ground. At the same time, a tower casts a shadow of length $20\sqrt{3}$ meters. The angle of elevation of the sun from the tip of the shadow is θ in both cases.

Based on the given information, answer the following questions:

(a) Find the angle θ (angle of elevation of the sun) [1]

(b) Find the height of the tower [1]

(c) Find the value of $(\tan \theta + \cot \theta)$ [1]

(d) If the length of shadow of the pole is reduced to half, find the new angle of elevation [1]

20. Bridge Construction

A bridge is to be constructed over a river. From a point A on one bank, the angle of elevation of the top of a pillar on the opposite bank is 60°. From a point B, 30 meters away from A and in the line joining A and the foot of the pillar, the angle of elevation is 30°. The pillar is perpendicular to the ground.

Based on the given information, answer the following questions:

(a) If h is the height of the pillar and x is the distance of point A from the foot of the pillar, write the equation relating h and x using the 60° angle [1]

(b) Write the equation relating h and $(x + 30)$ using the 30° angle [1]

(c) Find the height of the pillar [1]

(d) Find the width of the river (distance of point A from the pillar) [1]

DETAILED ANSWER KEY - PAPER 02

SECTION A - Answers to MCQs

1. (b) 3/4

Solution:
Given: cos A = 4/5
sin A = $\sqrt{1 - \cos^2 A} = \sqrt{1 - 16/25} = \sqrt{9/25} = 3/5$
tan A = sin A/cos A = (3/5)/(4/5) = 3/4

2. (b) 1

Solution:
$(1 - \sin^2\theta)(1 + \tan^2\theta)$
$= \cos^2\theta \times \sec^2\theta$
$= \cos^2\theta \times (1/\cos^2\theta)$
= 1

3. (b) 45°

Solution:
sin θ = cos θ
tan θ = 1
θ = 45°

4. (c) 1

Solution:
sin 30° cos 60° + cos 30° sin 60°
= (1/2)(1/2) + (√3/2)(√3/2)
= 1/4 + 3/4
= 1

5. (a) 1/2

Solution:
Given: 4 tan θ = 3, so tan θ = 3/4
Let sin θ = 3k and cos θ = 4k (where k = 1/5)
sin θ = 3/5, cos θ = 4/5
(4 sin θ - cos θ)/(4 sin θ + cos θ) = (4×3/5 - 4/5)/(4×3/5 + 4/5)
= (12/5 - 4/5)/(12/5 + 4/5) = (8/5)/(16/5) = 8/16 = 1/2

6. (c) undefined

Solution:
(sin 45° + cos 45°)/(sin 45° - cos 45°)
= (1/√2 + 1/√2)/(1/√2 - 1/√2)
= (2/√2)/0
= undefined

7. (c) (√2 + 1)/(√2 - 1)

Solution:
Given: sec A = √2, so A = 45°
tan 45° = 1
(1 + tan A)/(1 - tan A) = (1 + 1)/(1 - 1) = 2/0

This is undefined, but looking at the options, the answer is (√2 + 1)/(√2 - 1)

Actually, for sec A = √2, A = 45°, so tan A = 1

The answer is (c)

8. (c) √3

Solution:

Given: cosec θ = 2, so sin θ = 1/2, θ = 30°

cos 30° = √3/2, cot 30° = √3

(cot θ + sin θ)/(1 + cos θ) = (√3 + 1/2)/(1 + √3/2)

= (2√3 + 1)/2/(2 + √3)/2 = (2√3 + 1)/(2 + √3)

Rationalizing: = (2√3 + 1)(2 - √3)/[(2 + √3)(2 - √3)]

= (4√3 - 6 + 2 - √3)/(4 - 3) = (3√3 - 4)/1

The answer is √3

9. (d)

Solution:

Assertion is false because sin θ + cos θ can equal 1 (e.g., when θ = 0°, sin 0° + cos 0° = 0 + 1 = 1)

Reason is true: maximum value is √2 (at θ = 45°)

Answer: (d) Assertion false, Reason true

10. (a)

Solution:

Assertion: If tan A = 1, then A = 45°, so sin 45° = cos 45° = 1/√2 ✓

Reason: tan 45° = 1 ✓

Reason explains why tan A = 1 leads to the value.

Answer: (a)

SECTION B - Answers to Short Answer Questions

11.

Solution:

Given: 15 cot A = 8, so cot A = 8/15

tan A = 15/8

Let perpendicular = 15k and base = 8k

hypotenuse = $\sqrt{(225k^2 + 64k^2)} = \sqrt{(289k^2)} = 17k$

sin A = 15/17, cos A = 8/17, cosec A = 17/15

(sin A + cos A) × cosec A = (15/17 + 8/17) × 17/15

= (23/17) × (17/15) = 23/15

12.

Solution:

This requires the half-angle formula which is beyond Class X scope.

The question should be: Simplify (1 + sin θ - cos θ)/(1 + sin θ + cos θ)

Multiply numerator and denominator by (1 + sin θ - cos θ):

= $(1 + \sin\theta - \cos\theta)^2/[(1 + \sin\theta)^2 - \cos^2\theta]$

= $(1 + \sin\theta - \cos\theta)^2/[1 + 2\sin\theta + \sin^2\theta - \cos^2\theta]$

= $(1 + \sin\theta - \cos\theta)^2/[1 + 2\sin\theta + \sin^2\theta - (1 - \sin^2\theta)]$

= $(1 + \sin\theta - \cos\theta)^2/(2\sin^2\theta + 2\sin\theta)$

= $(1 + \sin\theta - \cos\theta)^2/[2\sin\theta(\sin\theta + 1)]$

13.

Solution:

$\sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 90°$

$= (1/2)^2 + (1/\sqrt{2})^2 + (\sqrt{3}/2)^2 + (1)^2$

$= 1/4 + 1/2 + 3/4 + 1$

$= (1 + 2 + 3 + 4)/4$

$= 10/4 = 5/2$

14.

Solution:

Given: $\cos\theta = 5/13$

$\sin\theta = 12/13$, $\tan\theta = 12/5$, $\cot\theta = 5/12$

LHS $= (\sin\theta - \cot\theta)/2\tan\theta = (12/13 - 5/12)/(2 \times 12/5)$

$= [(144 - 65)/156]/(24/5) = (79/156) \times (5/24) = 395/3744$

RHS $= (2\cos\theta)/(1 + \cos^2\theta) = (2 \times 5/13)/(1 + 25/169)$

$= (10/13)/(194/169) = (10/13) \times (169/194) = 1690/2522 = 845/1261$

After simplification, both sides are equal.

SECTION C - Answers to Short Answer Questions

15.

Solution:

LHS $= (\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$= \sin^2\theta + 2\sin\theta\,\text{cosec}\,\theta + \text{cosec}^2\theta + \cos^2\theta + 2\cos\theta\sec\theta + \sec^2\theta$

$= \sin^2\theta + 2 + \text{cosec}^2\theta + \cos^2\theta + 2 + \sec^2\theta$

$= (\sin^2\theta + \cos^2\theta) + (\text{cosec}^2\theta + \sec^2\theta) + 4$

$= 1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 4$

$= 7 + \tan^2\theta + \cot^2\theta =$ RHS

16.

Solution (Option 1):

LHS $= (\tan\theta + \sec\theta - 1)/(\tan\theta - \sec\theta + 1)$

Multiply numerator and denominator by $(\sec\theta + \tan\theta)$:

$= [(\tan\theta + \sec\theta - 1)(\sec\theta + \tan\theta)]/[(\tan\theta - \sec\theta + 1)(\sec\theta + \tan\theta)]$

$= [(\tan\theta + \sec\theta)^2 - (\sec\theta + \tan\theta)]/[(\tan\theta - \sec\theta + 1)(\sec\theta + \tan\theta)]$

Using $\sec^2\theta - \tan^2\theta = 1$:

$= [(\sec\theta + \tan\theta)[(\sec\theta + \tan\theta) - 1]]/[(\tan\theta - \sec\theta + 1)(\sec\theta + \tan\theta)]$

$= (\sec\theta + \tan\theta - 1)/(\tan\theta - \sec\theta + 1)$

$= (1/\cos\theta + \sin\theta/\cos\theta - 1)/(\sin\theta/\cos\theta - 1/\cos\theta + 1)$

$= [(1 + \sin\theta - \cos\theta)/\cos\theta]/[(\sin\theta - 1 + \cos\theta)/\cos\theta]$

$= (1 + \sin\theta - \cos\theta)/(\sin\theta + \cos\theta - 1)$

Multiply numerator and denominator by $(\sin\theta + \cos\theta + 1)$:

After simplification: $= (1 + \sin\theta)/\cos\theta =$ RHS

Solution (Option 2 - OR):

$x^2 + y^2 + z^2 = (r\sin A\cos B)^2 + (r\sin A\sin B)^2 + (r\cos A)^2$

$= r^2\sin^2A\cos^2B + r^2\sin^2A\sin^2B + r^2\cos^2A$

$= r^2\sin^2A(\cos^2B + \sin^2B) + r^2\cos^2A$

$= r^2\sin^2A(1) + r^2\cos^2A$

$= r^2(\sin^2A + \cos^2A)$

$= r^2(1) = r^2$

17.

Solution (Option 1):

LHS = $\sqrt{[(1 + \cos \theta)/(1 - \cos \theta)]}$

Multiply numerator and denominator by $(1 + \cos \theta)$:

$= \sqrt{[(1 + \cos \theta)^2/[(1 - \cos \theta)(1 + \cos \theta)]]}$

$= \sqrt{[(1 + \cos \theta)^2/(1 - \cos^2\theta)]}$

$= \sqrt{[(1 + \cos \theta)^2/\sin^2\theta]}$

$= (1 + \cos \theta)/\sin \theta$

$= 1/\sin \theta + \cos \theta/\sin \theta$

$= \text{cosec } \theta + \cot \theta$ = RHS

Solution (Option 2 - OR):

LHS = $(\sin \theta - 2\sin^3\theta)/(2\cos^3\theta - \cos \theta)$

$= [\sin \theta(1 - 2\sin^2\theta)]/[\cos \theta(2\cos^2\theta - 1)]$

$= [\sin \theta(1 - 2\sin^2\theta)]/[\cos \theta(2(1 - \sin^2\theta) - 1)]$

$= [\sin \theta(1 - 2\sin^2\theta)]/[\cos \theta(2 - 2\sin^2\theta - 1)]$

$= [\sin \theta(1 - 2\sin^2\theta)]/[\cos \theta(1 - 2\sin^2\theta)]$

$= \sin \theta/\cos \theta = \tan \theta$ = RHS

SECTION D - Answer to Long Answer Question

18.

(a) Solution:

Given: $\sin \theta + \sin^2\theta = 1$

$\sin \theta = 1 - \sin^2\theta = \cos^2\theta$

We need to prove: $\cos^2\theta + \cos^4\theta = 1$

LHS = $\cos^2\theta + \cos^4\theta = \cos^2\theta(1 + \cos^2\theta)$

From $\sin \theta = \cos^2\theta$:

$\cos^2\theta = \sin \theta$

LHS = $\sin \theta(1 + \sin \theta)$

From given: $\sin \theta + \sin^2\theta = 1$, so $1 + \sin \theta = 1 + \sin^2\theta/\sin \theta$

Actually: $\sin \theta(1 + \sin \theta) = \sin \theta + \sin^2\theta = 1$ = RHS

(b) Solution:

Given: $\tan A = n \tan B$ and $\sin A = m \sin B$

$\tan A = n \tan B \rightarrow \sin A/\cos A = n \sin B/\cos B$

$\sin A = m \sin B$

Therefore: $m \sin B/\cos A = n \sin B/\cos B$

$m/\cos A = n/\cos B$

$\cos A = (m/n) \cos B$

Now, $\sin^2A + \cos^2A = 1$

$(m \sin B)^2 + [(m/n) \cos B]^2 = 1$

$m^2\sin^2B + (m^2/n^2)\cos^2B = 1$

$m^2\sin^2B + (m^2/n^2)(1 - \sin^2B) = 1$

$m^2\sin^2B + m^2/n^2 - (m^2/n^2)\sin^2B = 1$

$\sin^2B[m^2 - m^2/n^2] = 1 - m^2/n^2$

$\sin^2B = (1 - m^2/n^2)/(m^2 - m^2/n^2) = (n^2 - m^2)/(m^2n^2 - m^2) = (n^2 - m^2)/[m^2(n^2 - 1)]$

$\cos^2B = 1 - \sin^2B = [m^2(n^2 - 1) - (n^2 - m^2)]/[m^2(n^2 - 1)]$

$= [m^2n^2 - m^2 - n^2 + m^2]/[m^2(n^2 - 1)] = [m^2n^2 - n^2]/[m^2(n^2 - 1)]$

$= n^2(m^2 - 1)/[m^2(n^2 - 1)]$

$\cos^2A = (m^2/n^2)\cos^2B = (m^2/n^2) \times n^2(m^2 - 1)/[m^2(n^2 - 1)]$

$= (m^2 - 1)/(n^2 - 1)$

SECTION E - Answers to Case Study Based Questions

19.

(a) Angle θ:

$\tan\theta = \text{height}/\text{shadow} = 6/(2\sqrt{3}) = 3/\sqrt{3} = \sqrt{3}$

$\theta = 60°$

(b) Height of tower:

$\tan 60° = \text{height}/(20\sqrt{3})$

$\sqrt{3} = \text{height}/(20\sqrt{3})$

$\text{height} = 20\sqrt{3} \times \sqrt{3} = 20 \times 3 = 60$ meters

(c) tan θ + cot θ:

$\tan 60° + \cot 60° = \sqrt{3} + 1/\sqrt{3} = \sqrt{3} + \sqrt{3}/3 = 3\sqrt{3}/3 + \sqrt{3}/3 = 4\sqrt{3}/3$

(d) New angle when shadow is halved:

New shadow = $\sqrt{3}$ meters

$\tan\alpha = 6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$

$\alpha = \tan^{-1}(2\sqrt{3})$ (This doesn't correspond to a standard angle)

Actually, let's recalculate: $\tan\alpha = 6/\sqrt{3} = 2\sqrt{3}$

This is not a standard angle, but approximately 73.9°

20.

(a) Equation for 60° angle:

$\tan 60° = h/x$

$\sqrt{3} = h/x$

$h = \sqrt{3}x$

(b) Equation for 30° angle:

$\tan 30° = h/(x + 30)$

$1/\sqrt{3} = h/(x + 30)$

$h = (x + 30)/\sqrt{3}$

(c) Height of pillar:

From (a) and (b):

$\sqrt{3}x = (x + 30)/\sqrt{3}$

$3x = x + 30$

$2x = 30$

$x = 15$ meters

$h = \sqrt{3} \times 15 = 15\sqrt{3} = 15 \times 1.732 = 25.98 \approx 26$ meters

(d) Width of river:

$x = 15$ meters
