UNIQUE STUDY POINT

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Class: X	**Subject:** Mathematics	**Session:** 2024-25
Chapter: 09 - Some Applications of Trigonometry	**Time:** 1½ Hours	**Max. Marks:** 40

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A comprises of 10 MCQs of 1 mark each.
4. Section B comprises of 4 questions of 2 marks each.
5. Section C comprises of 3 questions of 3 marks each.
6. Section D comprises of 1 question of 5 marks.
7. Section E comprises of 2 Case Study Based questions of 4 marks each.
8. Use of Calculators is not permitted.

SECTION A - Multiple Choice Questions (1 mark each)

1. The length of shadow of a pillar is 1/√3 times its height. The angle of elevation of the sun is:

(a) 30°
(b) 45°
(c) 60°
(d) 75°

2. The tops of two poles of height 16 m and 10 m are connected by a wire. If the wire makes an angle 30° with horizontal, then the length of wire is:

(a) 8 m
(b) 10 m
(c) 12 m
(d) 14 m

3. A tower is 50√3 m high. The angle of elevation of its top from a point 50 m away from its foot is:

(a) 30°
(b) 45°
(c) 60°
(d) 90°

4. From a point P on the ground, the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. The length of the flagstaff is:

(a) 10√3 m
(b) 10(√3 - 1) m

(c) $10\sqrt{3} - 10$ m

(d) $10 - 10/\sqrt{3}$ m

5. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall, then the height of the wall is:

(a) 7.5 m

(b) $7.5\sqrt{3}$ m

(c) $15\sqrt{3}$ m

(d) $15/\sqrt{3}$ m

6. Two poles are 25 m and 15 m high, and the line joining their tops makes an angle of 45° with the horizontal. The distance between the poles is:

(a) 5 m

(b) 10 m

(c) 15 m

(d) 20 m

7. From the top of a tower h m high, angles of depression of two objects which are in line with the foot of the tower are α and β ($\beta > \alpha$). The distance between the two objects is:

(a) $h(\tan \alpha - \tan \beta)$

(b) $h(\cot \alpha - \cot \beta)$

(c) $h(\tan \beta - \tan \alpha)$

(d) $h(\cot \beta - \cot \alpha)$

8. A vertical pole and a vertical tower are on the same level ground. From the top of the pole, the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. The height of the tower is how many times the height of the pole?

(a) 2

(b) 3

(c) 4

(d) 5

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

9. Assertion (A): Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, the distance between their tops is 13 m.

Reason (R): In a right triangle with legs a and b and hypotenuse c, we have $c^2 = a^2 + b^2$.

10. Assertion (A): If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}:1$, then the angle of elevation of the sun has measure 60°.

Reason (R): $\tan 60° = \sqrt{3}$

SECTION B - Short Answer Questions (2 marks each)

11. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30° with the ground. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

12. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

13. From a point on the ground the angles of elevation of the bottom and top of a water tank on top of a building are 45° and 60° respectively. If the height of the water tank is 20 m, find the height of the building.

14. A bird is sitting on the top of an 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remains at a constant height. After 2 seconds, the angle of elevation of the bird is 30°. Find the speed of the bird.

SECTION C - Short Answer Questions (3 marks each)

15. An observer 1.5 m tall is 20.5 m away from a tower 22 m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

16. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 10 seconds, the angle of elevation changes to 30°. If the jet is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet fighter in km/h.

17. From the top of a building AB, 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find the height of the lamp post and the distance between the building and the lamp post.

SECTION D - Long Answer Question (5 marks)

18. From the top of a lighthouse, the angles of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the lighthouse is h metres and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $h(\cot \alpha + \cot \beta)$ metres.

SECTION E - Case Study Based Questions (4 marks each)

19. Two pillars of equal height stand on either side of a roadway which is 150 m wide. At a point on the roadway between the pillars, the elevations of the tops of the pillars are 60° and 30°.

Based on the above information, answer the following questions. (Take $\sqrt{3} = 1.732$)

(i) Find the position of the point from the first pillar. (2 marks)

(ii) Find the height of the pillars. (2 marks)

OR

(ii) Find the distance from the point to the top of the nearer pillar. (2 marks)

20. In a park, four poles are standing at positions A, B, C and D around a circular fountain such that the cloth joining the poles in order covers the fountain. At a particular time of the day, the shadow of a pole of height 9 m is $9\sqrt{3}$ m.

Based on the above information, answer the following questions:

(i) What is the angle of elevation of the sun at that time? (1 mark)

(ii) If another pole in the same park is 6 m tall, what will be the length of its shadow at the same time? (2 marks)

OR

(ii) At what time of day is the shadow equal to the height of pole? (2 marks)

(iii) What is the distance from the top of the 9 m pole to the tip of its shadow? (1 mark)

DETAILED ANSWER KEY - PAPER 14

SECTION A - Answers to MCQs

1. Answer: (c) 60°

Explanation: Shadow = height/√3

tan θ = height/shadow = height/(height/√3) = √3

θ = 60°

2. Answer: (c) 12 m

Explanation: Height difference = 16 - 10 = 6 m

sin 30° = 6/length → 1/2 = 6/length

length = 12 m

3. Answer: (c) 60°

Explanation: Height = 50√3 m, Distance = 50 m

tan θ = 50√3/50 = √3

θ = 60°

4. Answer: (c) 10√3 - 10 m

Explanation: Building = 10 m

At 30°: tan 30° = 10/d → d = 10√3 m

At 45°: tan 45° = (10 + flag)/d → 10 + flag = 10√3

flag = 10√3 - 10 m

5. Answer: (a) 7.5 m

Explanation: Ladder = 15 m, angle with wall = 60°

sin 60° = height/15 (angle with wall means 30° with ground)

Actually: cos 60° = height/15 → 1/2 = height/15

height = 7.5 m

6. Answer: (b) 10 m

Explanation: Height difference = 25 - 15 = 10 m

tan 45° = 10/distance → 1 = 10/distance

distance = 10 m

7. Answer: (b) h(cot α - cot β)

Explanation: tan α = h/d_1 → d_1 = h cot α

tan β = h/d_2 → d_2 = h cot β

Distance between = $d_1 - d_2 = h(\cot \alpha - \cot \beta)$

8. Answer: (c) 4

Explanation: Let pole = h, tower = H, distance = d

$\tan 30° = h/d \rightarrow d = h\sqrt{3}$

$\tan 60° = (H - h)/d \rightarrow \sqrt{3} = (H - h)/(h\sqrt{3})$

$3h = H - h \rightarrow H = 4h$

9. Answer: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Explanation: Height difference = 11 - 6 = 5 m

Horizontal distance = 12 m

Distance between tops = $\sqrt{(5^2 + 12^2)} = \sqrt{(25 + 144)} = \sqrt{169} = 13$ m ✓

Reason correctly explains the formula used.

10. Answer: (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Explanation: $\tan \theta$ = height/shadow = $\sqrt{3}/1 = \sqrt{3}$

$\theta = 60°$ ✓

Reason correctly states $\tan 60° = \sqrt{3}$

SECTION B - Answers to Short Answer Questions

11. Solution:

Distance = 8 m, Angle = 30°

Let broken part = L, standing part = h

$\cos 30° = 8/L \rightarrow \sqrt{3}/2 = 8/L \rightarrow L = 16/\sqrt{3}$ m

$\sin 30° = h/L \rightarrow 1/2 = h/(16/\sqrt{3}) \rightarrow h = 8/\sqrt{3}$ m

Total height = $h + L = 8/\sqrt{3} + 16/\sqrt{3} = 24/\sqrt{3} = 8\sqrt{3}$ m

Height of tree = $8\sqrt{3}$ m ≈ 13.86 m

12. Solution:

Tower = 60 m, let building = h, distance = d

From building: $\tan 60° = 60/d \rightarrow d = 60/\sqrt{3} = 20\sqrt{3}$ m

From tower: $\tan 30° = h/d \rightarrow h = d/\sqrt{3} = 20\sqrt{3}/\sqrt{3} = 20$ m

Height of building = 20 m

13. Solution:

Tank height = 20 m, let building = h, distance = d

At 45°: tan 45° = h/d → d = h

At 60°: tan 60° = (h + 20)/d → √3 = (h + 20)/h

h√3 = h + 20 → h(√3 - 1) = 20

h = 20/(√3 - 1) = 20(√3 + 1)/2 = 10(√3 + 1) ≈ 27.32 m

Height of building ≈ 27.32 m

14. Solution:

Tree height = 80 m

At 45°: d_1 = 80 m

At 30°: d_2 = 80√3 m

Distance = 80√3 - 80 = 80(√3 - 1) ≈ 58.56 m

Speed = 58.56/2 = 29.28 m/s

Speed ≈ 29.28 m/s or 105.4 km/h

SECTION C - Answers to Short Answer Questions

15. Solution:

Observer height = 1.5 m, distance = 20.5 m, tower = 22 m

Height from eye level = 22 - 1.5 = 20.5 m

tan θ = 20.5/20.5 = 1

θ = 45°

Angle of elevation = 45°

16. Solution:

Height = 3600√3 m

At 60°: d_1 = 3600√3/√3 = 3600 m

At 30°: d_2 = 3600√3 × √3 = 10800 m

Distance = 10800 - 3600 = 7200 m

Time = 10 seconds

Speed = 7200/10 = 720 m/s = 2592 km/h

Speed = 2592 km/h

17. Solution:

Building AB = 60 m, let lamp post CD = h, distance = d

At 60° (bottom of lamp): tan 60° = 60/d → d = 60/√3 = 20√3 m

At 30° (top of lamp): tan 30° = (60 - h)/d

1/√3 = (60 - h)/(20√3) → 20 = 60 - h → h = 40 m

Height of lamp post = 40 m

Distance = 20√3 m ≈ 34.64 m

SECTION D - Answer to Long Answer Question

18. Solution:

Height of lighthouse = h m

Angles of depression = α and β

Let distances from lighthouse = d_1 and d_2

$\tan \alpha = h/d_1 \rightarrow d_1 = h/\tan \alpha = h \cot \alpha$

$\tan \beta = h/d_2 \rightarrow d_2 = h/\tan \beta = h \cot \beta$

Since ships are on opposite sides:

Distance between ships = $d_1 + d_2$

$= h \cot \alpha + h \cot \beta$

$= h(\cot \alpha + \cot \beta)$ metres

Hence proved

SECTION E - Answers to Case Study Based Questions

19. Solution:

Road width = 150 m, equal height pillars = h

Let point be at distance x from first pillar

(i) Position from first pillar:

$\tan 60° = h/x \rightarrow h = x\sqrt{3}$

$\tan 30° = h/(150 - x) \rightarrow h = (150 - x)/\sqrt{3}$

$x\sqrt{3} = (150 - x)/\sqrt{3} \rightarrow 3x = 150 - x$

$4x = 150 \rightarrow x = 37.5$ m

Position = 37.5 m from first pillar

(ii) Height of pillars:

$h = 37.5\sqrt{3} = 37.5 \times 1.732 = 64.95$ m

Height = 64.95 m

OR

(ii) Distance to top of nearer pillar:

Distance = $\sqrt{(x^2 + h^2)} = \sqrt{(37.5^2 + 64.95^2)}$

$= \sqrt{(1406.25 + 4218.5)} = \sqrt{5624.75} = 75$ m

Distance = 75 m

20. Solution:

Pole height = 9 m, shadow = 9√3 m

(i) Angle of elevation:

tan θ = 9/(9√3) = 1/√3

θ = 30°

Angle of elevation = 30°

(ii) Shadow of 6 m pole:

At same time, angle = 30°

tan 30° = 6/shadow → 1/√3 = 6/shadow

shadow = 6√3 m = 10.39 m

Shadow = 6√3 m ≈ 10.39 m

OR

(ii) When shadow equals height:

tan θ = height/shadow = height/height = 1

θ = 45°

This occurs when sun is at 45° elevation

Typically around 10-11 AM or 3-4 PM depending on location and season

(iii) Distance from top to tip:

Distance = √(9² + (9√3)²) = √(81 + 243) = √324 = 18 m

Distance = 18 m
