UNIQUE STUDY POINT

PA 1 PRACTISE PAPER BY SUMEET SAHU

Class 10 - Mathematics

1. Prove that $\sqrt{3}$ is an irrational number. **[2]**
2. Find HCF of 81445 and 687897. **[2]**
3. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number. **[2]**
4. Prove that $3\sqrt{2}$ is irrational **[2]**
5. Two alarm clocks ring their alarms at regular intervals of 20 minutes and 25 minutes respectively. If they first beep together at 12 noon, at what time will they beep again together next time? **[2]**
6. Prove that (3 - $\sqrt{5}$) is irrational. **[3]**
7. Show that 3 + 5$\sqrt{2}$ is an irrational number. **[3]**
8. Define HCF of two positive integers and find the HCF of the pair of numbers: 475 and 495. **[3]**
9. Maya has two pieces of cloth. One piece is 36 inches wide and the other piece is 24 inches wide. She wants to cut both pieces into strips of equal width that are as wide as possible. How wide should she cut the strips? **[3]**
10. Find the LCM of the following polynomials: $x\left(8x^3+27\right)$ and $2x^2\left(2x^2+9x+9\right)$ **[3]**
11. The product of three consecutive positive integers is divisible by 6. Is this statement true or false? Justify your answer. **[5]**
12. **Read the following text carefully and answer the questions that follow:** **[4]**

    To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

    i. What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B? (1)
    ii. If the product of two positive integers is equal to the product of their HCF and LCM is true then, the HCF (32, 36) is (1)
    iii. 7 × 11 × 13 × 15 + 15 is a (2)

    OR

    If p and q are positive integers such that p = ab^2 and q = a^2b, where a, b are prime numbers, then the LCM (p, q) is (2)
13. Richa is an artificial jewellery seller. She buys them from a dealer at a price based on the presence or absence of stones as shown in the table below. **[4]**

Item	With stones	Without stones
Ring	110	70
Earring pair	170	90
Bangle pair	90	120

i. For every ring with stones, two rings without stones are purchased by Richa's customers. Richa buys rings accordingly from the dealer for ₹ 10000. How many rings does she buy?

ii. In her next visit to the dealer, Richa finds that dealer has increased the price of rings with stones by ₹ 10. Richa still wants to spend ₹ 10000 on rings. What is the reduction in the number of rings she buys?

a. 10
b. 20
c. 40
d. 50

iii. The dealer increases the price of one type of earrings. Richa buys earrings (with stones and without stones) in the ratio of 5: 9 before the price hike. After the price hike, she buys earrings in the ratio of 8 : 13 with the same amount of money. Does she buy more earrings with stones than earrings without stones after the price hike? Give reasons.

iv. Richa purchases 37 pairs of bangles with stones and 33 pairs of bangles without stones. She divides them into two sets. Set 1 contains 40 pairs of bangles, and set 2 contains 30 pairs of bangles. What is the difference between the number of bangle pairs with stones in set 1 and the number of bangle pairs without stones in set 2?

a. 4
b. 7
c. 10
d. 30

v. Despite the price hike by the dealer, Richa increases the number of jewellery with stones she purchases for her customers. What could be the reason for her decision?

14. **Assertion (A):** H.C.F. of 12 and 77 is 1. **[1]**

Reason (R): L.C.M. of two coprime numbers is equal to their product.

a) Both A and R are true and R is the correct explanation of A.

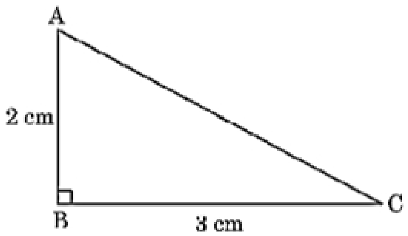
b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

15. **Assertion (A):** The perimeter of △ABC is a rational number. **[1]**

Reason (R): The sum of the squares of two rational numbers is always rational.



a) Both A and R are true and R is the correct explanation of A.

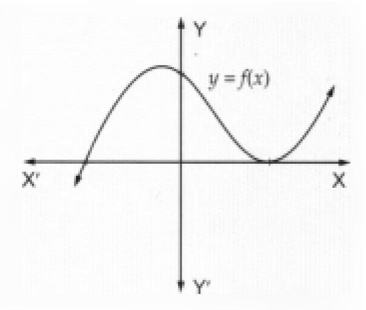
b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.
d) A is false but R is true.

16. **Assertion (A):** H.C.F. of smallest prime and smallest composite is 2. **[1]**
Reason (R): Smallest prime is 2 and smallest composite is 4 so their H.C.F. is 2.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

17. Find the number of real zeros of the polynomial y = f(x), having graph as shown in Fig. **[2]**



18. If α and β are the zeroes of a polynomial x^2- $4\sqrt{3}$ x + 3, then find the value of $\alpha + \beta - \alpha\beta$ **[2]**

19. For what value of k, is -2 a zero of the polynomial $3x^2$ + 4x + 2k? **[2]**

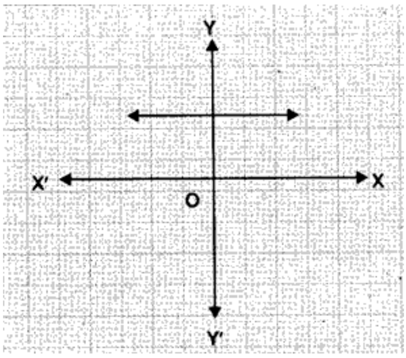
20. Find the zeroes of the polynomial $4x^2$ - 4x + 1 and verify the relationship between the zeroes and the coefficients. **[2]**

21. Zeroes of the quadratic polynomial x^2 - 3x + 2 are α and β. Construct a quadratic polynomial whose zeroes are 2 α + 1 and 2β + 1. **[2]**

22. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 +bx + c, then evaluate: $\alpha^2\beta + \alpha\beta^2$ **[2]**

23. If α and β are the zeros of the quadratic polynomial f(x) = ax^2 +bx + c, then evaluate: $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$. **[2]**

24. For a polynomial p(x), the graph of y = p(x) is given below. Find the number of zeroes of p(x). **[2]**



25. If 2 and 3 are zeroes of polynomial $3x^2$ - 2kx + 2m, find the values of k and m. **[2]**

26. Obtain the zeros of the quadratic polynomial $\sqrt{3}x^2 - 8x + 4\sqrt{3}$ and verify relation between its zeros and coefficients. **[2]**

27. If the coefficient of x in a quadratic polynomial is zero, then prove that zeros will be equal in magnitude and opposite in sign. **[3]**

28. Find the zeroes of the quadratic polynomial $7y^2$ - $\frac{11}{3}$y - $\frac{2}{3}$ and verify the relationship between the zeroes and the coefficients. **[3]**

29. Find the zeroes of the polynomial $2s^2$ + (1 + $2\sqrt{2}$)s + $\sqrt{2}$ by factorisation method and verify the relationship between the zeroes and coefficient of the polynomial. **[3]**

30. Find the zeros of f(x) = x^2 - 2x - 8 and verify the relationship between the zeros and its coefficients. **[3]**

31. Find the zeroes of the quadratic polynomial $3x^2 - 2$ and verify the relationship between the zeroes and the coefficients. **[3]**

32. If one root of the quadratic polynomial $2x^2 - 3x + p$ is 3, find the other root. Also, find the value of p. **[3]**

33. Find a quadratic polynomial whose sum and product of the zeroes are $-\frac{21}{8}$ and $\frac{5}{16}$ respectively. Also find the zeroes of the polynomial by factorisation. **[3]**

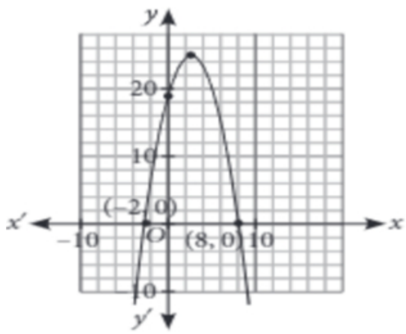
34. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $\frac{1}{3}$, respectively. **[3]**

35. Find the zeros of f(s) = $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$ and verify the relationship between the zeros and its coefficients. **[5]**

36. Find the zeros of q(y) = $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeros and its coefficients. **[5]**

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Rachna and her husband Amit who is an architect by profession, visited France. They went to see Mont Blanc Tunnel which is a highway tunnel between France and Italy, under the Mont Blanc Mountain in the Alps, and has a parabolic cross-section. The mathematical representation of the tunnel is shown in the graph.



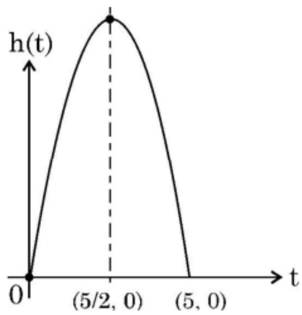
i. What will be the expression of the polynomial given in diagram? (1)
ii. What is the value of the polynomial, represented by the graph, when x = 4? (1)
iii. If the tunnel is represented by $-x^2 + 3x - 2$. Then what is its zeroes? (2)

OR

What is sum of zeros and product of zeros for $-x^2 + 3x - 2$? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A ball is thrown in the air so that t seconds after it is thrown, its height h metre above its starting point is given by the polynomial $h = 25t - 5t^2$.



Observe the graph of the polynomial and answer the following questions:

i. Write zeroes of the given polynomial. **(1)**
ii. Find the maximum height achieved by ball. **(1)**
iii. a. After throwing upward, how much time did the ball take to reach to the height of 30 m? **(2)**

OR

b. Find the two different values of t when the height of the ball was 20 m. **(2)**

39. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 **[2]**

km by rickshaw, and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

40. Solve the pair of linear equations by the substitution method: x + y = 14; x - y = 4. **[2]**

41. Solve for x and y: **[2]**

$3x - \frac{y+7}{11} + 2 = 10, 2y + \frac{x+11}{7} = 10$

42. Solve the following pair of linear equations by the substitution method: x + *y* = 14, *x* – *y* = 4 **[2]**

43. Find whether the lines represented by 2x + y = 3 and 4x + 2y = 6 are parallel, coincident or intersecting **[2]**

44. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the pair of linear equations are consistent, or inconsistent: 2x − 3y = 8, 4x − 6y = 9. **[2]**

45. Solve the pair of equations x = 3 and y = -4 graphically. **[2]**

46. Solve for x and y: x + y = 6, 2x - 3y = 4 **[2]**

47. 37 pens and 53 pencils together cost Rs 955, while 53 pens and 37 pencils together cost Rs 1115. Find the cost of a pen and that of a pencil. **[2]**

48. Solve: ax + by = a - b **[2]**

bx - ay = a + b

49. The area of a rectangle remains the same if the length is increased by 7 metres and the breadth is decreased by 3 metres. The area remains unaffected if the length is decreased by 7 metres and breadth is increased by 5 metres. Find the dimensions of the rectangle. **[3]**

50. Graph the following pairs of equations. State whether the equations are consistent, inconsistent or dependent. x + 2y = 3; 2x + 4y = 8 **[3]**

51. A man has only 20 paisa coins and 25 paisa coins in his purse. If he has 50 coins in all totalling to ₹ 11.25, how many coins of each kind d38196oes he have? **[3]**

52. Solve the system of equations graphically: **[3]**

3x - 4y =7

5x + 2y =3

Shade the region between the lines and the y-axis

53. The sum of digits of a two digit number is 15. The number obtained by reversing the order of digits of the given number exceeds the given number by 9. Find the given number. **[3]**

54. Solve the system of the equation: **[3]**

99x + 101y = 499

101x + 99y = 501

55. A two-digit number is 4 times the sum of its digits. If 18 is added to the number, the digits are reversed. Find the number. **[3]**

56. A is elder to B by 2 years. A's father F is twice as old38196 as A and B is twice as old as his sister S. If the ages of the father and sister differ by 40 years, find the age of A. **[3]**

57. Solve the pair of linear equations $\frac{3x}{2} - \frac{5y}{3} = -2$ and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ by substitution method. **[5]**

58. Solve the following system of equations by substitution method. 2x – 7y = 1; 4x + 3y = 15. Also verify the solution. **[5]**

59. **Read the following text carefully and answer the questions that follow:** **[4]**

Lokesh, a production manager in Mumbai, hires a taxi everyday to go to his office. The taxi charges in Mumbai consists of a fixed charges together with the charges for the distance covered. His office is at a distance of 10 km from his home. For a distance of 10 km to his office, Lokesh paid ₹ 105. While coming back home, he took another route. He covered a distance of 15 km and the charges paid by him were ₹ 155.

i. What are the fixed charges? (1)
ii. What are the charges per km? (1)
iii. If fixed charges are ₹ 20 and charges per km are ₹ 10, then how much Lokesh have to pay for travelling a distance of 10 km? (2)

OR

Find the total amount paid by Lokesh for travelling 10 km from home to office and 25 km from office to home. [Fixed charges and charges per km are as in (i) & (ii). (2)

60. **Read the following text carefully and answer the questions that follow:** **[4]**

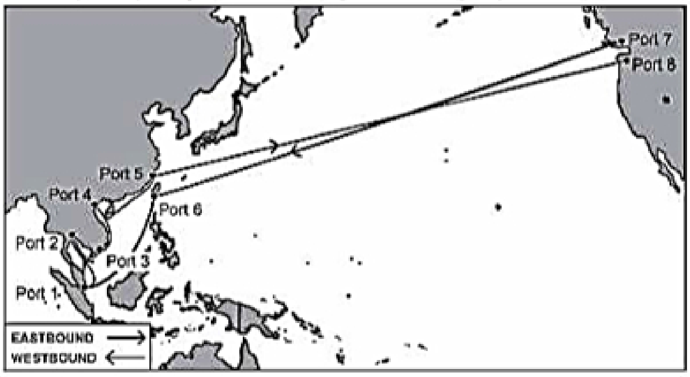
Two schools **P** and **Q** decided to award prizes to their students for two games of Hockey ₹ x per student and Cricket ₹ y per student. School **P** decided to award a total of ₹ 9,500 for the two games to 5 and 4 students respectively; while school **Q** decided to award ₹ 7,370 for the two games to 4 and 3 students respectively.

i. Represent the following information algebraically (in terms of x and y). (1)
ii. What is the prize amount for hockey? (1)
iii. Prize amount on which game is more and by how much? (2)

OR

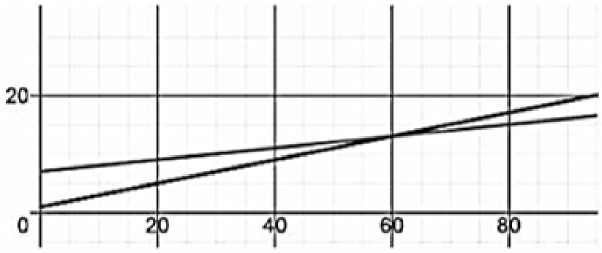
What will be the total prize amount if there are 2 students each from two games? (2)

61. Large cargo ships ferry cargo between ports. They take months to ferry them from one port to another. The distance in the sea is calculated in nautical miles (nm). The map shows the eastbound (grey) and westbound (black) cargo lines of a shipment company. **[4]**



The distance between port 6 and port 7 is approximately 20000 nm and a ship takes 95 days to travel between the ports in either direction.

The distance-time graph drawn by Pooja shows the journey of eastbound and westbound ships.



i. What do the two axes in the graph show?
ii. According to Pooja's graph, what is the approximate distance travelled by the eastbound ship during these 95 days?
iii. Anish looks at the graph and claims that an eastbound ship route meets the westbound route at 13000 nm. Is he correct? Give reasons.
iv. The eastbound and westbound journeys are of different length in 95 days. What can be the possible reason for it?

62. Solve for x: $\frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30}$, x # -4, 7 **[2]**

63. Find the roots of the quadratic equation $15x^2 - 10\sqrt{6}x + 10 = 0$. **[2]**

64. If the equation $kx^2 - 2kx + 6 = 0$ has equal roots, then find the value of k. **[2]**

65. Solve the quadratic equation $2x^2 - 5x - 1 = 0$ for x. **[2]**

66. For what value of m, the quadratic equation $mx^2 - 2(m - 1)x + (m + 2) = 0$ has two real and equal roots? **[2]**

67. The product of Shikha's age five years ago and her age 8 years later is 30, her age at both times being given in years. Find her present age. **[3]**

68. Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. **[3]**

69. The length of a rectangular park is 5 metres more than twice its breadth. If the area of the park is 250 sq m, find the length and breadth of the park. **[3]**

70. If a, b, c are real numbers such that $ac \neq 0$, then show that at least one of the equations $ax^2 + bx + c = 0$ and $-ax^2 + bx + c = 0$ has real roots. **[3]**

71. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream, than to return to the same point. Find the speed of the stream and total time of the journey. **[5]**

72. The difference of squares of two numbers is 204. The square of the smaller number is 4 less than 10 times the larger number. Find the two numbers. **[5]**

73. **Read the following text carefully and answer the questions that follow:** **[4]**

The tradition of pottery making in India is very old. In fact, it is older than Indus Valley Civilization. The shaping and baking of clay articles has continued through the ages. The picture of a potter is shown below:
A potter makes a certain number of pottery articles in a day. It was observed on a particular day the cost of production of each article(in ₹) was one more than twice the number of articles produced on that day. The total cost of production on that day was ₹ 210.

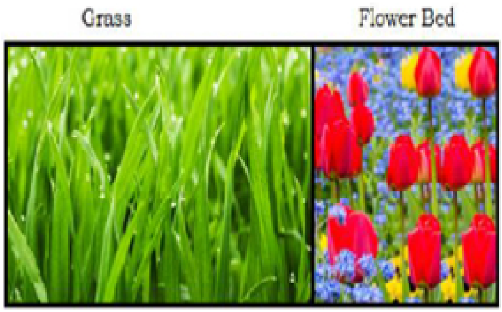
i. Taking number of articles produced on that day as x, form a quadratic equation in x. (1)
ii. Find the number of articles produced and the cost of each article. (1)
iii. Find the cost of production of 15 articles. (2)

OR

Find the number of articles made by Potter in a day if the total cost of production is ₹ 1575. (2)

74. **Read the following text carefully and answer the questions that follow:** **[4]**

Shreya has a field with a flowerbed and grassland. The grassland is in the shape of rectangle while flowerbed is in the shape of square. The length of the grassland is found to be 3 m more than twice the length of the flowerbed. Total area of the whole land is 1260 m^2.



i. If the length of the square is x m then find the total length of the field. (1)
ii. What will be the perimeter of the whole figure in terms of x? (1)
iii. Find the value of x if the area of total field is 1260 m^2. (2)

OR

Find area of grassland and the flowerbed separately. (2)

75. **Read the following text carefully and answer the questions that follow:** **[4]**

A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

i. Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information. **(1)**

ii. Write the corresponding quadratic equation in standard form. **(1)**

iii. a. Find the value of x, the length of side of a tile by factorisation. **(2)**

OR

b. Solve the quadratic equation for x, using quadratic formula. **(2)**

Solution

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Class 10 - Mathematics

1. Let $\sqrt{3}$ be a rational number.
   $\sqrt{3} = \frac{p}{q}$ p, q are coprime, q $\neq 0$
   $3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p$ Let p = 3 m
   $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$
   $\therefore$ 3 is common factor of p and q
   Contraction to our assumption
   Hence $\sqrt{3}$ is irrational No.
2. Two positive integers are 687897 and 81445.
   By applying Euclid's division lemma
   $687897 = 81445 \times 8 + 36337$
   $81445 = 36337 \times 2 + 8771$
   $36337 = 8771 \times 4 + 1253$
   $8771 = 1253 \times 7 + 0$
   $\therefore$ HCF = 1253
3. Let a and b are numbers and HCF = x
   Then LCM = 14x
   Now sum of HCF and LCM
   x+14x =600
   15x = 600
   x = 40
   Hence HCF=40 and LCM=14×40
   Given a=280 and b=?
   We know that
   a $\times$ b = HCF $\times$ LCM
   So b $= \frac{40 \times 14 \times 40}{280} = 2 \times 40 = 80$
   Hence the other number = 80
4. Let us assume, to the contrary, that $3\sqrt{2}$ is rational. Then, there exist co-prime positive integers a and b such that
   $3\sqrt{2} = \frac{a}{b}$
   $\Rightarrow \quad \sqrt{2} = \frac{a}{3b}$
   $\Rightarrow \quad \sqrt{2}$ is rational $\left[\because 3, a \text{ and } b \text{ are integers } \therefore \frac{a}{3b} \text{ is a rational number}\right]$
   This is a contradiction. Hence our assumption is wrong.
   So, $3\sqrt{2}$ is an irrational number.
5. LCM (20, 25) = 100
   $\therefore$ After 100 minutes from 12:00 noon
   $\Rightarrow$ They will beep again together at 1:40 pm
6. Let $3 - \sqrt{5} = \frac{p}{q}$
   $\therefore 3 - \sqrt{5} = \frac{P}{q}$ (where p and q are integers, co-prime and $q \neq 0$)
   $\Rightarrow \quad 3 - \frac{p}{q} = \sqrt{5}$
   $\Rightarrow \quad \frac{3q-p}{q} = \sqrt{5}$
   $(3q - p)$ and q are integers, so $\left(\frac{3q-p}{q}\right)$ is a rational number, but $\sqrt{5}$ is an irrational number. This contradiction arises because of our wrong assumption.
   So $(3 - \sqrt{5})$ is an irrational number
7. Let $3 + 5\sqrt{2}$ be rational and have only common factor 1.
   Let, $3 + 5\sqrt{2} = \frac{a}{b}$
   $5\sqrt{2} = \frac{a}{b} - 3$

$\sqrt{2} = \frac{a-3b}{5b}$

If $\frac{a-3b}{5b}$ is rational so, $\sqrt{2}$ is also rational number but it is not true as $\sqrt{2}$ is an irrational number.

So it is contradiction to our assumption,

Therefore, 3 + 5$\sqrt{2}$ is an irrational number.

8. **HCF (highest common factor) :** The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.

   We need to find H.C.F of 475 and 495.

   By applying Euclid's Division lemma, we have

   495 = 475 $\times$ 1 + 20.

   Since remainder ≠ 0, apply division lemma on 475 and remainder 20

   475 = 20 $\times$ 23 + 15.

   Since remainder ≠ 0, apply division lemma on 20 and remainder 15

   20 = 15 $\times$ 1 + 5.

   Since remainder ≠ 0, apply division lemma on 15 and remainder 5

   15 = 5 $\times$ 3+ 0.

   Therefore, H.C.F. of 475 and 495 = 5

9. This problem can be solved using H.C.F. because we are cutting or "dividing" the strips of cloth into smaller pieces of 36 and 24 and we are looking for the widest possible strips .

   So,

   H.C.F. of 36 and 24 is 12

   So we can say that

   Maya should cut each piece to be 12 inches wide.

10. $P(x) = x\left(8x^3 + 27\right)$

    $= x(2x+3)\left(4x^2 - 6x + 9\right)$ Using identity $a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)$

    $Q(x) = 2x^2\left(2x^2 + 6x + 3x + 9\right)$

    $= 2 \times x^2[2x(x+3) + 3(x+3)]$

    $= 2 \times x^2 \times (x+3)(2x+3)$

    Common on factors: x, $(2x+3)$

    Uncommon on factors: $\left(4x^2 - 6x + 9\right)$ and $2, x, (x+3)$

    $\therefore$ LCM of $P(x)$ and $Q(x) = 2 \times x^2(x+3)(2x+3)\left(4x^2 - 6x + 9\right)$

11. Let three consecutive numbers be x, (x + 1) and (x + 2)

    Let x = 6q + r $0 \leq r < 6$

    $\therefore x = 6q, 6q+1, 6q+2, 6q+3, 6q+4, 6q+5$

    product of $x(x+1)(x+2) = 6q(6q+1)(6q+2)$

    if x = 6q then which is divisible by 6

    if $x = 6q+1$

    $= (6q+1)(6q+2)(6q+3)$

    $= 2(3q+1).3(2q+1)(6q+1)$

    $= 6(3q+1) \cdot (2q+1)(6q+1)$

    which is divisible by 6

    if x = 6q + 2

    $= (6q+2)(6q+3)(6q+4)$

    $= 3(2q+1).2(3q+1)(6q+4)$

    $= 6(2q+1) \cdot (3q+1)(6q+1)$

    Which is divisible by 6

    if $x = 6q+3$

    $= (6q+3)(6q+4)(6q+5)$

    $= 6(2q+1)(3q+2)(6q+5)$

    which is divisible by 6

    if $x = 6q+4$

    $= (6q+4)(6q+5)(6q+6)$

    $= 6(6q+4)(6q+5)(q+1)$

    which is divisible by 6

if x = 6q + 5
$= (6q+5)(6q+6)(6q+7)$
$= 6(6q+5)(q+1)(6q+7)$
which is divisible by 6
$\therefore$ the product of any three natural numbers is divisible by 6.

12. i. The number of students in Section A is 32, and the number of students in Section B is 36.
Step 1: Find the prime factors of each number:
32 = $2 \times 2 \times 2 \times 2 \times 2$
36 = $2 \times 2 \times 3 \times 3$
Step 2: Identify the common and uncommon prime factors. The common ones are 2×2.
Step 3: Multiply the common and uncommon prime factors together to get the LCM:
LCM = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$ = 288
So, the minimum number of books needed to be acquired for the class library is 288, so they can be distributed equally among students of Section A or Section B.
ii. Step 1: Find the prime factors of each number:
32 = $2 \times 2 \times 2 \times 2$
36 = $2 \times 2 \times 3 \times 3$
Step 2: Identify the common prime factors and their minimum exponent:
The common prime factors are 2×2.
Step 3: Calculate the HCF by multiplying the common prime factors:
HCF = 2×2 = 4
So, the HCF of 32 and 36 is 4.
iii. Given number (7 $\times$ 11 $\times$ 13 $\times$ 15 + 15)
It can also be written as 15 (7 $\times$ 11 $\times$ 13 +1).
As it is a product of two composite numbers
hence it is a composite number.
OR
Given:
p = ab^2
q = a^2b
Take the highest power of each prime factor:
LCM = $a^2 \times b^2$
So, the LCM of p and q is a^2b^2.

13. i. 120
ii. (b) 20
iii. Yes, 8 : 13 > 5 :19, thus she had bought more earrings with stones after the price hike
iv. (b) 7
v.
- The demand for jewelry with stones among customers increases.
- People prefer to buy jewelry with stones.

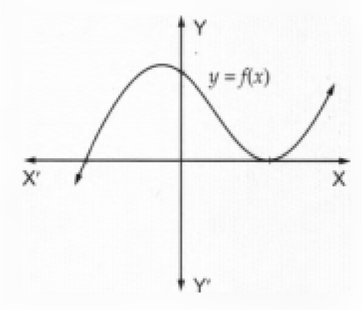
14.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

15.
(d) A is false but R is true.
Explanation:
A is false but R is true.

16. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:

Smallest prime is 2 and smallest composite is 4 so H.C.F. of 2 and 4 is 4.

17. A real number α is a zero of polynomial $f(x)$, if $f(\alpha) = 0$



In the above figure the curve intersects x-axis at one point and touches at one point.

Hence the number of real zeroes is 2.

18. we have $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}x+3$

then, $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow \alpha + \beta = -\frac{(-4\sqrt{3})}{1}$

$\alpha + \beta = 4\sqrt{3}$

Now, $\alpha\beta = \frac{c}{a}$

$\Rightarrow \alpha\beta = \frac{3}{1}$

$\Rightarrow \alpha\beta = 3$

$\therefore \alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$

19. Let $f(x)=3x^2 + 4x +2k$

If -2 is zero of f(x) then f(-2)=0

$\Rightarrow 3 \times (-2)^2 + 4 \times -2 + 2k = 0$

$\Rightarrow 12 - 8 + 2k = 0$

$\Rightarrow 4 + 2k = 0$

$\Rightarrow 2k = -4$

$\Rightarrow k = \frac{-4}{2}$

$\Rightarrow k = -2$

20. $p(x) = 4x^2 - 4x + 1 = (2x - 1) (2x - 1)$

$\therefore$ Zeroes are $\frac{1}{2}$ and $\frac{1}{2}$

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{(4)} = \frac{-\text{ Coeff. of } x}{\text{Coeff. of } x^2}$

Product of zeroes = $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} = \frac{\text{Constan term}}{\text{Coeff. of } x^2}$

21. $p(x) = x^2 - 3x + 2$

α, β are its zeroes

$\therefore \alpha + \beta = \frac{-b}{a} = 3$

$\alpha\beta = 2$

Required sum of zeroes = $(2\alpha + 1) + (2\beta + 1) = 2 (\alpha + \beta) + 2 = 8$

Required product of zeroes = $(2\alpha + 1) (2\beta + 1) = 4\alpha\beta + 2 (\alpha + \beta) +1$

$= 4 \times 2 + 2 \times 3 +1 = 15$

Required quadratic polynomial is $k (x^2 - 8x + 15)$ or $x^2 - 8x + 15$

22. α,β are zeros of ax^2+bx+c

Then $\alpha+\beta= -\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$\alpha^2\beta + \alpha\beta^2$

$= \alpha\beta (\alpha + \beta)$

$= \frac{c}{a}\left(\frac{-b}{a}\right)$

$= \frac{-bc}{a^2}$.

23. α,β are zeros of ax^2+bx+c

Then $\alpha+\beta= -\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$

$= \left[\frac{1}{\alpha} + \frac{1}{\beta}\right] - 2\alpha\beta$

$= \left[\frac{\alpha+\beta}{\alpha\beta}\right] - 2\alpha\beta$ ....(1)

$= \frac{-\frac{b}{a}}{\frac{c}{a}} - \frac{2c}{a} = -\frac{b}{c} - \frac{2c}{a} = -\frac{ab+2c^2}{ca}$.

24. There is no zero as the graph does not intersect the x-axis at any point.

25. Here P(x) =$3x^2$ - 2kx + 2m

Since, 2 is a zero of p(x), then

P(2) = $3(2)^2$ - 4k + 2m = 12 - 4k + 2m=0

12 - 4k + 2m = 0

4k - 2m = 12-----(1)

Since, 3 is a zero of p(x), then

P(3) = $3(3)^2$ - 2$\times$3k + 2m = 0

$\Rightarrow$ 27 - 6k + 2m = 0

$\Rightarrow$ 6k - 2m = 27.........(2)

On subraction of (1) from(2) we get

2k=15,k=$\frac{15}{2}$

From (1)

2(2k)-2m=12

2$\times$15-2m=12

2m=30-12=18

Hence m=9,k=$\frac{15}{2}$

26. We have

$f(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3} = \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$

$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$= (x - 2\sqrt{3})(\sqrt{3}x - 2)$

$\therefore f(x) = 0, if\ (x - 2\sqrt{3})(\sqrt{3}x - 2) = 0$

$\Rightarrow (x - 2\sqrt{3}) = 0$ or $(\sqrt{3}x - 2) = 0$

$\Rightarrow x = 2\sqrt{3}$ or $x = \frac{2}{\sqrt{3}}$

So, the zeros of f(x) are $2\sqrt{3}$ and $\frac{2}{\sqrt{3}}$

Verification:

Sum of zeros $= \left(2\sqrt{3} + \frac{2}{\sqrt{3}}\right) = \frac{8}{\sqrt{3}} = \frac{-(\text{ coefficient of } x)}{(\text{coefficient of } x^2)}$

product of zeros $= \left(2\sqrt{3} \times \frac{2}{\sqrt{3}}\right) = \frac{4\sqrt{3}}{\sqrt{3}} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

27. Consider general quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$

b = 0 (given)

Let α, β be the zeroes of p(x)

$\therefore$ Sum of zeroes = $\alpha + \beta = -\frac{b}{a} = \frac{0}{a} = 0$

$\Rightarrow \alpha + \beta = 0$

$\Rightarrow \alpha = -\beta$

In other words $\beta = -\alpha$

$\therefore$ The zeroes are α, - α.

Hence, the zeroes are equal in magnitude but opposite in sign.

28. p(y) = $7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}[(7y + 1)(3y - 2)]$

$\therefore$ Zeroes are $\frac{2}{3}, -\frac{1}{7}$

Sum of Zeroes = $\frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a}$ = $\frac{11}{21}$
$\therefore$ sum of zeroes = $\frac{-b}{a}$
Product of Zeroes = ($\frac{2}{3}$)(-$\frac{-1}{7}$) = -$\frac{2}{21}$
$\frac{c}{a}$ = - $\frac{2}{3}$($\frac{1}{7}$) = -$\frac{2}{21}$
$\therefore$ Product = $\frac{c}{a}$

29. $2s^2 + (1+2\sqrt{2})s + \sqrt{2}$
$= 2s^2 + s + 2\sqrt{2}s + \sqrt{2}$
$= s(2s+1) + \sqrt{2}(2s+1)$
$= (2s+1)(s+\sqrt{2})$
$\Rightarrow s = -\frac{1}{2}, -\sqrt{2}$ are zeroes of the polynomial.
Sum of zeroes = $-[\frac{1}{2} + \sqrt{2}] = -\frac{1+2\sqrt{2}}{2}$
Also, $\frac{-b}{a} = -\frac{1+2\sqrt{2}}{2}$
$\Rightarrow$Sum of zeroes = $\frac{-b}{a}$
Product of zeroes = $\frac{-1}{2} \times -\sqrt{2} = \frac{\sqrt{2}}{2}$
and $\frac{c}{a} = \frac{\sqrt{2}}{2}$
$\Rightarrow$Product of zeroes = $\frac{c}{a}$

30. f(x) = x^2 - 2x - 8
= x^2 - 4x + 2x - 8
= x(x - 4) + 2(x - 4)
= (x + 2) (x - 4)
f(x) = 0 if x+2 = 0 or x-4 = 0
x = -2 or 4
So the zeroes of the polynomials are -2 and 4.
For the Polynomial f(x)=x^2 - 2x - 8
a=1,b=-2, c=-8
Sum of the zeroes = - 2 + 4 = 2 = -$\frac{b}{a}$
Product of zeros =(-2)(4) = -8 = $\frac{c}{a}$
Hence, the relationship between the zeros and coefficients is verified.

31. Here, p(x) = $3x^2$ - 2.
Now p(x) = 0
$\Rightarrow \quad 3x^2 - 2 = 0$
$\Rightarrow 3x^2 = 2$
$\Rightarrow \quad x^2 = \frac{2}{3}$
$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$
Therefore, zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.
If p(x) = $3x^2$ - 2 , then a = 3, b = 0 and c = -2
Now, sum of zeroes = $\sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right)$= 0 .... (i)
Also, $\frac{-b}{a} = \frac{-0}{3} = 0$ .... (ii)
From (i) and (ii)
Sum of zeroes$=\frac{-b}{a}$
and product of zeroes = $\sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3}$ .... (iii)
Also, $\frac{c}{a} = \frac{-2}{3}$ .... (iv)
From (iii) and (iv)
Product of zeroes = $\frac{c}{a}$

32. The given quadratic polynomial is p(x) = $2x^2$ - 3x + p
Since, 3 is a root (zero) of p(x)
$\Rightarrow$ $2(3)^2$ - 3 $\times$ 3 + p = 0
$\Rightarrow$18 - 9 + p = 0

$\Rightarrow$9 + p = 0

$\Rightarrow$ p = - 9

Now p(x) = $2x^2$ - 3x - 9

= $2x^2$- 6x + 3x - 9

= 2x (x - 3) + 3 (x - 3)

= (x - 3) (2x + 3)

For roots of polynomial, p(x) = 0

$\Rightarrow$(x - 3) (2x + 3) = 0

$\Rightarrow$ x = 3 or x = -$\frac{3}{2}$

Hence the other root is -$\frac{3}{2}$.

33. We know, quadratic polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

Given, Sum of zeroes = $-\frac{21}{8}$ and Product of zeroes = $\frac{5}{16}$

$\therefore$ Quadratic Polynomial = $x^2+\frac{21}{8}x+\frac{5}{16}$

= $\frac{1}{16}(16x^2+42x+5)$

$\Rightarrow$ Quadratic polynomial is $16x^2$ + 42x + 5

Now, we rewrite the polynomial as $16x^2+2x+40x+5$

= $2x\cdot(8x+1)+5\cdot(8x+1)$

= $(2x+5)\cdot(8x+1)$

Now, for Zeros, $(8x+1)\cdot(2x+5)=0$

$\Rightarrow x=\frac{-1}{8},\frac{-5}{2}$

34. Let the polynomial be ax^2 + bx + c.

and its zeroes be α and β.

Then, $\alpha+\beta=\sqrt{2}=-\frac{b}{a}$ and $\alpha\beta=\frac{1}{3}=\frac{c}{a}$

If a = 3, then $b=-3\sqrt{2}$ and c = 1.

So, one quadratic polynomial which fits the given conditions is $3x^2-3\sqrt{2}x+1$.

It is given that $\alpha+\beta=\sqrt{2}$ and $\alpha\beta=\frac{1}{3}$

Now, standard form of quadratic polynomial is given by $x^2-(\alpha+\beta)x+\alpha\beta$

$=x^2-(\alpha+\beta)x+\alpha\beta$

$=x^2-\sqrt{2}x+\frac{1}{3}$

$=\frac{1}{3}\left(3x^2-3\sqrt{2}x+1\right)$

Hence the required quadratic polynomial is $3x^2-3\sqrt{2}x+1$

35. Here the given polynomial is

f(s) = $2s^2-(1+2\sqrt{2})s+\sqrt{2}$

$=s(2s-1)-\sqrt{2}(2s-1)$

$=(2s-1)(s-\sqrt{2})$

Hence f(s) = 0 if 2s - 1 = 0 or $s-\sqrt{2}=0$

$s=\frac{1}{2}$ or $s=\sqrt{2}$

Verification of the relation between α, β, a, b and c

$\alpha=\frac{1}{2},\beta=\sqrt{2}$,, a = 2, $b=-(1+2\sqrt{2})$, $c=\sqrt{2}$

$\alpha+\beta=\frac{-b}{a}$

$\Rightarrow \frac{1}{2}+\sqrt{2}=\frac{+(1+2\sqrt{2})}{2}$

$\Rightarrow \frac{1}{2}+\sqrt{2}=\frac{1}{2}+\frac{2\sqrt{2}}{2}$

$\Rightarrow \frac{1}{2}+\sqrt{2}=\frac{1}{2}+\sqrt{2}$

$\Rightarrow$ LHS = RHS

Now, $\alpha\times\beta=\frac{c}{a}$

$\Rightarrow \left(\frac{1}{2}\right)\left(\sqrt{2}\right)=\frac{\sqrt{2}}{2}$

$\Rightarrow \frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$

$\Rightarrow$ LHS = RHS

Hence verified.

36. Given quadratic polynomial is

f(y) = $7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of f(y), put f(y) = 0

$\Rightarrow 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow$ 21y^2 - 11y - 2 = 0

$\Rightarrow$ 21y^2 - 14y + 3y - 2 = 0 (by splitting the middle term method)

$\Rightarrow$ 7y(3y - 2) + 1(3y - 2) = 0

$\Rightarrow$ (3y - 2)(7y + 1) = 0

Therefore, either 3y - 2 = 0 or 7y + 1 = 0

$\Rightarrow y = \frac{2}{3}$ or $y = \frac{-1}{7}$

Now Verification of the relations between α, β ,a, b, and c:

We have $\alpha = \frac{2}{3},\ \beta = \frac{-1}{7},$ a = 7, $b = -\frac{11}{3},\ c = \frac{-2}{3}$

$\Rightarrow \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \left(\frac{2}{3}\right) - \frac{1}{7} = \frac{+\frac{11}{3}}{7}$

$\Rightarrow \frac{14-3}{21} = \frac{11}{3} \times \frac{1}{7}$

$\Rightarrow \frac{11}{21} = \frac{11}{21}$

$\Rightarrow$ LHS = RHS

Hence verified.

Also, we know that $\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow \left(\frac{2}{3}\right) \times \left(\frac{-1}{7}\right) = \frac{\frac{-2}{3}}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{3} \times \frac{1}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{21}$

$\Rightarrow$ LHS = RHS

Hence verified.

37. i. Zeroes are -2 and 8

$\alpha + \beta$ = -2 + 8 = 6

$\alpha\beta$ =-2 $\times$ 8 = -16

expression of polynomial

x^2 - $(\alpha + \beta)$x + $\alpha\beta$

x^2 - 6x - 16

ii. P(x) = x^2 - 6x - 16

P(4) = 4^2 - 6(4) - 16

= 16 - 24 - 16

= -24

iii. P(x) = -x^2 + 3x - 2

$\alpha + \beta = \frac{-3}{-1}$

$\alpha + \beta$ = 3 ...(i)

$\alpha\beta$ = $\frac{-2}{-1}$

$\alpha\beta$ = 2 ...(ii)

$(\alpha - \beta)^2 = (\alpha + \beta)^2$ - 4$\alpha\beta$

$(\alpha - \beta)^2$ = (3)^2 - 4(2)

$(\alpha - \beta)^2$ = 9 - 8

$\alpha - \beta = \pm\sqrt{1}$

$\alpha - \beta = \pm 1$

Taking

$\alpha - \beta = 1$

$\alpha + \beta = 3$

2α = 4

α = 2

Put α = 2 in, $\alpha - \beta = 1$
2 - β = 1
β = 1
OR
$\alpha + \beta$ = $\frac{-3}{-1}$ = 3
$\alpha\beta = \frac{-2}{-1} = 2$

38. i. Zeroes of the polynomial are 0 and 5
ii. Maximum height achieved by ball
$= 25 \times \frac{5}{2} - 5 \times \left(\frac{5}{2}\right)^2$
$= \frac{125}{4}$ or 31.25 m
iii. a. $-5t^2 + 25t = 30$
$\Rightarrow t^2 - 5t + 6 = 0$
$\Rightarrow$ (t - 2)(t - 3) = 0
$t \neq 3, t = 2$
OR
b. $-5t^2 + 25t = 20$
$\Rightarrow t^2 - 5t + 4 = 0$
$\Rightarrow$ (t - 4)(t - 1) = 0
$\Rightarrow$ t = 4, 1

39. Let the speed of the rickshaw and the bus are x and y km/h, respectively.
Now, she has taken time to travel 2 km by rickshaw, $t_1 = \frac{2}{x} hr$
Because $\left[\text{speed } = \frac{\text{distance}}{\text{time}}\right]$
She has taken time to travel remaining distance i.e., (14 - 2) = 12km
By bus, $t_2 = \frac{12}{y} hr$
By first condition,
$t_1 + t_2 = \frac{1}{2} = \frac{2}{x} + \frac{12}{y}$ ...(i)
Now, she has taken time to travel 4 km by rickshaw, $t_3 = \frac{4}{x} hr$
and she has taken time to travel remaining distance i.e., (14 - 4) = 10km,
by bus = $t_4 = \frac{10}{y} hr$
By second condition,
$t_3 + t_4 = \frac{1}{2} + \frac{9}{60}$
$\frac{4}{x} + \frac{10}{y} = \frac{13}{20}$ ...(ii)
Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ then Eqs. (i) and (ii) becomes
$2u + 12v = \frac{1}{2}$ ...(iii)
and $4u + 10v = \frac{13}{20}$ ...(iv)
On multiplying in Eq. (iii) by 2 and then subtracting equation (iv) from it, we get
$(4u + 24v) - (4u + 10v) = 1 - \frac{13}{20}$
$14v = \frac{7}{20}$
$v = \frac{1}{40}$
Now, put the value of v in Eq. (iii), we get
$2u + 12\left(\frac{1}{40}\right) = \frac{1}{2}$
$2u = \frac{2}{10}$
$u = \frac{1}{10}$
$x = \frac{1}{u} = \frac{10km}{hr}$
$y = \frac{1}{v} = 40km/hr$
Hence, the speed of rickshaw and the bus are 10 km/h and 40 km/h, respectively.

40. x + y = 14 ....(1)
x - y = 4 ....(2)
x = 4 + y from equation (2)

Putting this in equation (1), we get

4+y +y =14

⇒ 2y =10

⇒ y = 5

Putting value of y in equation (1), we get

x + 5 = 14

⇒ x = 14 - 5 = 9

Therefore, x =9 and y =5

41. Given, $3x - \frac{y+7}{11} + 2 = 10$

$\Rightarrow \frac{33x-(y+7)+22}{11} = 10$

$\Rightarrow 33x - y - 7 + 22 = 110$

$\Rightarrow 33x - y = 95$ ..(i)

Also, $2y + \frac{x+11}{7}$ = 10

$\Rightarrow \frac{14y+x+11}{7}$ = 10

$\Rightarrow 14y + x + 11 = 70$

$14y + x = 59$

$14y = 59 - x$

⇒ y = $\frac{59-x}{14}$

eq. (i) becomes

33x - $\left(\frac{59-x}{14}\right)$ = 95

$\Rightarrow \frac{462x-59+x}{14}$ = 95

⇒ 463x - 59 = 1330

⇒ x = 3

When x = 3 eq. (i) becomes

33(3) - y = 95

⇒ y =4

42. x + y = 14 …(1)

x – y = 4 … (2)

If x - y = 4

x = 4 + y .... (3)

Put (3) in (1) we get

4 + y + y = 14

⇒ 4 + 2y = 14

⇒ 2y = 10

⇒ y = 5

Putting value of y in equation (3), we get

x = 4 + y

⇒ x = 4 + 5 = 9

Therefore, x = 9 and y = 5

43. The given lines represented by 2x + y = 3 and 4x + 2y = 6

compare with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

Here $a_1 = 2, b_1 = 1, c_1 = -3$

and $a_2 = 4, b_2 = 2, c_2 = -6$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the lines are parallel.

Clearly $\frac{2}{4} = \frac{1}{2} = \frac{3}{6}$

Hence lines are coincident.

44. Given equations are as:

2x − 3y = 8

4x − 6y = 9

Comparing equation 2x − 3y = 8 with $a_1x + b_1y + c_1 = 0$

and 4x − 6y = 9 with

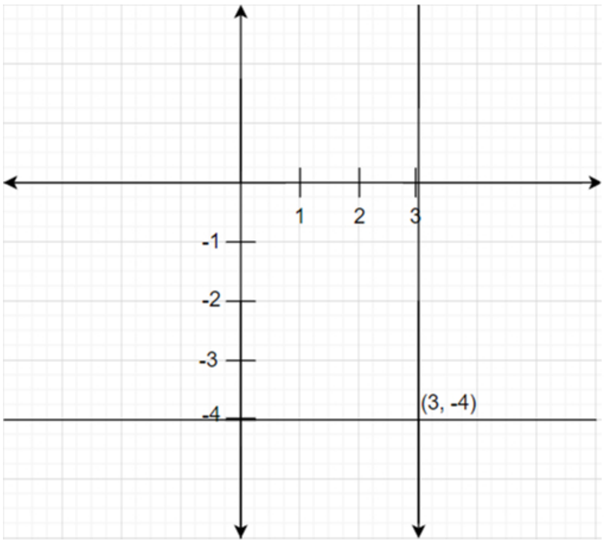
$a_2x + b_2y + c_2 = 0$,

We get, $a_1 = 2$, $b_1 = -3$, $c_1 = -8$, $a_2 = 4$, $b_2 = -6$, $c_2 = -9$

Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ because $\frac{2}{4} = \frac{-3}{-6} \neq \frac{-8}{-9} \Rightarrow \frac{1}{2} = \frac{1}{2} \neq \frac{-8}{-9}$

Therefore, equations have no solution because they are parallel.

Hence, they are inconsistent.

45. 

x = 3, corresponds to a line parallel to y-axis

y = -4, corresponds to a line parallel to x-axis

Solution of equation = (3, -4)

46. x + y = 6, ...(i)

2x - 3y = 4 ...(ii)

Multiplying by 3 in equation (i)

3x + 3y = 18 ...(iii)

Adding equation (ii) and (iii)

5x = 22

x = $\frac{22}{5}$

Putting in equation (i)

$y = 6 - \frac{22}{5} = \frac{30-22}{5} = \frac{8}{5}$

47. Let the cost of each pen be Rs x and that of a pencil be Rs y.

As per given condition cost of 37 pens and 53 pencils is Rs 955,

Then, 37x + 53y = 955 ..... (i)

And the cost of 53 pens and 37 pencils is Rs 1115.

And, 53x + 37y = 1115 ..... (ii)

Adding (i) and (ii), we get

(37x + 53y) + (53x + 37y) = 955 + 1115

$\Rightarrow$ 90x + 90y = 2070

$\Rightarrow$ 90(x + y) = 2070

$\Rightarrow$ x + y = 23 .... (iii)

On subtracting (i) from (ii), we get

(37x + 53y) - (53x + 37y) = 955 - 1115

$\Rightarrow$ 16x - 16y = 160

$\Rightarrow$ 16(x - y) = 160

$\Rightarrow$ x - y = 10. ...(iv)

Adding (iii) and (iv), we get

(x + y) + (x- y) = 23 + 10

$\Rightarrow$ x + y + x - y =33

$\Rightarrow$ 2x = 33

$\Rightarrow x = \frac{33}{2} = 16.50$

Subtracting (iv) from (iii), we get

(x + y) - (x- y) = 23 - 10

⇒ x + y - x + y = 13
⇒ 2y = 13
⇒ $x = \frac{33}{2} = 16.50$
∴ Cost of each pen = Rs 16.50 and cost of each pencil = Rs 6.50.

48. ax + by = a - b multiply by a
bx - ay = a + b multiply by b
Now, adding the resulting equations gives

$$\begin{array}{l} a^2x + aby = a^2 - ab \\ b^2x - aby = ab + b^2 \\ \hline (a^2+b^2)x = a^2 + b^2 \end{array}$$

⇒ x = 1
∴ a + by = a - b
⇒ by = -b
⇒ y = -1
∴ x = 1 and y = -1

49. Let us suppose that the length and breadth of the rectangle be x m and y m respectively.
Then, Area of rectangle = xy meter^2
Now, according to question if length is increased by 7m and the breadth is decreased by 3m, the area remains same
∴ xy = (x + 7)(y - 3)
⇒ xy = xy - 3x + 7y - 21
⇒ 3x - 7y = -21 ....(i)
Again, according to question when length is decreased by 7m and breadth is increased by 5m, then area remains unaffected
∴ xy = (x - 7)(y + 5)
⇒ xy = xy + 5x - 7y - 35
⇒ 35 = 5x - 7y
⇒ 5x - 7y = 35 ....(ii)
Subtracting equation (i) from (ii), we get
5x - 7y -(3x- 7y) = 35 - (-21)
or, 5x - 7y -3- + 7y = 35 + 21
⇒ 2x = 56
⇒ $x = \frac{56}{2} = 28$
Put the value of x = 28 in equation (ii), we get
5 × 28 - 7y = 35
⇒ 140 - 7y = 35
⇒ -7y = 35 - 140
⇒ -7y = -105
⇒ $y = \frac{105}{7} = 15$
Therefore, dimensions of the rectangle are 28m and 15m respectively.

50. x + 2y = 3 ⇒ $y = \frac{3-x}{2}$

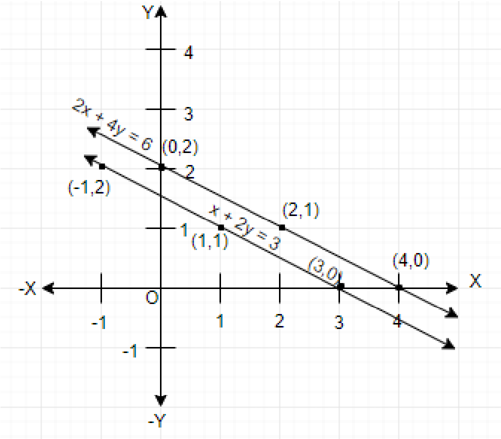
x	1	-1	3
y	1	2	0

Points are (1, 1), (-1, 2) and (3, 0)
2x + 4y = 8 ⇒ $y = \frac{8-2x}{4}$

x	0	2	4
y	2	1	0

Points are (0, 2), (2, 1) and (4, 0)
The graph of the two equations represent a pair of parallel lines.
There is no common solution.

Hence, the pair of equations is inconsistent.



51. Let the number of 20 paisa coins be x and that of 25 paisa coins be y. Then,

x + y = 50 .... (i)

Total value of 20 paisa coins = 20 x paisa

Total value of 25 paisa coins = 25 y paisa

∴20 x+25 y = 1125 ...(because Rs11.25 = 1125 paisa)

⇒ 4x + 5y = 225 ... (ii)

Thus, we get the following system of linear equations

x + y - 50 = 0

4x + 5y - 225 = 0

By using cross-multiplication, we have

$\frac{x}{-225+250} = \frac{-y}{-225+200} = \frac{1}{5-4}$

⇒ $\frac{x}{25} = \frac{y}{25} = \frac{1}{1}$ ⇒ x = 25 and y = 25

Hence, there are 25 coins of each kind.

52. $3x - 4y = 7 \; and \; 5x + 2y = 3$

The given system of linear equation is $3x - 4y = 7 \; and \; 5x + 2y = 3$

Now, $3x - 4y = 7$

$y = \frac{3x-7}{4}$

When x = 1 then, y = -1

When x = -3 then y = -4

x	1	-3
y	-1	-4

Now, 5x + 2y = 3

$y = \frac{3-5x}{2}$

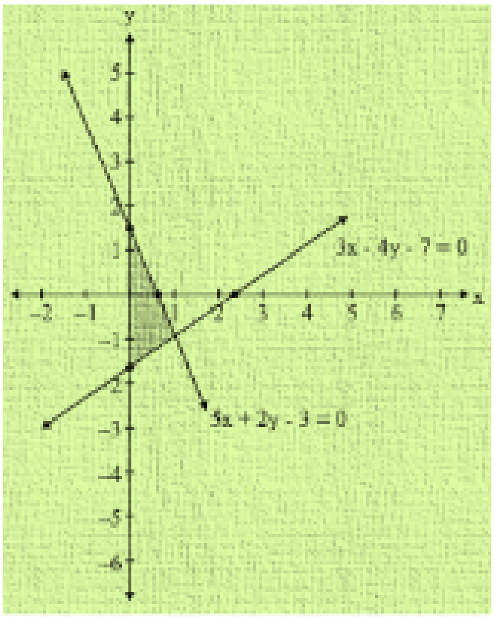
When x = 1 then, y = -1

When x = 3 then y = -6

Thus, we have the following table

x	1	3
y	-1	-6

Graph of the given system of equations are



Clearly the two lines intersect at A(1, -1)

Hence, x = 1 and y = -1 is the solution of the given system of equations.

53. Let the ten's digit of required number be x and its unit digit be y respectively.

Then, As per given condition

The sum of digits of a two digit number is 15.

x + y = 15 ....(i)

Required number = 10x + y

Number formed on reversing the digits = 10y + x

So, as per given condition, the number obtained by reversing the order of digits of the given number exceeds the given number by 9.

$\therefore$ 10y + x -(10x + y) = 9

$\therefore$ 10y + x - 10x - y = 9

9y - 9x = 9

- x + y = 1.............(ii)

Adding (i) and (ii), we get

$2y = 16$

$\Rightarrow y = \frac{16}{2} = 8$

Putting y = 8 in (i), we get

x + 8 = 15

x = 15 - 8 = 7

Number = 10x + y

= 10 $\times$ 7 + 8

= 70 + 8

= 78

Hence the given two digit number is 78.

54. Given, 99x + 101y = 499 ....(i)

101x + 99y = 501... (ii)

Adding eqn. (i) and (ii),

(99x + 101y) + (101x + 99y) = 499 + 501

99x + 101y + 101x + 99y = 1000

200x + 200y = 1000

x + y = 5 ...(iii)

Subtracting eqn. (ii) from eqn. (i), we get

(99x + 101y) - (101x + 99y) = 499 - 501

99x + 101y - 101x - 99y = -2

-2x + 2y = - 2

or, x - y= 1 .... (iv)
Adding equations (iii) and (iv)
x + y + x - y = 5 + 1
2x = 6
$\therefore$ x = 3
Substituting the value of x in eqn. (iii), we get
3 + y = 5
y = 2
Hence the value of x and y of given equation are 3 and 2 respectively.

55. Let us suppose that the digit at unit place be x
Suppose the digit at tens place be y.
Thus, the number is 10y + x.
According to question it is given that the number is 4 times the sum of the two digits.
Therefore, we have
10y + x = 4(x + y)
$\Rightarrow$ 10y + x = 4x + 4y
$\Rightarrow$ 4x + 4y - 10y - x = 0
$\Rightarrow$ 3x - 6y = 0
$\Rightarrow$ 3(x - 2y) = 0
$\Rightarrow$ x - 2y = 0
After interchanging the digits, the number becomes 10x + y.
Again according to question If 18 is added to the number, the digits are reversed.
Thus, we have
(10y + x) + 18 = 10x + y
$\Rightarrow$ 10x + y - 10y - x = 18
$\Rightarrow$ 9x - 9y = 18
$\Rightarrow$ 9(x - y) = 18
$\Rightarrow x - y = \frac{18}{9}$
$\Rightarrow$ x - y = 2
Therefore, we have the following systems of equations
x - 2y = 0 ....(1)
x - y = 2...............(2)
Here x and y are unknowns. Now let us solve the above systems of equations for x and y.
Subtracting the equation (1) from the (2), we get
(x - y) - (x - 2y) = 2 - 0
$\Rightarrow$ x - y - x + 2y = 2
$\Rightarrow$ y = 2
Now, substitute the value of y in equation (1), we get
x - 2 $\times$ 2 = 0
$\Rightarrow$ x - 4 = 0
$\Rightarrow$ x = 4
Therefore the number is 10 $\times$ 2 + 4 = 24
Thus the number is 24

56. Let age of A be A years, age of B be B years, age of F be F years and age of S be S years.
According to question we have
A = B + 2 ....(1)
F = 2A ......(2)
B = 2S ....(3)
F - S = 40 ....(4)
Substituting equations (2) & (3) in (4), we get
2A - $\frac{B}{2}$ = 40
$\Rightarrow$ 4A - B = 80 ....(5)
Solving eqn (1) & (5), we get

3A = 78

⇒ A = 26

57. $\frac{3x}{2} - \frac{5y}{3} = -2; \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

The given system of linear equation is

$\frac{3x}{2} - \frac{5y}{3} = -2 \quad \ldots\ldots\ldots\ldots (1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ .... (2)

⇒ $\quad 9x - 10y = -12$ .... (3)

2x + 3y = 13 ...... (4)

From equation (3)

9x - 10y = -12

9x = 10y - 12

$x = \frac{10y-12}{9}$

Substituting the value of y in equation (4), we get

$2\left(\frac{10y-12}{9}\right) + 3y = 13$

$20y - 24 + 27y = 117$

$47y = 117 + 24$

$y = \frac{141}{47}$

y = 3

Substituting the value of y in equation (4), we get

$2x + 3 \times 3 = 13$

$2x + 9 = 13$

$2x = 13 - 9$

$x = \frac{4}{2} = 2$

Therefore, the solution is

x = 2, y = 3

Verification, Substituting x = 2 and y = 3, we find that both the equations (1) and (2) are satisfied as shown below:

$\frac{3}{2}x - \frac{5y}{3} = \frac{3}{2}(2) - \frac{5}{3}(3) = 3 - 5 = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{2}{3} + \frac{3}{2} = \frac{13}{6}$

This verifies the solution.

58. The given system of equation is

2x - 7y = 1 .... (1)

4x + 3 y = 15 ...... (2)

From equation (1),

7y = 2x - 1

⇒ $\quad y = \frac{2x-1}{7}$ .... (3)

Substitute this value of y in equation (2), we get

$4x + 3\left(\frac{2x-1}{7}\right) = 15$

⇒ $\quad 28x + 6x - 3 = 105$

⇒ $\quad 34x - 3 = 105$

⇒ $\quad 34x = 105 + 3$

⇒ $\quad x = \frac{108}{34}$

⇒ $\quad x = \frac{54}{17}$

Substituting this value of x in equation (3), we get

$y = \frac{2\left(\frac{54}{17}\right)-1}{7} = \frac{\frac{108}{17}-1}{7} = \frac{108-17}{119}$

$= \frac{91}{119} = \frac{13}{17}$

Therefore the solution is $x = \frac{54}{17}, y = \frac{13}{17}$.

Verification, Substituting $x = \frac{54}{17}$ and, $y = \frac{13}{17}$ we find that both

the equation (1) and (2) are satisfied as shown below:

$2x - 7y = 2\left(\frac{54}{17}\right) - 7\left(\frac{13}{17}\right) = \frac{108}{17} - \frac{21}{17}$

$= \frac{108-91}{17} = \frac{17}{17} = 1$

$4x + 3y = 4\left(\frac{54}{17}\right) + 3\left(\frac{13}{17}\right) = \frac{216}{17} + \frac{39}{17}$

$= \frac{216+39}{17} = \frac{255}{17} = 15$ This verifies the solution.

59. i. Let the fixed charge be ₹ x and per kilometer charge be ₹ y

$\therefore$ x + 10y = 105 ...(i)

x + 15y = 155 ...(ii)

From (i) and (ii)

5y = 50

y = $\frac{50}{5}$ = 10

From equation (i)

x + 100 = 105

x = 105 - 100 = 5

Fixed charges = ₹ 5

ii. Let the fixed charge be ₹ x and per kilometer charge be ₹ y

$\therefore$ x + 10y = 105 ...(1)

x + 15y = 155 ...(2)

From (1) and (2)

5y = 50

y = $\frac{50}{5}$ = 10

From equation (1)

x + 100 = 105

x = 105 - 100 = 5

Per km charges = ₹ 10

iii. Let the fixed charge be ₹ a and per kilometer charge be ₹ b

a + 10b

20 + 10 $\times$ 10 = ₹ 120

OR

Total amount = x + 10y + x + 25y

= 2x + 35y

= 2 $\times$ 5 + 35 $\times$ 10

= 10 + 350

= ₹ 360

60. i. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

$\therefore$ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

ii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

$\therefore$ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

Multiply by 3 in equation (i) and by 4 in equation (ii)

15x + 12y = 28,500 ...(iii)

16x + 12y = 29480 ...(iv)

On subtracting equation (iii) from equation (iv), we get

x = 980

$\therefore$ Prize amount for hockey = ₹ 980

iii. Given, prize amount for Hockey ₹ x and ₹ y for cricket per student

$\therefore$ Algebraic equations are

5x + 4y = 9500 ...(i)

and 4x + 3y = 7370 ...(ii)

Now, put this value in equation (i), we get

5 $\times$ 980 + 4y = 9500

$\Rightarrow$ 4y = 9500 - 4900 = 4600

$\Rightarrow$ y = 1150

$\therefore$ Prize amount for cricket = ₹ 1150

Difference = 1150 - 980 = 170

$\therefore$ Prize amount for cricket is ₹ 170 more than hockey.

OR

Total prize amount for 2 students each from two games

= 2x + 2y

= 2(x + y)

= 2(980 + 1150)

= 2 $\times$ 2130

= ₹ 4260

61. i. Mentions that the y-axis represents distance with or without units (in 1,000 nm) and the x-axis represents the number of days
ii. Accept any number between 8000 and 9000 with or without units nm.
iii. No, a distance-time graph cannot be related to a route map.
iv. The relation between speed, time and distance
    - The cargo ships' speeds differ in the two routes.
    - The westbound cargo ships sail at greater speed.
    - The ocean current helps westbound ships to travel faster.

62. $\frac{1}{x+4}-\frac{1}{x+7}=\frac{11}{30}$

$\Rightarrow \frac{(x+7)-(x+4)}{(x+4)(x+7)}=\frac{11}{30}$

$\Rightarrow \frac{3}{x^2+4x+7x+28}=\frac{11}{30}$

$\Rightarrow \frac{3}{x^2+11x+28}=\frac{11}{30}$

$\Rightarrow$ $11x^2$ + 121x + 308 = 90

$\Rightarrow$ $11x^2$ + 121x + 218 = 0

$\Rightarrow$ $11(x^2$ + 11x + 18) = 0

$\Rightarrow$ x^2 + 11x + 18 = 0

$\Rightarrow$ x^2 + 9x + 2x + 18 = 0

$\Rightarrow$ x^2(x + 9) + 2(x + 9) = 0

$\Rightarrow$ (x + 2)(x + 9) = 0

$\Rightarrow$ x = -2, -9

63. $15x^2 - 10\sqrt{6}x + 10 = 0$.

$3x^2 - 2\sqrt{6}x + 2 = 0$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$\sqrt{3}x(\sqrt{3}x-\sqrt{2})-\sqrt{2}(\sqrt{3}x-\sqrt{2})=0$

$(\sqrt{3}x-\sqrt{2})(\sqrt{3}x-\sqrt{2})=0$

$\therefore \quad x=\frac{\sqrt{2}}{\sqrt{3}},\frac{\sqrt{2}}{\sqrt{3}}$

64. Since the roots are equal, then

D = 0

$D=b^2-4ac=0$

or, $(-2k)^2-4(k)(6)=0$

or, $4k^2-24k=0$

or, 4k(k - 6) = 0

or, k = 0,6

But $k\neq 0$ as coefficient of x^2 can not be zero.

k= 6

65. Use the quadratic formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Once in standard form, identify a, b and c from the original equation and plug them into the quadratic formula.

$2x^2$ - 5x - 1 = 0

a = 2
b = -5
c = -1
x = $\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot2(-1)}}{2\cdot2}$
x = $\frac{5+\sqrt{33}}{4}$
x = $\frac{5-\sqrt{33}}{4}$
x = $\frac{5\pm\sqrt{33}}{4}$

66. $mx^2 - 2(m - 1)x + (m + 2) = 0$
For a quadratic equation to have real and equal roots,
$b^2 - 4ac = 0$
So,
$(2(m - 1))^2 - 4(m + 2)(m) = 0$
$(2m- 2)^2 - 4(m^2 + 2m) = 0$
$4m^2 - 8m + 4 - 4m^2 - 8m = 0$
-8m = -4
m = $\frac{4}{8}$
m = $\frac{1}{2}$

67. Let Shikha's present age be x.
Then,
Her age 5 years ago = (x - 5) years,
Her age 8 years later = (x + 8) years,
It is given that the product of these ages is 30.
$\therefore$ (x - 5)(x + 8) = 30
$\Rightarrow x^2 + 8x - 5x - 40 = 30$
$\Rightarrow x^2 + 3x - 40 - 30 = 0$
$\Rightarrow x^2 + 3x - 70 = 0$
$\Rightarrow x^2 + 10x - 7x - 70 = 0$
$\Rightarrow$ x(x + 10) - 7(x + 10) = 0
$\Rightarrow$ (x - 7)(x + 10) = 0
$\Rightarrow$ x - 7 = 0 [$\because$ age can never be negative $\therefore$ x + 10 $\neq$ 0]
$\Rightarrow$ x = 7 years
Hence, Shikha's present age is 7 years.

68. Let the present age of one friend be x years.
Also, sum of ages of both friends = 20 years
hence age of 2nd friend will be (20 - x) years.
4 years ago, age of 1st friend = (x - 4) years.
age of 2nd friend= (20-x)- 4 = (16-x) years.
According to the question;
(x - 4)(1 6 - x) = 48
$\Rightarrow x^2 - 20x + 112 = 0$
Let D be the discriminant of this quadratic. Then,
D = $b^2 - 4ac$ = 400 - 448 = -48 < 0. (here, a = 1, b = -20, c = 112)
So, above equation does not have real roots. Hence, the given situation is not possible.

69. Let the breadth of the park is x and the length is 2x + 5.
Area of the park = x $\times$ (2x + 5) = 250
$\Rightarrow 2x^2 + 5x - 250 = 0$
$\Rightarrow 2x^2 + 25x - 20x - 250 = 0$
$\Rightarrow$ x(2x + 25) -10(2x + 25) = 0
$\Rightarrow$ (x - 10) (2x + 25)

$\Rightarrow$ x = 10 or

2 x=-25

x = $\frac{-25}{2}$ = -12.5

$\therefore$ Breadth of the park = 10 m

Length = 12.5 m

70. According to the question,the given two equations are,

$ax^2 + bx + c = 0$ ....(i)

And $-ax^2 + bx + c = 0$ ....(ii)

The discriminant of equation (i) is given by

$D_1 = b^2 - 4ac$

Now, the discriminant of equation (ii) is given by

$D_2 = b^2 - 4(-a)c$

$\Rightarrow D_2 = b^2 + 4ac$

Now, $D_1 + D_2 = b^2 - 4ac + b^2 + 4ac$

$\Rightarrow D_1 + D_2 = 2b^2 \geqslant 0$ [since, b is real]

$\Rightarrow$ At least one of D_1 and D_2 is greater than or equal to zero.

Therefore, at least one of the two equations has real roots.

71. Given:-

Speed of boat =18 km/hr

Distance = 24 km

Let x be the speed of stream.

Let t_1 and t_2 be the time for upstream and downstream As we know that,

speed = $\frac{\text{distance}}{\text{time}}$

$\Rightarrow$ time = $\frac{\text{distance}}{\text{speed}}$

For upstream, Speed = (18 - x) km/hr

Distance =24 km

Time = t_1

Therefore,

$t_1 = \frac{24}{18-x}$

For downstream,

Speed = (18 + x) km/hr

Distance = 24 km

Time = t_2

Therefore,

$t_2 = \frac{24}{18+x}$

Now according to the question-

$t_1 = t_2 + 1$

$\frac{24}{18-x} = \frac{24}{18+x} + 1$

$\Rightarrow \frac{1}{18-x} - \frac{1}{18+x} = \frac{1}{24}$

$\Rightarrow \frac{(18+x)-(18-x)}{(18-x)(18+x)} = \frac{1}{24}$

$\Rightarrow$ 48x = (18 - x)(18 + x)

$\Rightarrow 48x = 324 + 18x - 18x - x^2$

$\Rightarrow x^2 + 48x - 324 = 0$

$\Rightarrow x^2 + 54x - 6x - 324 = 0$

$\Rightarrow$ x(x + 54) -6(x + 54) = 0

$\Rightarrow$ (x + 54)(x - 6) = 0

$\Rightarrow$ x = -54 or x = 6

Since speed cannot be negative.

$\Rightarrow x \neq -54$

$\therefore x = 6$

Thus the speed of stream is 6 km/hr.

Total time of Journey = $t_1 + t_2$

$= \frac{24}{18-x} + \frac{24}{18+x}$

$= \frac{24}{12} + \frac{24}{24} = 2 + 1 = 3$ hrs.

72. Let the numbers are x and (x > y)

$x^2 - y^2 = 204$ …(i)

$y^2 = 10x - 4$ …(ii)

By (i) and (ii)

$x^2 - 10x + 4 - 204 = 0$

$x^2 - 10x + 200 = 0$

$(x - 20)(x + 10) = 0$

x = 20, x = -10 (rejected)

y = 14

73. i. Let the no of articles produced be x

Price of each article = 2x + 1

Price of all articles produced = ₹ 210

$x(2x + 1) = 210$

$2x^2 + x - 210 = 0$

ii. On solving

$2x^2 + x - 210 = 0$

$x = \frac{-1\pm\sqrt{1-(4)(2)(-210)}}{2\times2}$

$x = \frac{-1\pm\sqrt{1680}}{4}$

$x = \frac{-1\pm\sqrt{1681}}{4}$

$x = \frac{-1\pm41}{4}$

$x = \frac{40}{4} = 10$

x = 10

$x = \frac{-42}{4}$

neglected as no of article's cannot be -ve

$\therefore$ no of articles = 10

$\therefore$ cost of each article = 2x + 1

$= 2 \times 10 + 1$

= ₹ 21

iii. Since cost of 1 article = 21

$\therefore$ cost of 15 article = 21×15

= ₹ 315

OR

Since 21 is the manufacturing cost of 1 article

$\therefore$ 1 is the manufacturing cost of $\frac{1}{21}$ article

$\therefore$ 1575 is the manufacturing cost $\frac{1}{21} \times 1575$

= 75 article

74. i. Length of flower bed = x m

Length of grassland = 2x + 3

Total length of field = length of grassland + length of flower bed =(2x + 3) + x

= (3x + 3) m

ii. Perimeter of whole figure = 2(l + b)

= 2(3x + 3 + x)

= 2(4x + 3)
= (8x + 6)m

iii. A = (3x + 3)x

$1260 = 3x^2 + 3x$

$420 = x^2 + x$

$x^2 + x - 420 = 0$

(x + 21) (x - 20) = 0

They x = 20 is only possible value.

OR

Area of grassland = $l \times b$

$= 43 \times 20$

$= 860\ m^2$

Area of flowerbed = $(side)^2$

$= (20)^2$

$= 400\ m^2$

75. i. $200x^2 = 128(x + 1)^2$

ii. $25x^2 = 16x^2 + 32x + 16$

$\Rightarrow 9x^2 - 32x - 16 = 0$

iii. a. $9x^2 - 32x - 16 = 0$

$\Rightarrow (9x + 4)(x - 4) = 0$

$x \neq \frac{-4}{9}$ so, x = 4

OR

b. x = $\frac{32 \pm \sqrt{1024+576}}{18} = \frac{32 \pm 40}{18}$

$x \neq \frac{-4}{9}$ so, x = 4