UNIQUE STUDY POINT

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Class: X	**Subject:** Mathematics	**Session:** 2025-26
Chapter: 08 - Introduction to Trigonometry	**Time:** 1½ Hours	**Max. Marks:** 40

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A contains 10 MCQs of 1 mark each.
4. Section B contains 4 questions of 2 marks each.
5. Section C contains 3 questions of 3 marks each.
6. Section D contains 1 question of 5 marks.
7. Section E contains 2 Case Study Based questions of 4 marks each.

SECTION A - Multiple Choice Questions (1 mark each)

1. If $\tan\theta = 4/3$, then the value of $(3\sin\theta + 2\cos\theta)/(3\sin\theta - 2\cos\theta)$ is:

(a) 2

(b) 3

(c) 4

(d) 5

2. The value of $(1 - \cos^2\theta)(1 + \cot^2\theta)$ is:

(a) 0

(b) 1

(c) $\sin^2\theta$

(d) $\cos^2\theta$

3. If $\sin(A - B) = \sqrt{3}/2$ and $\cos(A + B) = 0$, where $A > B$ and $A + B \leq 90°$, then the value of A is:

(a) 30°

(b) 45°

(c) 60°

(d) 75°

4. The value of $(\cos^2 0° + \cos^2 45° + \cos^2 60° + \cos^2 90°)$ is:

(a) 1

(b) 7/4

(c) 9/4

(d) 2

5. If $3\tan^2\theta = 1$, then the value of $(2\sin^2\theta + 3\cos^2\theta)$ is:

(a) 7/3

(b) 8/3

(c) 10/3

(d) 11/3

6. If cosec A = √10, then the value of (sin A + cos A) is:

(a) √10

(b) (1 + 3)/√10

(c) 4/√10

(d) 1

7. The value of (tan 15° + cot 75°) is:

(a) 1

(b) 2 tan 15°

(c) 2 cot 75°

(d) 0

8. If sin A = 12/13, then the value of (13 sec A - 12 tan A) is:

(a) 1

(b) 5

(c) 12

(d) 13

9. Assertion (A): If 3 tan θ = 4, then (sin θ + cos θ) = 7/5

Reason (R): If 3 tan θ = 4, then sin θ = 4/5 and cos θ = 3/5

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

10. Assertion (A): The value of sin 60° cos 30° + sin 30° cos 60° is equal to 1

Reason (R): sin(A + B) = sin A cos B + cos A sin B

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

SECTION B - Short Answer Questions (2 marks each)

11. If cot A = 12/5, find the value of (2 sin A + 3 cos A)/(4 sin A + 3 cos A).

12. Evaluate: $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

13. If $3 \sin^2 \theta = 2 \cos^2 \theta$, find the value of cot θ.

14. Prove that: $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$

SECTION C - Short Answer Questions (3 marks each)

15. Prove that: (tan A + sec A - 1)/(tan A - sec A + 1) = (1 + sin A)/cos A

16. Prove that: $(\sin\theta + 1 - \cos\theta)/(\cos\theta - 1 + \sin\theta) = (1 + \sin\theta)/\cos\theta$

OR

If $\text{cosec}\,\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$, prove that $a^2b^2(a^2 + b^2) = 1$

17. Prove that: $(\cos\theta/(1 - \tan\theta)) + (\sin\theta/(1 - \cot\theta)) = \sin\theta + \cos\theta$

OR

Prove that: $\tan\theta/(1 - \cot\theta) + \cot\theta/(1 - \tan\theta) = 1 + \sec\theta\,\text{cosec}\,\theta$

SECTION D - Long Answer Question (5 marks)

18. (a) Prove that: $(1 + \sin A)/(1 - \sin A) = (\sec A + \tan A)^2$ [3]

(b) If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$ [2]

SECTION E - Case Study Based Questions (4 marks each)

19. Leaning Tower Problem

A boy standing at the base of a leaning tower observes that at a distance of 10 meters from the tower's base, the angle of elevation to the top is 60°. The tower leans at an angle such that it makes 75° with the ground. He wants to calculate the actual length of the tower and its vertical height.

Based on the given information, answer the following questions:

(a) Find the vertical height of the point directly above the 10-meter mark [1]

(b) Find the value of $\tan 60° + \cot 30°$ [1]

(c) If the tower were vertical, what would be its height for the same angle of elevation? [1]

(d) Verify that $\tan^2 60° - \sec^2 60° = -1$ [1]

20. Satellite Dish Installation

A satellite dish needs to be installed on the roof of a building. The technician uses a ladder that is 20 meters long. The ladder is placed such that it makes an angle of 60° with the ground. After installing, he adjusts it to make an angle of 45° with the ground without changing the point where the ladder touches the ground.

Based on the given information, answer the following questions:

(a) Find the height reached by the ladder at 60° angle (Take $\sqrt{3} = 1.732$) [1]

(b) Find the height reached by the ladder at 45° angle (Take $\sqrt{2} = 1.414$) [1]

(c) Find the difference in heights [1]

(d) Find $\sin^2 60° + \cos^2 60°$ [1]

DETAILED ANSWER KEY - PAPER 04

SECTION A - Answers to MCQs

1. (b) 3

Solution:

Given: $\tan\theta = 4/3$

Let $\sin\theta = 4/5$ and $\cos\theta = 3/5$

$(3\sin\theta + 2\cos\theta)/(3\sin\theta - 2\cos\theta)$

$= (3\times4/5 + 2\times3/5)/(3\times4/5 - 2\times3/5)$

$= (12/5 + 6/5)/(12/5 - 6/5)$

$= (18/5)/(6/5)$

$= 18/6 = 3$

2. (b) 1

Solution:

$(1 - \cos^2\theta)(1 + \cot^2\theta)$

$= \sin^2\theta \times \text{cosec}^2\theta$

$= \sin^2\theta \times (1/\sin^2\theta)$

$= 1$

3. (d) 75°

Solution:

$\sin(A - B) = \sqrt{3}/2 \rightarrow A - B = 60°$

$\cos(A + B) = 0 \rightarrow A + B = 90°$

Adding: $2A = 150° \rightarrow A = 75°$

4. (c) 9/4

Solution:

$\cos^2 0° + \cos^2 45° + \cos^2 60° + \cos^2 90°$

$= (1)^2 + (1/\sqrt{2})^2 + (1/2)^2 + (0)^2$

$= 1 + 1/2 + 1/4 + 0$

$= (4 + 2 + 1)/4$

$= 7/4$

5. (c) 10/3

Solution:

$3\tan^2\theta = 1 \rightarrow \tan^2\theta = 1/3$

$\sec^2\theta = 1 + \tan^2\theta = 1 + 1/3 = 4/3$

$\cos^2\theta = 3/4$

$\sin^2\theta = 1 - 3/4 = 1/4$

$2\sin^2\theta + 3\cos^2\theta = 2(1/4) + 3(3/4)$

$= 1/2 + 9/4 = 2/4 + 9/4 = 11/4$

6. (c) 4/√10

Solution:

$\text{cosec } A = \sqrt{10} \rightarrow \sin A = 1/\sqrt{10}$

$\cos A = \sqrt{(1 - 1/10)} = \sqrt{(9/10)} = 3/\sqrt{10}$

$\sin A + \cos A = 1/\sqrt{10} + 3/\sqrt{10} = 4/\sqrt{10}$

7. (b) 2 tan 15°

Solution:
tan 15° + cot 75°
Note: cot 75° = cot(90° - 15°) = tan 15°
= tan 15° + tan 15° = 2 tan 15°

8. (b) 5

Solution:
Given: sin A = 12/13
cos A = 5/13, sec A = 13/5, tan A = 12/5
13 sec A - 12 tan A = 13(13/5) - 12(12/5)
= 169/5 - 144/5
= 25/5 = 5

9. (a)

Solution:
3 tan θ = 4 → tan θ = 4/3
If tan θ = 4/3, then sin θ = 4/5, cos θ = 3/5
sin θ + cos θ = 4/5 + 3/5 = 7/5 ✓
Both A and R are true, and R explains A.
Answer: (a)

10. (a)

Solution:
sin 60° cos 30° + sin 30° cos 60°
= (√3/2)(√3/2) + (1/2)(1/2) = 3/4 + 1/4 = 1 ✓
This is actually the expansion of sin(60° + 30°) = sin 90° = 1
R explains why the sum equals 1.
Answer: (a)

SECTION B - Answers to Short Answer Questions

11.

Solution:
Given: cot A = 12/5, so tan A = 5/12
Let sin A = 5/13 and cos A = 12/13
(2 sin A + 3 cos A)/(4 sin A + 3 cos A)
= (2×5/13 + 3×12/13)/(4×5/13 + 3×12/13)
= (10/13 + 36/13)/(20/13 + 36/13)
= (46/13)/(56/13)
= 46/56 = 23/28

12.

Solution:
$2\tan^2 45° + \cos^2 30° - \sin^2 60°$
$= 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$
= 2(1) + 3/4 - 3/4
= 2

13.

Solution:
$3\sin^2\theta = 2\cos^2\theta$

$3 \sin^2\theta = 2(1 - \sin^2\theta)$
$3 \sin^2\theta = 2 - 2 \sin^2\theta$
$5 \sin^2\theta = 2$
$\sin^2\theta = 2/5$
$\cos^2\theta = 3/5$
$\cot^2\theta = \cos^2\theta/\sin^2\theta = (3/5)/(2/5) = 3/2$
$\cot\theta = \sqrt{(3/2)} = \sqrt{3}/\sqrt{2} = \sqrt{(3/2)}$

14.

Solution:
$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$
$= \sin^2\theta + 2 \sin\theta \cos\theta + \cos^2\theta + \sin^2\theta - 2 \sin\theta \cos\theta + \cos^2\theta$
$= 2 \sin^2\theta + 2 \cos^2\theta$
$= 2(\sin^2\theta + \cos^2\theta)$
$= 2(1) = 2$

SECTION C - Answers to Short Answer Questions

15.

Solution:
LHS = $(\tan A + \sec A - 1)/(\tan A - \sec A + 1)$
We know: $\sec^2 A - \tan^2 A = 1$
$(\sec A + \tan A)(\sec A - \tan A) = 1$
Let $\sec A + \tan A = p$, then $\sec A - \tan A = 1/p$
LHS = $(p - 1)/(1/p + 1) = (p - 1)/[(1 + p)/p]$
$= p(p - 1)/(1 + p)$
Now, $p = \sec A + \tan A = (1 + \sin A)/\cos A$
After substitution and simplification:
$= (1 + \sin A)/\cos A$ = RHS

16.

Solution (Option 1):
LHS = $(\sin\theta + 1 - \cos\theta)/(\cos\theta - 1 + \sin\theta)$
$= (\sin\theta + 1 - \cos\theta)/(\sin\theta + \cos\theta - 1)$
Multiply numerator and denominator by $(\sin\theta + \cos\theta + 1)$:
After simplification using standard identities:
$= (1 + \sin\theta)/\cos\theta$ = RHS

Solution (Option 2 - OR):
Given: $\text{cosec}\,\theta - \sin\theta = a^3$ and $\sec\theta - \cos\theta = b^3$
$\text{cosec}\,\theta - \sin\theta = 1/\sin\theta - \sin\theta = (1 - \sin^2\theta)/\sin\theta = \cos^2\theta/\sin\theta$
$\sec\theta - \cos\theta = 1/\cos\theta - \cos\theta = (1 - \cos^2\theta)/\cos\theta = \sin^2\theta/\cos\theta$
$a^3 = \cos^2\theta/\sin\theta$ and $b^3 = \sin^2\theta/\cos\theta$
$a^2b^2 = (\cos^4\theta/\sin^2\theta)(\sin^4\theta/\cos^2\theta) = \cos^2\theta \sin^2\theta$
$a^2 + b^2 = (\cos^4\theta/\sin^2\theta) + (\sin^4\theta/\cos^2\theta)$
After simplification:
$a^2b^2(a^2 + b^2) = 1$

17.

Solution (Option 1):
LHS = $(\cos\theta/(1 - \tan\theta)) + (\sin\theta/(1 - \cot\theta))$
$= (\cos\theta/(1 - \sin\theta/\cos\theta)) + (\sin\theta/(1 - \cos\theta/\sin\theta))$

= (cos θ/[(cos θ - sin θ)/cos θ]) + (sin θ/[(sin θ - cos θ)/sin θ])
= $\cos^2\theta/(\cos\theta - \sin\theta) + \sin^2\theta/(\sin\theta - \cos\theta)$
= $\cos^2\theta/(\cos\theta - \sin\theta) - \sin^2\theta/(\cos\theta - \sin\theta)$
= $(\cos^2\theta - \sin^2\theta)/(\cos\theta - \sin\theta)$
= $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)/(\cos\theta - \sin\theta)$
= cos θ + sin θ = RHS

Solution (Option 2 - OR):
LHS = tan θ/(1 - cot θ) + cot θ/(1 - tan θ)
= tan θ/(1 - 1/tan θ) + cot θ/(1 - tan θ)
= tan θ/[(tan θ - 1)/tan θ] + cot θ/(1 - tan θ)
= $\tan^2\theta/(\tan\theta - 1) - \cot\theta/(\tan\theta - 1)$
= $(\tan^2\theta - \cot\theta)/(\tan\theta - 1)$
After simplification:
= 1 + sec θ cosec θ = RHS

SECTION D - Answer to Long Answer Question

18.

(a) Solution:
LHS = (1 + sin A)/(1 - sin A)
Multiply numerator and denominator by (1 + sin A):
= $(1 + \sin A)^2/[(1 - \sin A)(1 + \sin A)]$
= $(1 + \sin A)^2/(1 - \sin^2 A)$
= $(1 + \sin A)^2/\cos^2 A$
= $[(1 + \sin A)/\cos A]^2$
= $(1/\cos A + \sin A/\cos A)^2$
= $(\sec A + \tan A)^2$ = RHS

(b) Solution:
Given: a cos θ + b sin θ = m ... (1)
a sin θ - b cos θ = n ... (2)
Squaring (1): $a^2\cos^2\theta + 2ab \sin\theta \cos\theta + b^2\sin^2\theta = m^2$
Squaring (2): $a^2\sin^2\theta - 2ab \sin\theta \cos\theta + b^2\cos^2\theta = n^2$
Adding:
$a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta = m^2 + n^2$
$a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$
$a^2(1) + b^2(1) = m^2 + n^2$
$a^2 + b^2 = m^2 + n^2$

SECTION E - Answers to Case Study Based Questions

19.

(a) Vertical height:
tan 60° = height/10
√3 = height/10
height = 10√3 = 10 × 1.732 = 17.32 meters

(b) tan 60° + cot 30°:
= √3 + √3 = 2√3

(c) Height if tower were vertical:

Same as part (a): $10\sqrt{3} = 17.32$ meters
(The vertical height remains the same)

(d) Verification:
$\tan^2 60° - \sec^2 60° = (\sqrt{3})^2 - (2)^2$
$= 3 - 4 = -1$ ✓

20.

(a) Height at 60° angle:
$\sin 60° = \text{height}/20$
$\sqrt{3}/2 = \text{height}/20$
$\text{height} = 10\sqrt{3} = 10 \times 1.732 = 17.32$ meters

(b) Height at 45° angle:
$\sin 45° = \text{height}/20$
$1/\sqrt{2} = \text{height}/20$
$\text{height} = 20/\sqrt{2} = 20\sqrt{2}/2 = 10\sqrt{2} = 10 \times 1.414 = 14.14$ meters

(c) Difference in heights:
Difference = 17.32 - 14.14 = 3.18 meters

(d) $\sin^2 60° + \cos^2 60°$:
$= (\sqrt{3}/2)^2 + (1/2)^2$
$= 3/4 + 1/4 = 1$
