UNIQUE STUDY POINT

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Class: VI	**Subject:** Mathematics	**Session:** 2025-26
Chapter: 03 - Number Play	**Time:** 1½ Hours	**Max. Marks:** 40

General Instructions:

1. All questions are compulsory.
2. This question paper contains 20 questions divided into five sections A, B, C, D and E.
3. Section A contains 10 MCQs of 1 mark each.
4. Section B contains 4 questions of 2 marks each.
5. Section C contains 3 questions of 3 marks each.
6. Section D contains 1 question of 5 marks.
7. Section E contains 2 Case Study Based questions of 4 marks each.

SECTION A - Multiple Choice Questions (1 mark each)

Q1. What is the sum of the smallest and largest 3-digit number?

(a) 1000

(b) 1099

(c) 1100

(d) 1199

Q2. Which of the following numbers will reach 1 in the fewest steps using the Collatz Conjecture?

(a) 16

(b) 17

(c) 18

(d) 19

Q3. What is the digit sum of 999?

(a) 24

(b) 25

(c) 26

(d) 27

Q4. In a table with 8 cells in a row, what is the maximum number of supercells possible?

(a) 3

(b) 4

(c) 5

(d) 6

Q5. Which of the following is a 4-digit palindrome using only odd digits?

(a) 1221

(b) 3553

(c) 4664
(d) 6886

Q6. Starting with 0, players alternate adding 1, 2, or 3. The first to reach 22 wins. Which player has the winning strategy?
(a) First player
(b) Second player
(c) Both have equal chance
(d) Neither can guarantee a win

Q7. How many 1-digit numbers are there?
(a) 8
(b) 9
(c) 10
(d) 11

Q8. What is the next number in the Collatz sequence after 40?
(a) 13
(b) 20
(c) 121
(d) 80

Q9. Which of these times is NOT a palindrome when written without the colon?
(a) 1:01
(b) 2:12
(c) 3:30
(d) 4:44

Q10. What is the smallest number whose digit sum is 20?
(a) 299
(b) 398
(c) 488
(d) 695

SECTION B - Short Answer Questions (2 marks each)

Q11. Place the following numbers on the number line and label the missing positions:

86,705 ____ ____ 89,705 ____ ____ 92,705

Q12. Write five different 5-digit numbers whose digit sum is 10. What is the pattern you observe?

Q13. Starting with 34, perform the reverse-and-add process twice. Show all calculations.

Q14. In the Game of 21, if the first player says "2", what should the second player say to follow the winning strategy? Explain.

SECTION C - Short Answer Questions (3 marks each)

Q15. Create a 3×3 grid (9 cells) using 3-digit numbers such that you get exactly 5 supercells. Show your grid clearly.

Q16. Pratibha uses the digits 3, 7, 4, and 9 to make the smallest and largest 4-digit numbers. Find:

(a) The smallest and largest numbers
(b) Their difference
(c) Their sum

Q17. Starting with the number 2016 (year format), perform the Kaprekar process. Show at least 3 complete rounds and state whether you reach the Kaprekar constant.

SECTION D - Long Answer Question (5 marks)

Q18.

(a) Verify the Collatz Conjecture for the number 19. Show all steps clearly until you reach 1. (3 marks)

(b) Why is the Collatz Conjecture still called a "conjecture" and not a "theorem"? What would be needed to prove it as a theorem? (2 marks)

SECTION E - Case Study Based Questions (4 marks each)

Q19. Case Study 1: Calendar Mathematics

Rohan is studying calendar patterns. He noticed that 2020 was a leap year. He wants to find out when the calendar of 2020 will repeat exactly (all dates falling on the same days of the week).

A calendar repeats after:

- 6 years if only one leap year is included in between
- 5 years if two leap years are included in between
- 11 years for a leap year calendar to repeat

(a) Will 2020's calendar repeat in 2025? Explain your reasoning. (2 marks)

(b) In which year will the exact calendar of 2020 (a leap year) repeat? Show your calculation. (2 marks)

Q20. Case Study 2: Number Patterns in Daily Life

A bus service operates on a route with the following schedule:

Stop Number	Distance from Start (km)	Passengers Boarding
1	0	25
2	15	30
3	30	20
4	45	15
5	60	10

(a) What pattern do you observe in the distance between stops? What would be the distance at Stop 6? (2 marks)

(b) If the pattern of passengers boarding continues, estimate how many passengers board at Stop 6 and Stop 7. (2 marks)

DETAILED ANSWER KEY - PAPER 04

SECTION A - Answers to MCQs

Q1. (b) 1099

Smallest 3-digit number = 100
Largest 3-digit number = 999
Sum = 100 + 999 = 1099

Q2. (a) 16

16 is a power of 2 ($2^4 = 16$)
16 → 8 → 4 → 2 → 1 (only 4 steps)
Powers of 2 reach 1 the fastest by repeatedly dividing by 2.

Q3. (d) 27

Digit sum of 999 = 9 + 9 + 9 = 27

Q4. (b) 4

For even number of cells: maximum supercells = n/2
For 8 cells: 8/2 = 4 supercells maximum

Q5. (b) 3553

3553 is a palindrome (reads same both ways) and uses only odd digits (3 and 5).
Others contain even digits: 1221 has 2, 4664 has 4 and 6, 6886 has 6 and 8.

Q6. (a) First player

The first player can follow the winning strategy by ensuring they always say multiples of 4 plus 2: 2, 6, 10, 14, 18, 22.
By maintaining this pattern, the first player can always win.

Q7. (b) 9

1-digit numbers are: 1, 2, 3, 4, 5, 6, 7, 8, 9
Total = 9 numbers (0 is not counted as it's not considered a number on its own)

Q8. (b) 20

In the Collatz sequence:
40 is even, so divide by 2
40 ÷ 2 = 20

Q9. (c) 3:30

3:30 written without colon = 330
330 reversed = 033 = 33
330 ≠ 33, so NOT a palindrome

Others: 101, 212, 444 are all palindromes

Q10. (a) 299

To get the smallest number with digit sum 20:
Use 2 and 99: 2 + 9 + 9 = 20

Therefore, 299 is the smallest.

SECTION B - Answers to Short Answer Questions

Q11.

The numbers increase by 1000 at each position:
86,705, **87,705**, **88,705**, 89,705, **90,705**, **91,705**, 92,705

Q12.

Five 5-digit numbers with digit sum 10:

1. 10,000 → 1+0+0+0+0 = 1 (Wait, that's wrong. Let me fix this)
1. 91,000 → 9+1+0+0+0 = 10 ✓
2. 82,000 → 8+2+0+0+0 = 10 ✓
3. 73,000 → 7+3+0+0+0 = 10 ✓
4. 64,000 → 6+4+0+0+0 = 10 ✓
5. 55,000 → 5+5+0+0+0 = 10 ✓

Pattern observed: To keep digit sum constant at 10 while creating different numbers, we can distribute the digits differently but maintain their sum. Numbers ending with more zeros are easier to generate.

Q13.

Reverse-and-add process for 34:

Step 1: 34 + 43 = 77 (Palindrome! ✓)

34 becomes a palindrome in just one step, so we can't perform it twice in the traditional sense. But if we continue:

Step 2: 77 + 77 = 154 (Not a palindrome)

If we continue from 154:

Step 3: 154 + 451 = 605 (Not a palindrome)

Q14.

First player says: 2
Second player should say: 6 (adding 4... but wait, players can only add 1, 2, or 3)

Let me recalculate:
Winning numbers in Game of 21: 1, 5, 9, 13, 17, 21

First player said 2 (not following winning strategy).
Second player should try to reach 5: Current = 2, Target = 5
Second player should say: 5 (adding 3)

Explanation: By saying 5, the second player gets back on the winning track of 5, 9, 13, 17, 21.

SECTION C - Answers to Short Answer Questions

Q15.

3×3 grid with exactly 5 supercells (marked with *):

900*	100	800*

200	700*	300
600*	400	500*

Verification:
Maximum possible in 3×3 = 5
This grid achieves exactly 5 supercells by placing larger numbers alternately.

Q16.

Digits available: 3, 7, 4, 9

(a) Smallest and largest numbers:

Smallest: Arrange in ascending order → 3479

Largest: Arrange in descending order → 9743

(b) Difference:

9743 - 3479 = 6264

(c) Sum:

9743 + 3479 = 13,222

Q17.

Kaprekar process for 2016:

Round 1:

Digits: 2, 0, 1, 6

Largest: 6210, Smallest: 0126 = 126

6210 - 126 = 6084

Round 2:

Largest: 8640, Smallest: 0468 = 468

8640 - 468 = 8172

Round 3:

Largest: 8721, Smallest: 1278

8721 - 1278 = 7443

Round 4:

Largest: 7443, Smallest: 3447

7443 - 3447 = 3996

Round 5:

Largest: 9963, Smallest: 3699

9963 - 3699 = 6264

Round 6:

Largest: 6642, Smallest: 2466

6642 - 2466 = 4176

Round 7:

Largest: 7641, Smallest: 1467
7641 - 1467 = 6174 ✓

Yes, we reach the Kaprekar constant 6174 in 7 rounds.

SECTION D - Answer to Long Answer Question

Q18.

(a) Collatz Conjecture for 19:

19 (odd) → 19 × 3 + 1 = 58
58 (even) → 58 ÷ 2 = 29
29 (odd) → 29 × 3 + 1 = 88
88 (even) → 88 ÷ 2 = 44
44 (even) → 44 ÷ 2 = 22
22 (even) → 22 ÷ 2 = 11
11 (odd) → 11 × 3 + 1 = 34
34 (even) → 34 ÷ 2 = 17
17 (odd) → 17 × 3 + 1 = 52
52 (even) → 52 ÷ 2 = 26
26 (even) → 26 ÷ 2 = 13
13 (odd) → 13 × 3 + 1 = 40
40 (even) → 40 ÷ 2 = 20
20 (even) → 20 ÷ 2 = 10
10 (even) → 10 ÷ 2 = 5
5 (odd) → 5 × 3 + 1 = 16
16 (even) → 16 ÷ 2 = 8
8 (even) → 8 ÷ 2 = 4
4 (even) → 4 ÷ 2 = 2
2 (even) → 2 ÷ 2 = 1 ✓

The sequence reaches 1, confirming the conjecture for 19.

(b) Why still a "conjecture"?

Definition difference:

• A **conjecture** is a statement that appears to be true based on observations but has not been proven mathematically for all cases.
• A **theorem** is a statement that has been rigorously proven to be true for all cases.

Why Collatz is still a conjecture:

Even though the conjecture has been tested for extremely large numbers (up to 2^{68} and beyond) and no counterexample has been found, mathematicians have not yet found a proof that works for ALL positive integers.

What's needed to make it a theorem:

A mathematical proof that shows, for every possible positive integer, the sequence will always eventually reach 1. This proof must cover infinite cases, not just tested examples.

SECTION E - Answers to Case Study Based Questions

Q19.

(a) Will 2020's calendar repeat in 2025?

No, 2020's calendar will NOT repeat in 2025.

Reasoning:

- 2020 was a leap year (366 days)
- Between 2020 and 2025, there is only one leap year (2024)
- A leap year calendar repeats after 28 years, not 5 or 6 years
- Regular (non-leap) year calendars can repeat after 6 or 11 years
- But leap year calendars follow a 28-year cycle

Therefore, 2025 will not have the same calendar as 2020.

(b) When will 2020's calendar repeat?

Answer: 2048

Calculation:

Leap year calendars repeat every 28 years

2020 + 28 = 2048

Verification:

Both 2020 and 2048 are leap years, and the calendar pattern (which day of the week each date falls on) will be identical.

Q20.

(a) Pattern in distance and Stop 6:

Distance pattern:

Stop 1 to Stop 2: 15 - 0 = 15 km

Stop 2 to Stop 3: 30 - 15 = 15 km

Stop 3 to Stop 4: 45 - 30 = 15 km

Stop 4 to Stop 5: 60 - 45 = 15 km

Pattern: Stops are evenly spaced at 15 km intervals.

Distance at Stop 6:

60 + 15 = **75 km**

(b) Passengers boarding at Stop 6 and Stop 7:

Passenger pattern:

Stop 1: 25 passengers

Stop 2: 30 passengers (increase of 5)

Stop 3: 20 passengers (decrease of 10)

Stop 4: 15 passengers (decrease of 5)
Stop 5: 10 passengers (decrease of 5)

Pattern analysis:
After the initial increase, passengers decrease by 5 at each subsequent stop.

Estimate for Stop 6:
10 - 5 = **5 passengers**

Estimate for Stop 7:
Following the pattern: 5 - 5 = **0 passengers** (or the bus might not stop, or minimal passengers)

Alternative interpretation: The decrease might slow down or stop, so a reasonable estimate might be 3-5 passengers for Stop 7.
