CBSE Class 10 Mathematics
Important Questions
Chapter 9
Some Applications of Trigonometry

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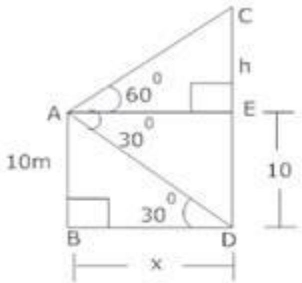
1 Marks Questions

1. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.



Ans. Let AB be the ship and CD be the hill.

Let BD = x m

In right ΔABD,

$$\frac{10}{x} = \tan 30^\circ$$

$$\Rightarrow \frac{10}{x} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow x = 10\sqrt{3}m$$

In right ΔCEA,

$$\frac{h}{AE} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{10\sqrt{3}} = \frac{\sqrt{3}}{1}$$
$$\Rightarrow h = 30m$$

Height of the hill = h + 10

= 30 +10 = 40 m

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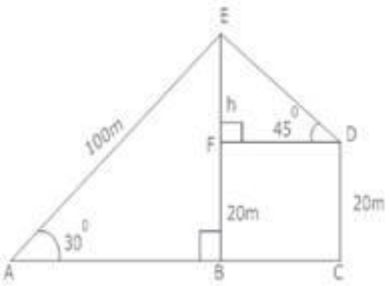
2. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30°. Another boy is standing on the roof of a 20 m high building and is flying his kite at an elevation of 45°. Both the boys are on the opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.



Ans. AE = 100 m (Given) and DC = 20 m (Given)

Let EF = h m

In right Δ ABE,

$$\frac{h+20}{100} = \sin 30^\circ$$

$$\Rightarrow \frac{h+20}{100} = \frac{1}{2}$$
$$\Rightarrow h = 30\ m$$

Now in right ΔEFD,

$$\frac{h}{ED} = \sin 45^\circ$$

$$\Rightarrow \frac{30}{ED} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow ED = 30\sqrt{2}\ m$$

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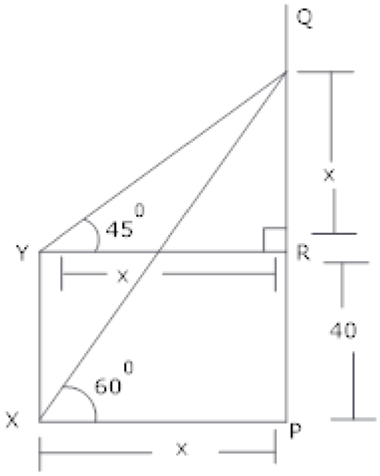
3. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. At a point Y, 40 m vertically above X, the angle of elevation is 45°, find the height of the tower PQ and the distance XQ.



Ans. In right ΔQRY,

$$\frac{QR}{x} = \tan 45°$$

$$\Rightarrow \frac{QR}{x} = \frac{1}{1}$$
$$\Rightarrow QR = x$$

In right ΔQPX,

$$\frac{x+40}{x} = \tan 60°$$
$$\Rightarrow \frac{x+40}{x} = \frac{\sqrt{3}}{1}$$
$$\Rightarrow \sqrt{3}x = x + 40$$

$$\Rightarrow x = \frac{40}{\sqrt{3}-1}$$
$$\Rightarrow x = \frac{40}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$
$$\Rightarrow x = 20\left(\sqrt{3}+1\right) = 54.64$$

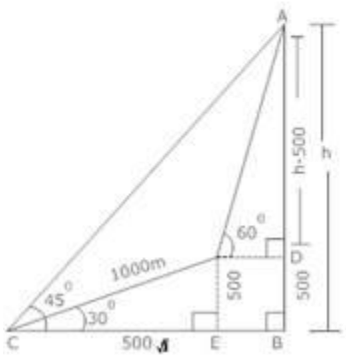
$$PQ = x + 40$$
$$= 54.64 + 40 = 94.64\ m$$

In right ΔQPX,

$$\frac{x+40}{XQ} = \sin 60°$$

$$\Rightarrow \frac{54.64+40}{XQ} = \frac{\sqrt{3}}{2}$$
$$\Rightarrow XQ = 109.3\ m$$

4. At the foot of a mountain the elevation of its summit is 45°. After ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.



Ans. In right ΔOEC,

$$\frac{CE}{1000} = \cos 30^\circ$$

$$\Rightarrow \frac{OE}{1000} = \frac{1}{2}$$
$$\Rightarrow OE = 500\ m$$

In right ΔADO,

$$\frac{h-500}{OD} = \tan 60^\circ$$

$$\Rightarrow \frac{h-500}{OD} = \frac{\sqrt{3}}{1}$$
$$\Rightarrow OD = \frac{h-500}{\sqrt{3}}$$

In right ΔABC,

$$\frac{h}{500\sqrt{3} + \frac{h-500}{\sqrt{3}}} = \tan 45^\circ$$

$$\Rightarrow \frac{\sqrt{3}h}{1500 + h - 500} = \frac{1}{1}$$
$$\Rightarrow \sqrt{3h} = 1000 + h$$

$$\Rightarrow h = \frac{100}{\sqrt{3}-1}$$
$$\Rightarrow h = 1369.86\ m$$

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5. If the angles of elevation of the top of a tower from two points at distances a and b from the base and in the same straight line with it are complementary, then the height of the tower is

(a) $\sqrt{ab}$

(b) ab

(c) $\frac{a}{b}$

(d) $\sqrt{\frac{a}{b}}$

Ans. a) $\sqrt{ab}$

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6. If the height of tower is half the height of the flagstaff on it and the angle of elevation of the top of the tower as seen from a point on the ground is 30°, then the angle of elevation of the top of the flagstaff as seen from the same point is

(a) 30°

(b)45°

(c) 60°

(d) 90°

Ans. (b)45°

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7. As observed from the top of a 75 m tall lighthouse the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, then distance between the two ships is

(a) $75\left(\sqrt{3}-1\right)$

(b) $75\sqrt{3}$

(c) 75

(d) $\frac{75}{\sqrt{3}}$

Ans. (a) $75\left(\sqrt{3}-1\right)$

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8. The angle of elevation of the top a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, then the height of the building is

(a) 50

(b) $\frac{50}{3}$

(c) $50\sqrt{3}$

(d) $\frac{50}{\sqrt{3}}$

Ans. (b) $\frac{50}{3}$

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9. A 1.6 m tall girl stands at a distance of 3.2 m from a lamp post and casts a shadow of 4.8 m on the ground, then the height of the lamp post is

(a) $\frac{8}{3}$

(b) $\frac{3}{8}$

(c) $8\sqrt{3}$

(d) $\frac{8}{13}$

Ans. (a) $\frac{8}{3}$

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10. A tree breaks due to storm and broken part bends so that the top of the tree touches the ground making an angle of 30° with ground. If the distance between the foot of the tree to the point where the top touches the ground is 8 m, then the height of the tree is

(a) $\frac{8}{3}$

(b) $\frac{3}{8}$

(c) $8\sqrt{3}$

(d) $\frac{8}{\sqrt{3}}$

Ans. (c) $8\sqrt{3}$

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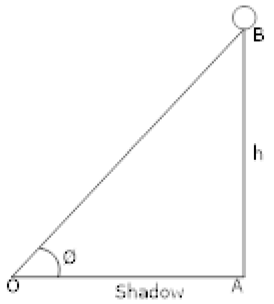
11. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times, then height of the pole is



(a) 30°

(b) 45°

(c) 60°

(d) 75°

Ans. (a) 30°

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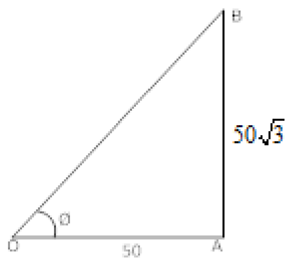
12. From the figure, the angle of elevation θ is



(a) 30°

(b) 60°

(c) 45°

(d) 75°

Ans. (b) 60°

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13. A person walking 20 m towards a chimney in a horizontal line through its bare observer that its angle of elevation changes from 30° to 45°, then height of chimney is

(a) $\frac{20}{\sqrt{3}+1}$

(b) $\frac{20}{\sqrt{3}-1}$

(c) $20\left(\sqrt{3}-1\right)$

(d) $20\left(\sqrt{3}+1\right)$

Ans. (b) $\frac{20}{\sqrt{3}-1}$

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14. Find height of the tower if length of shadow = 10 m and sun's altitude = 45°.

(a) 10 m

(b) 12 m

(c) 20 m

(d) none of these

Ans. (a) 10 m

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15. If the angles of elevation of a tower from two points at distances a and b where a > b, from its foot and in the same straight line from it are 30° and 60°, then height of the tower is

(a) $\sqrt{a+b}$

(b) $\sqrt{ab}$

(c) $\sqrt{a-b}$

(d) $\sqrt{\frac{a}{b}}$

Ans. (b) $\sqrt{ab}$

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16. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower is 30°, then height of tower is

(a) 10 m

(b) $10\sqrt{3}$ **m**

(c) $\frac{10}{\sqrt{3}}$ **m**

(d) $3\sqrt{10}$ **m**

Ans. (a) 10 m

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17. If height of the tower = shadow of the tower, then angle of elevation is

(a) 30°

(b) 45°

(c) 60°

(d) 90°

Ans. (b) 45°

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18. If length of shadow = 20 m and angle of elevation = 60°. Then height of tower is

(a) 20 m

(b) $20\sqrt{2}m$

(c) $\frac{20}{\sqrt{3}}m$

(d) $20\sqrt{3}m$

Ans. (d) $20\sqrt{3}m$

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19. A little boy is flying a kite. The string of kite makes an angle of 30° with the ground. If length of the kite is h = 21 m, then length of string is

(a) 63 m

(b) 42 m

(c) 35 m

(d) 21 m

Ans. (b) 42 m

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20. Line joining an eye and the object to be viewed is called

(a) Horizontal

(b) line of sight

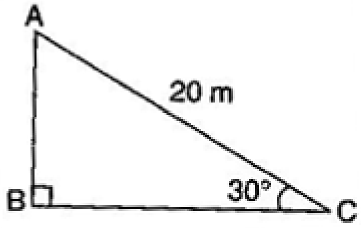
(c) Vertical line

(d) None of these

Ans. (b) line of sight

2 Marks Questions

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see figure).



Ans. In right triangle ABC,

$\sin 30^{\circ} = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{AB}{20}$

$\Rightarrow$ AB = 10 m

Hence, the height of the pole is 10 m.

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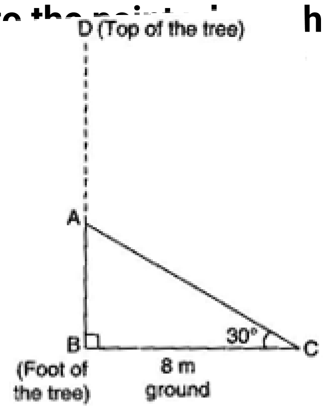
2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.



Ans. In right triangle ABC,

$\cos 30^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{AC}$

$\Rightarrow AC = \frac{16}{\sqrt{3}}$ m

Again, $\tan 30^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{8}$

$\Rightarrow$

$\therefore AB = \frac{8}{\sqrt{3}}$ m

Height of the tree = AB + AC

$= \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$

$= \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 8\sqrt{3}$ m

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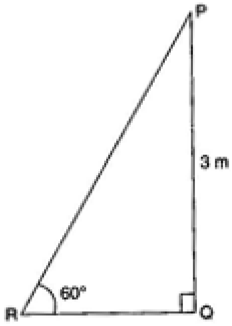
3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?



Ans. In right triangle ABC,

$\sin 30^\circ = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{1.5}{AC}$

$\Rightarrow$ AC = 3 m

In right triangle PQR,

$\sin 60^\circ = \frac{PQ}{PR}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{PR}$

$\Rightarrow$ PR = $2\sqrt{3}$ m

Hence, the lengths of the slides are 3 m and $2\sqrt{3}$ m respectively.

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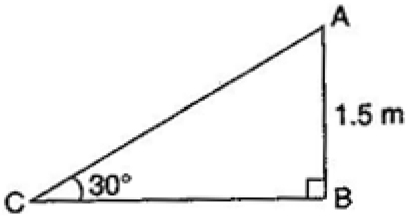
4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the lower is 30°. Find the height of the tower.



Ans. In right triangle ABC,

$\tan 30^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\Rightarrow$ AB = $\frac{30}{\sqrt{3}}$ m

$\Rightarrow \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 10\sqrt{3}$ m

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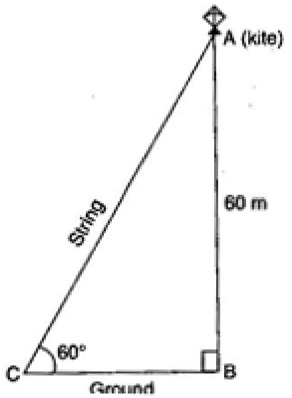
5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.



Ans. In right triangle ABC,

$$\sin 60^\circ = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{60}{AC}$$

$\Rightarrow$ AC = $40\sqrt{3}$ m

Hence the length of the string is $40\sqrt{3}$ m.

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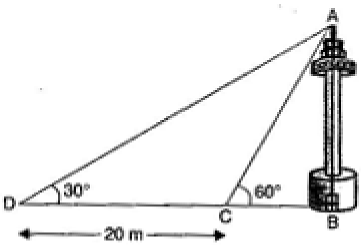
6. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30° (see figure). Find the height of the tower and the width of the canal.



Ans. In right triangle ABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BC}$$

$\Rightarrow$ AB = $BC\sqrt{3}$ m ……….(i)

In right triangle ABD,

$$\tan 30^\circ = \frac{AB}{BD}$$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$

$\Rightarrow$ AB = $\frac{BC + 20}{\sqrt{3}}$ m ……….(ii)

From eq. (i) and (ii),

$BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$

$\Rightarrow$ 3BC = BC + 20

$\Rightarrow$ BC = 10 m

From eq. (i), AB = $10\sqrt{3}$ m

Hence height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

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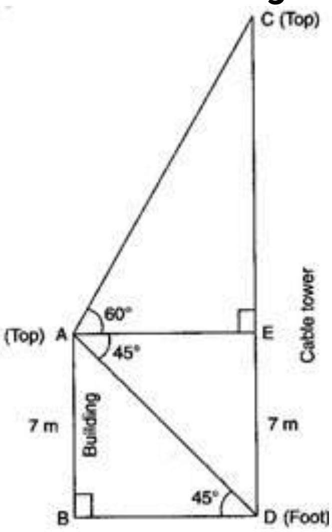
7. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.



Ans. In right triangle ABD,

$\tan 45^\circ = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{7}{BD}$

$\Rightarrow$ BD = 7 m

$\Rightarrow$ AE = 7 m

In right triangle AEC,

$\tan 60^\circ = \frac{CE}{AE}$

$\Rightarrow \sqrt{3} = \frac{CE}{7}$

$\Rightarrow$ CE = $7\sqrt{3}$ m

$\therefore$ CD = CE + ED

= CE + AB

= $7\sqrt{3} + 7$

= $7\left(\sqrt{3}+1\right)$ m

Hence height of the tower is $7\left(\sqrt{3}+1\right)$ m.

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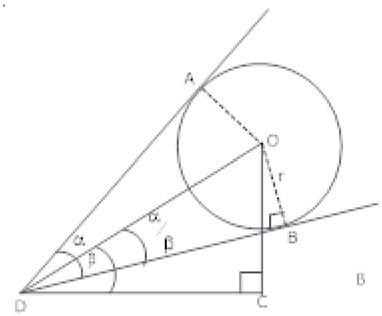
8. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of the elevation of its centre is β. Prove that the height of the centre of the balloon is r sin β cosec $\frac{\alpha}{2}$.



Ans. In right ΔOBD,

$\frac{OD}{r} = \operatorname{cosec}\frac{\alpha}{2}$

$\Rightarrow OD = r \operatorname{cosec}\frac{\alpha}{2}$

In right ΔOCD,

$$\frac{OC}{OD} = \text{Sin}\beta$$

$$\Rightarrow \frac{OC}{r\cos ec\frac{\alpha}{2}} = \frac{\sin\beta}{1}$$

$$\Rightarrow OC = r\sin\beta.\cos ec\frac{\alpha}{2}$$

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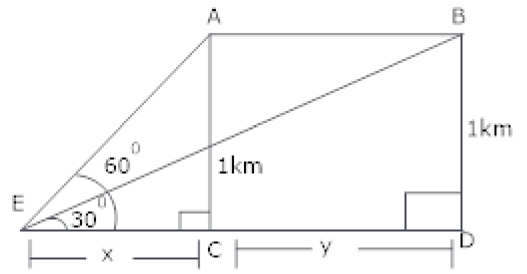
9. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of $60°$. After 10 seconds, its elevation is observed to $30°$. Find the speed of the aeroplane in km/hr.



Ans. In right $\Delta ACE,$

$$\frac{AC}{x} = \tan 60°$$

$$\Rightarrow \frac{1}{x} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}m$$

In right $\Delta BDE,$

$$\frac{1}{x+y} = \tan 30°$$

$$\Rightarrow \frac{1}{\frac{1}{\sqrt{3}}+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\sqrt{3}}{1+\sqrt{3}y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 1+\sqrt{3}y = 3$$

$$\Rightarrow y = \frac{2}{\sqrt{3}}km$$

Speed = $\frac{\text{distance}}{\text{time}}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{10}{3600}}$

$= 240\sqrt{3} km / hr$

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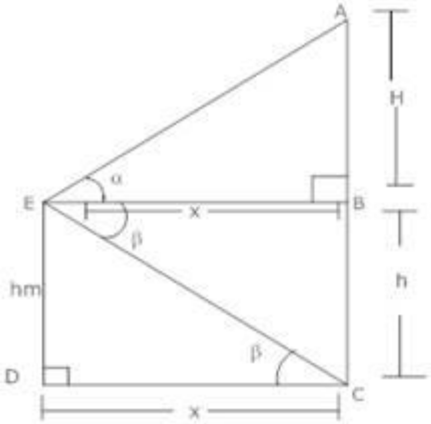
10. From a window (h m high above the ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are α and β respectively, show that the height of the opposite house is $h(1+\tan\alpha.\cot\beta)m.$



Ans. Let DE = h m

DC = x m

In right $\Delta EDC,$

$\frac{h}{x} = \tan\beta$

$\Rightarrow \frac{h}{\tan\beta} = x......(i)$

In right $\Delta ABE,$

$\frac{H}{x} = \tan\alpha$

$\Rightarrow \frac{H}{\frac{h}{\tan\beta}} = \tan\alpha$ [from (i)

$\Rightarrow H\tan\beta = h\tan\alpha$

$\Rightarrow H = h\tan\alpha.\cot\beta$

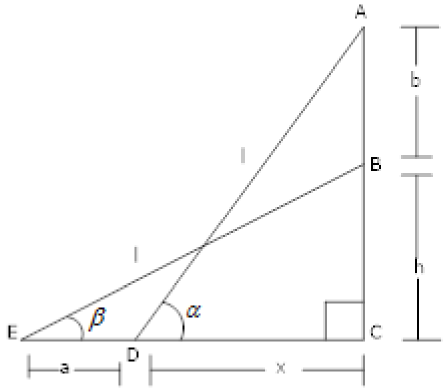
$AC = H + h$

$= h\tan\alpha.cot\beta + h$

$= h(\tan\alpha.\cot\beta + 1)$

Hence Proved.

11. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance 'a' so that it slides a distance 'b' down the wall making an angle 'β' with the horizontal. Show that $\frac{a}{b}=\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$.



Ans. In right $\Delta ACD,$

$$\frac{b+h}{l}=\sin\alpha \text{ and } \frac{x}{l}=\cos\alpha$$

Similarly, in right $\Delta BCE,$

$$\text{Sin}\beta=\frac{h}{l},\cos\beta=\frac{a+x}{l}$$

$$R.H.S=\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$$

$$=\frac{\frac{x}{l}-\frac{a+x}{l}}{\frac{h}{l}-\frac{b+h}{l}}=\frac{\frac{x-a-x}{l}}{\frac{h-b-h}{l}}=\frac{a}{b}$$

Hence Proved.

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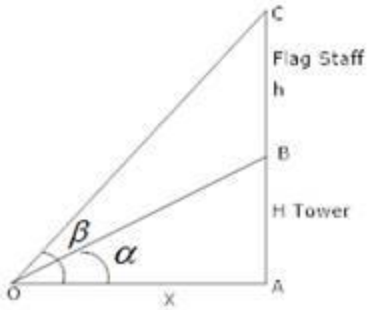
12. A vertical tower stands on a horizontal plane and surmounted by vertical flagstaff of height h. At a point on the plane, the angles of elevation of the bottom and the top of the flagstaff are α and βrespectively, Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta-\tan\alpha}$.



Ans. Let AB = Height of tower = H

Let BC = Height of flagstaff = h

In right-angled triangle OAB and OAC,

$$\frac{x}{H} = \cot\alpha$$
$$\Rightarrow x = H\cot\alpha$$

And $\frac{x}{H+h} = \cot\beta \Rightarrow x = (H+h)\cot\beta$

Equating value of x, we get

$$H\cot\alpha = (H+h)\cot\beta$$
$$\Rightarrow H(\cot\alpha - \cot\beta) = h\cot\beta$$
$$\Rightarrow H = \frac{h\cot\beta}{\cot\alpha - \cot\beta}$$

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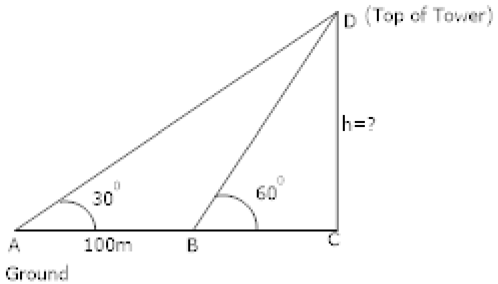
13. The angle of elevation of the top of a tower at a point on the level ground is 30°. After walking a distance of 100 m towards the foot of the tower along the horizontal line through the foot of the tower on the same level ground the angle of elevation to the top of the tower is 60°, find the height of the tower.



Ans. In ΔBCD, $\frac{h}{x} = \tan 60° = \sqrt{3}$

$$h = \sqrt{3}x \ldots\ldots(i)$$

In ΔACD, $\frac{h}{100+x} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow h\sqrt{3} = 100 + x$$
$$\Rightarrow h\sqrt{3} = 100 + \frac{h}{\sqrt{3}}$$
$$\Rightarrow h\left[\sqrt{3} - \frac{1}{\sqrt{3}}\right] = 100$$

$$\Rightarrow h\left[\frac{3-1}{\sqrt{3}}\right] = 100$$

$$\Rightarrow h = \frac{100\sqrt{3}}{2} = 50\sqrt{3}$$

$$= 50 \times 1.732 = 86.6m$$

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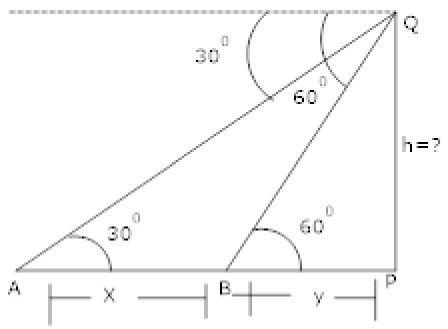
14. As observed from the top of light house 100 m high above sea level the angle of depression of a ship sailing directly towards it changes from 30° to 60°. Determine the distance travelled by the ship during the period of observation. (Use $\sqrt{3} = 1.732$)



Ans. Let PQ be the light house such that PQ = 100 m. Let A and B be the positions of ship when the angle of depression changes from 30° to 60° respectively.

Let $AB = x\,m$ and BP = y m

From right-angled $\Delta APQ,$

$$\frac{100}{x+y} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow x + y = 100\sqrt{3} ....(i)$$

From right-angled triangle BPQ,

$$\frac{100}{y} = \tan 30° = \sqrt{3}$$

$$\therefore y = \frac{100}{\sqrt{3}} = \frac{100}{3}\sqrt{3} ....(ii)$$

From (i) and (ii),

$$x + \frac{100}{3}\sqrt{3} = 100\sqrt{3}cm$$

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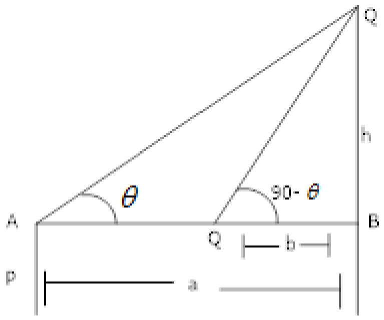
15. The angles of elevation of the top of a tower from two points P and Q at distances of a and b respectively from the base and in the same straight line with are complementary. Prove that the height of the tower is $\sqrt{ab},$ where a > b.



Ans. Let AB be tower of height h. Let P and Q be the given points in the same straight line with the foot B of the tower.

Let BP = a, BQ = b

$\angle APB = Q, \angle AQB = 90^\circ - \theta$

From right angled $\Delta APB,$

$\frac{h}{a} = \tan\theta$

$\Rightarrow h = a\tan\theta ....(i)$

From right-angled $\Delta AQB,$

$\frac{h}{b} = \tan(90^\circ - \theta)$

$\Rightarrow \frac{h}{b} = \cot\theta$

$\Rightarrow h = b\cot\theta = \frac{b}{\tan\theta} ....(ii)$

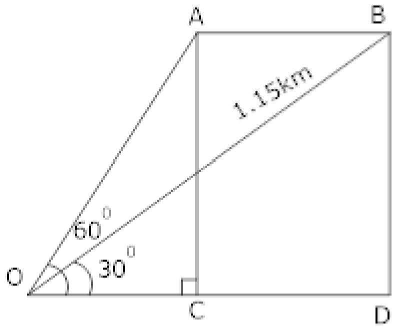
Multiplying (i) and (ii), we get

$h^2 = ab$

$\Rightarrow h = \sqrt{ab}$

---

16. An aeroplane flying horizontally at a height of 1.5 km above the ground is observed at a certain point on the earth to subtend an angle of 60°. After 15 seconds, its angle of elevation is observed to be 30°. Calculate the speed of aeroplane in km/hr.



Ans. Let O be the observation point.

Let A be the position of aeroplane such that $\angle AOC = 60°$ and $AC = 1.5 km = BD$

Let B be the position of aeroplane after 15 seconds. $\angle BOD = 30°, OC = x\ km, CD = y\ km$

In right $\Delta OCA, \frac{x}{1.5} = \cot 60° = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{1.5}{\sqrt{3}}$

$\Rightarrow x = \frac{15}{10\sqrt{3}} = \frac{\sqrt{3}}{2}$.......$(i)$

In right $\Delta ADB,$

$\frac{x+y}{1.5} = \cot 30° = \sqrt{3}$

$\Rightarrow x+y = \sqrt{3}(1.5)$

$\Rightarrow x+y = \sqrt{3} \times \frac{3}{2}$.......$(ii)$

$eq.(ii) - eq.(i)$

$y = \frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = (3-1)\frac{\sqrt{3}}{2} = \sqrt{3}$

$\therefore$Distance covered in 15 seconds $y = \sqrt{3}\ km$

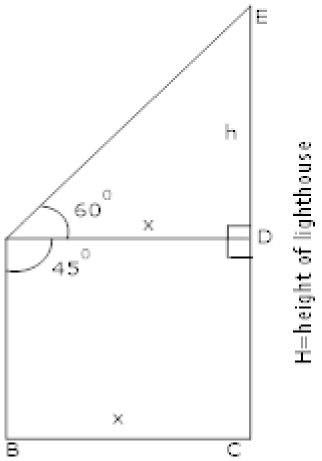
Distance covered in 1 second = $\frac{\sqrt{3}}{15} km$

Distance covered in 3600 seconds = $\frac{\sqrt{3}}{15} \times 3600 = 240\sqrt{3}\ km$

---

17. A man is standing on the deck of a ship which is 25 m above water level. He observes the angle of elevation of the top of a lighthouse as 60° and the angle of depression of the

base of the lighthouse as 45°. Calculate the height of the lighthouse.



Ans. H = Height of lighthouse = h + 25 ....(i)

In right $\Delta ADC, \frac{x}{25} = \cot 45^\circ = 1$

$\Rightarrow x = 25\,m$

In right $\Delta ADE, \frac{x}{h} = \cot 60^\circ = \frac{1}{\sqrt{3}}$

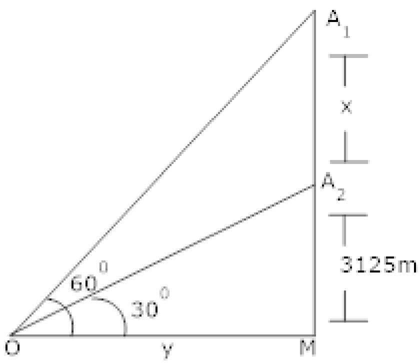
$\Rightarrow \frac{25}{h} = \frac{1}{\sqrt{3}} \Rightarrow h = 25\sqrt{3}$

Now $H = h + 25 = 25\sqrt{3} + 25$

$= 25\left(\sqrt{3}+1\right)m$

---

18. An aeroplane when flying at a height of 3125 m form the ground passes vertically below another aeroplane at an instant when the angle of elevation of the two planes from the same point on the ground are 30° and 60° respectively. Find the distance between the two aeroplanes at that instant.



Ans. Let A_1 and A_2 be the positions of the two aeroplanes

Let $A_1A_2 = x?$

And OM = y

$$\frac{y}{3125} = \cot 30° = \sqrt{3}$$

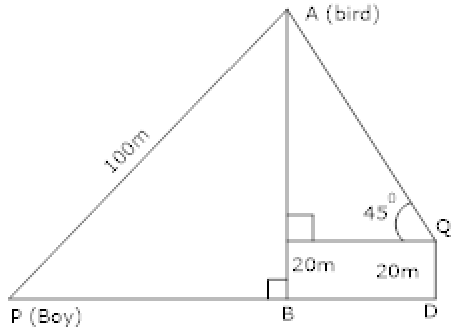
$$\Rightarrow y = (3125)\sqrt{3}.......(i)$$

Also $\frac{y}{3125+x} = \cot 60° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{(3125)\sqrt{3}}{3125+x} = \frac{1}{\sqrt{3}}$$
$$\Rightarrow 3125 + x = (3125)(3)$$
$$\Rightarrow x = 3125(3-1) = 3125 \times 2 = 6250\ m$$

---

19. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 m high building finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite side of the bird. Find the distance of the bird from the girl.



Ans. Positions of bird at A, boy at P and girl at B are as shown in figure.

In ΔABP,

$$\frac{AB}{100} = \sin 30° = \frac{1}{2}$$
$$\Rightarrow AD = \frac{100}{2} = 50m$$

Also BC = DQ = 20m

$$\therefore AC = AB - BC = 50 - 20 = 30m$$

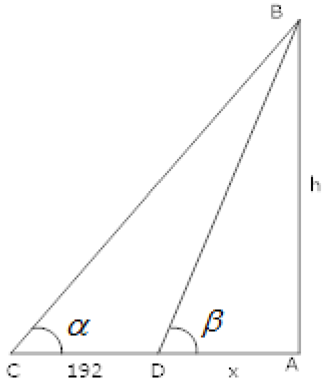
In $\Delta ACQ, \frac{AC}{AQ} = \sin 45°$

$$\Rightarrow \frac{30}{AQ} = \frac{1}{\sqrt{2}} \Rightarrow AQ = 30\sqrt{2}m$$

Hence, the bird is 30$\sqrt{2}$ m away from the girl.

---

20. At a point on level ground, the angle of a elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$; on walking 19.2 m towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of tower.



Ans. Suppose height of tower is h meter

In ΔABD,

$$tan\beta = \frac{h}{x} = \frac{3}{4}......(i)$$

In ΔABC,

$$\tan\alpha = \frac{h}{192+x}$$

$$\Rightarrow \frac{h}{192+x} = \frac{5}{12}........(ii)$$

$(i) \div (ii)$

$$\frac{\frac{h}{x}}{\frac{h}{192+x}} = \frac{\frac{3}{4}}{\frac{5}{12}}$$

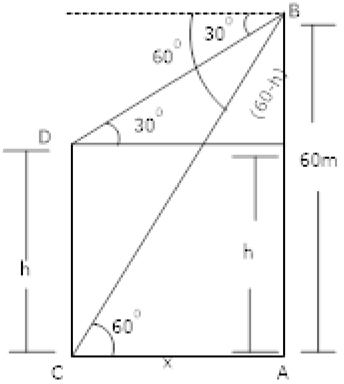
$$\Rightarrow x = 240$$

---

21. From the top of a building 60 m high, the angles of depression of the top and bottom of a vertical lamp post are observed to be 30° and 60° respectively. Find:

(i) the horizontal distance between the building and the lamp post

(ii) the height of the lamp post. [Take $\sqrt{3} = 1.732$]



Ans. Suppose height of lamppost is h meter.

In ΔDEB,

$$\frac{x}{60-h} = \cot 30° = \sqrt{3}$$
$$\Rightarrow x = (60-h)\sqrt{3} \ldots (i)$$

In right $\Delta CAB, \frac{x}{60} = \cot 60° = \frac{1}{\sqrt{3}}$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 20\sqrt{3}$$
$$\Rightarrow x = 20 \times 1.732 = 34.64\, m \ldots (ii)$$

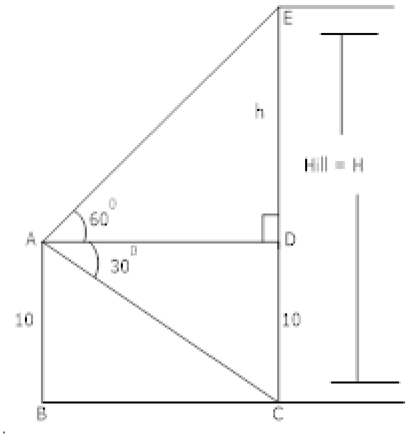
By eq. (i) and (ii)

(60 − h)$\sqrt{3}$ = 20$\sqrt{3}$

$$\Rightarrow 60 - h = 20$$
$$\Rightarrow h = 40\, m$$

---

22. A man standing on the deck of a ship which is 10 m above the water level observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.



Ans. Let H = Height of hill

$CE = CD + DE$

$= 10 + h$........(i)

In right $\Delta ADE,$

$\frac{x}{10} = \frac{1}{\sqrt{3}}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$

In right $\Delta ADC, \frac{x}{10} = \cot 30° = \sqrt{3}$

$\Rightarrow x = 10\sqrt{3}$

Equating the values of x, we get

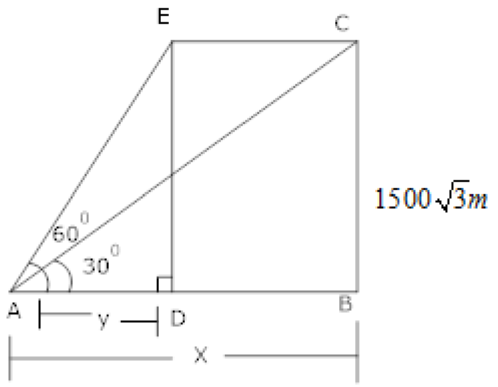
$\frac{h}{\sqrt{3}} = 10\sqrt{3} \Rightarrow h = 30cm$

∴From H = 10 + h = 10 + 30 = 40 m

And x = distance of hill from ship = 10 $\sqrt{3}$ m

---

23. The angle of elevation of a jet-plane from a point P on the ground is 60°. After a flight of 15 seconds the angle of elevation (change to 30°). If the jet plane is flying at a constant height of $1500\sqrt{3}$, find the speed of the jet plane in km/hour.



Ans. Let A be the point on the ground E is the position of aeroplane such that $\angle EAD = 60°$ and $ED = 1500\sqrt{3} = CB$

C is the position of plane after 15 seconds

$\therefore \angle CAB = 30°, AB = x, AD = y$

In right $\Delta ABC, \frac{x}{1500\sqrt{3}} = \cot 30°$

$\Rightarrow \frac{x}{1500\sqrt{3}} = \sqrt{3}$

$\Rightarrow x = 4500m$

In $\Delta ADE, \frac{y}{1500\sqrt{3}} = \cot 60°$

$\Rightarrow \frac{y}{1500\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow y = 1500m$

Distance $BD = x - y = 4500 - 1500 = 3000m$

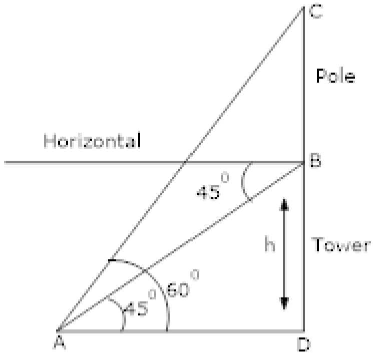
$S = \frac{D}{T}$

Speed = $\frac{3000}{15} = 200m/\sec$

$= \frac{200 \times 3600}{1000} = 200km/hr.$

---

24. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of pole observed from a point A on the ground is 60° and the angle of depression of the point A from the top of the tower is 45°. Find the height of the tower. (Take $\sqrt{3} = 1.732$)



Ans. In ΔADB,

$$\frac{AD}{h} = \cot 45°$$
$$\Rightarrow \frac{AD}{h} = 1$$
$$\Rightarrow AD = h$$

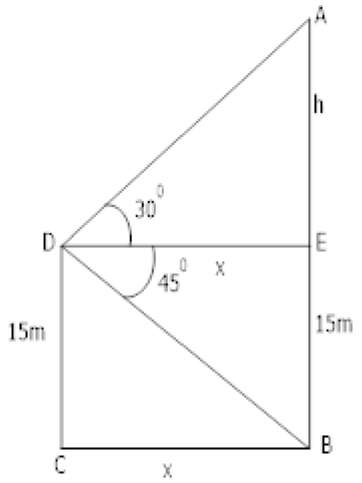
In ΔADC,

$$\frac{AD}{h+5} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{h+5} = \frac{1}{\sqrt{3}} \quad [\because AD = h]$$
$$\Rightarrow h+5 = \sqrt{3}h$$
$$\Rightarrow 5 = \left(\sqrt{3}-1\right)h$$
$$\Rightarrow h = \left(\frac{5}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \frac{5(1.732+1)}{3-1}$$
$$= 6.83\ m$$

---

25. From a window 15 m high above the ground in a street. The angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30° and 45° respectively. Show that the height of the apposite house is 23.66 m.
[Take $\sqrt{3} = 1.732$]



Ans. Suppose $AE = h\,m$ and $BC = x\,m$

Then, AB = $(15+h)m$

In $\Delta AED,$

$$\tan 30° = \frac{h}{x}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$
$$\Rightarrow x = \sqrt{3}h...(i)$$

In $\Delta DEB,$

$$\tan 45° = \frac{15}{x}$$
$$\Rightarrow 1 = \frac{15}{x}$$
$$\Rightarrow x = 15\,m$$

Then $h = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ by.......(i)

$$\Rightarrow h = \frac{15\sqrt{3}}{3} = 5\sqrt{3}m$$

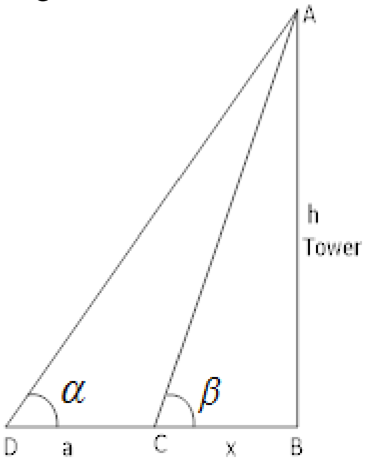
Height of another house $= 15 + 5\sqrt{3} = 15 + 5 \times 1.732$

$$= 15 + 8.660$$
$$= 23.66m$$

---

26. The angle of elevation of the top of a tower as observed from a point on the ground is $'\alpha'$ and on moving 'a' meter towards the tower. The angle of elevation is $'\beta'$ prove that

height of tower is $\frac{a\tan\alpha\tan\beta}{\tan\beta-\tan\alpha}$.



Ans. Let AB b tower and height of tower = h m

In $\Delta ABC,$

$$\tan\beta=\frac{h}{x}$$

$$\Rightarrow x=\frac{h}{\tan\beta}\text{.........}(i)$$

In $\Delta ABD,$

$$\tan\alpha=\frac{h}{x+a}$$

$$\Rightarrow h=(x+a)\tan\alpha$$

$$\Rightarrow h=\left[\frac{h}{\tan\beta}+a\right]\tan\alpha\ \text{ by ....}(i)$$

$$\Rightarrow h=\frac{h\tan\alpha}{\tan\beta}+a\tan\alpha$$

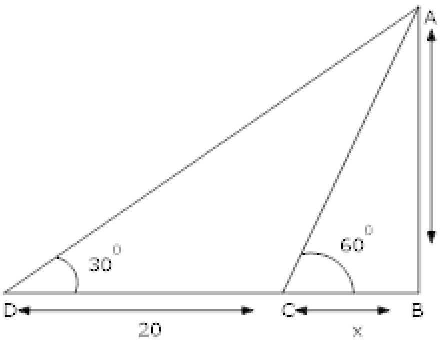
$$\Rightarrow h\tan\beta=h\tan\alpha+a\tan\alpha\tan\beta$$

$$\Rightarrow h(\tan\beta-\tan\alpha)=a\tan\alpha\tan\beta$$

$$\Rightarrow h=\frac{a\tan\alpha\tan\beta}{\tan\beta-\tan\beta}$$

---

27. A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° from a point 20 m away from this point on the same back the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Ans. Let h be the height of tower and x be the width of the river

In $\Delta ABC, \frac{h}{x} = \tan 60°$

$\Rightarrow h = \sqrt{3}x ...(i)$

In $\Delta ABD, \frac{h}{x+20} = \tan 30°$

$\Rightarrow h = \frac{x+20}{\sqrt{3}} ....(ii)$

Equating (i) and (ii),

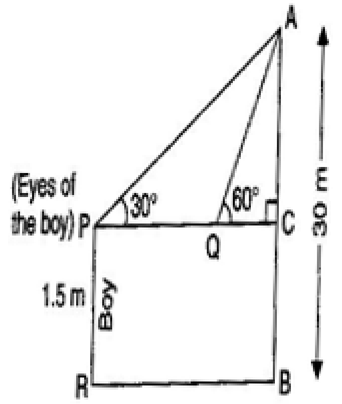
$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$
$\Rightarrow 3x = x+20$
$\Rightarrow 2x = 20$
$\Rightarrow x = 10m$

Put $x = 10$ in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}m$

3 Marks Questions

1. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.



Ans. AB = 30 m and PR = 1.5 m

AC = AB – BC

= AB – PR

= 30 – 1.5

= 28.5 m

In right triangle ACQ,

$$\tan 60^\circ = \frac{AC}{QC}$$

$$\Rightarrow \sqrt{3} = \frac{28.5}{QC}$$

$\Rightarrow$ QC = $\frac{28.5}{\sqrt{3}}$ m

In right triangle ACP,

$$\tan 30^\circ = \frac{AC}{PC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ + QC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5}{PQ + \frac{28.5}{\sqrt{3}}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{28.5 \times \sqrt{3}}{PQ\sqrt{3} + 28.5}$$

$$\Rightarrow PQ\sqrt{3} + 28.5 = 85.5$$

$$\Rightarrow PQ\sqrt{3} = 57$$

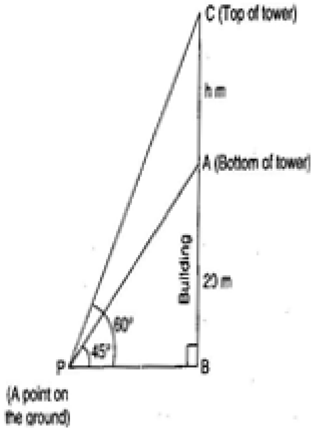
$\Rightarrow$ PQ = $\frac{57}{\sqrt{3}}$

$\Rightarrow$ PQ = $\frac{57}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ = $19\sqrt{3}$ m

Hence, the walked towards the building is $19\sqrt{3}$ m.

---

2. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.



Ans. Let the height of the tower be h m. Then,

in right triangle CBP,

$\tan 60^\circ = \frac{BC}{BP}$

$\Rightarrow \sqrt{3} = \frac{AB + AC}{BP}$

$\Rightarrow \sqrt{3} = \frac{20 + h}{BP}$ ....(i)

In right triangle ABP,

$\tan 45^\circ = \frac{AB}{BP}$

$\Rightarrow 1 = \frac{20}{BP}$

$\Rightarrow$ BP = 20 m

Putting this value in eq. (i), we get,

$\sqrt{3} = \frac{20 + h}{20}$

$\Rightarrow\ 20\sqrt{3} = 20 + h$

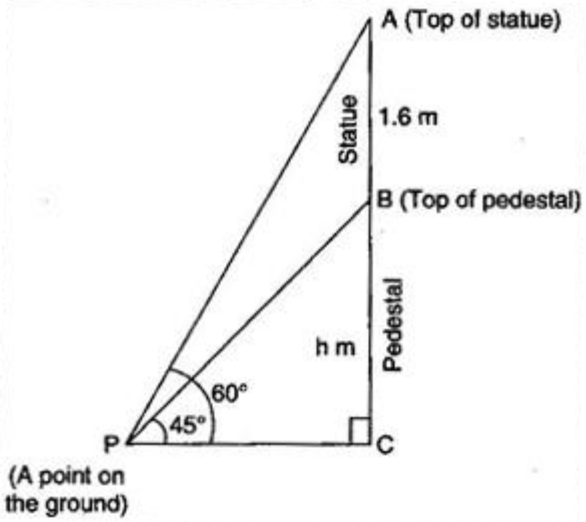
$\Rightarrow\ h = 20\sqrt{3} - 20$

$\Rightarrow\ h = 20\left(\sqrt{3} - 1\right)$ m

∴ The height of the tower is $20\left(\sqrt{3} - 1\right)$ m.

---

3. A statue, 1.6 m tall, stands on the top of a postal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.



Ans. Let the height of the pedestal be h m.

∴ BC = h m

In right triangle ACP,

$\tan 60^\circ = \dfrac{AC}{PC}$

$\Rightarrow\ \sqrt{3} = \dfrac{AB + BC}{PC}$

$\Rightarrow\ \sqrt{3} = \dfrac{1.6 + h}{PC}$ ....(i)

In right triangle BCP,

$\tan 45^\circ = \dfrac{BC}{PC}$

$\Rightarrow\ 1 = \dfrac{h}{PC}$

$\Rightarrow$ PC = h

$\therefore$ $\sqrt{3} = \frac{1.6 + h}{h}$ [From eq. (i)]

$\Rightarrow$ $\sqrt{3}h = 1.6 + h$

$\Rightarrow$ $h\left(\sqrt{3} - 1\right) = 1.6$

$\Rightarrow$ $h = \frac{1.6}{\sqrt{3} - 1}$

$\Rightarrow$ $h = \frac{1.6\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)}$

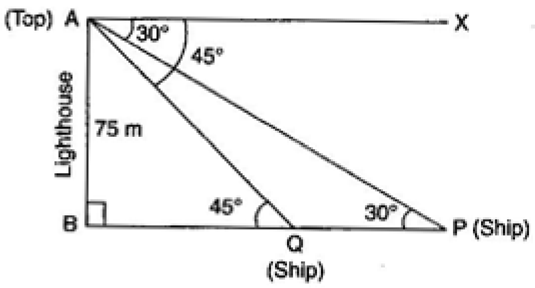
$\Rightarrow$ $h = \frac{1.6\sqrt{3} + 1}{3 - 1}$

$\Rightarrow$ $h = \frac{1.6\left(\sqrt{3} + 1\right)}{2}$

$\Rightarrow$ $h = 0.8\left(\sqrt{3} + 1\right)$ m

---

4. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.



Ans. In right triangle ABQ,

$\tan 45^\circ = \frac{AB}{BQ}$

$\Rightarrow$ $1 = \frac{75}{BQ}$

$\Rightarrow$ BQ = 75 m ……….(i)

In right triangle ABP,

$\tan 30^\circ = \frac{AB}{BP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BQ + QP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{75 + QP}$ [From eq. (i)]

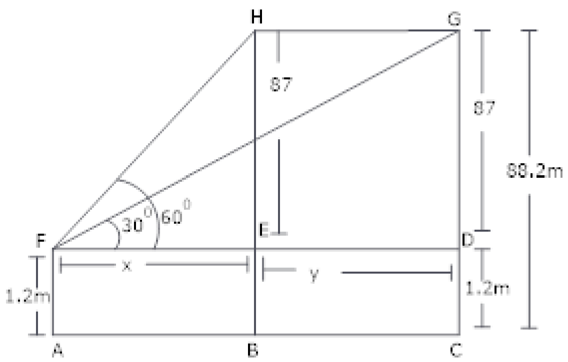
$\Rightarrow$ 75 + QP = $75\sqrt{3}$

$\Rightarrow$ QP = $75\left(\sqrt{3}-1\right)$ m

Hence the distance between the two ships is $75\left(\sqrt{3}-1\right)$ m.

---

5. A girl who is 1.2 m tall, spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eye of the girl at any instant is 60° . After sometime, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.



Ans. In right ΔHEF,

$\frac{87}{x} = \tan 60^\circ$

$\Rightarrow \frac{87}{x} = \frac{\sqrt{3}}{1}$

$\Rightarrow x = \frac{87}{\sqrt{3}}$

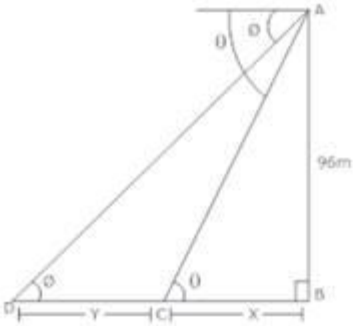
$\Rightarrow x = \frac{87}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{87\sqrt{3}}{3} = 29\sqrt{3}m$

In right ΔGDF,

$$\frac{87}{x+y}=\tan 30^\circ$$
$$\Rightarrow \frac{87}{29\sqrt{3}+y}=\frac{1}{\sqrt{3}}$$
$$\Rightarrow y=87\sqrt{3}-29\sqrt{3}=58\sqrt{3}m$$

---

6. From the top of a tower 96 m high, the angles of depression of two cars on a road at the same level as the base of the tower and on same side of it are θ and ϕ, where $\tan\theta=\frac{3}{4}$ and $\tan\phi=\frac{1}{3}$. Find the distance between the two cars.



Ans. In right ΔABC,

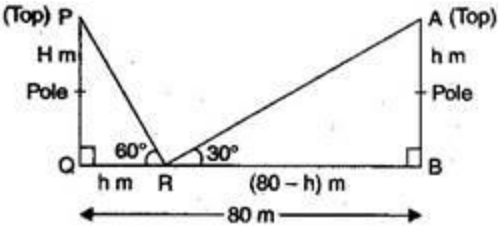
$$\frac{96}{x}=\tan\theta$$
$$\Rightarrow \frac{96}{x}=\frac{3}{4}$$
$$\Rightarrow x=\frac{96\times 4}{3}\quad [\tan\theta=\frac{3}{4}$$
$$\Rightarrow x=32\times 4=128m$$

In right ΔABD,

$$\frac{96}{x+y}=\tan\phi$$
$$\Rightarrow \frac{96}{x+y}=\frac{1}{3}\quad [\because \tan\phi=\frac{1}{3}$$
$$\Rightarrow \frac{96}{128+y}=\frac{1}{3}$$
$$\Rightarrow 128+y=228$$
$$\Rightarrow y=288-128=160m$$

4 Marks Questions

1. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.



Ans . Let the height of the building be h m.

$\tan 60^\circ = \frac{PQ}{BQ}$

$\Rightarrow \sqrt{3} = \frac{50}{BQ}$

$\Rightarrow$ BQ = $\frac{50}{\sqrt{3}}$ m ……….(i)

In right triangle ABQ,

$\tan 30^\circ = \frac{AB}{BQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{BQ}$

$\Rightarrow$ BQ = $h\sqrt{3}$ m ……….(ii)

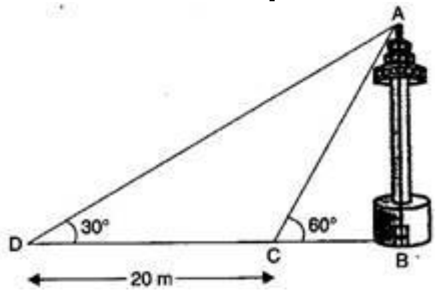
From eq. (i) and (ii),

$h\sqrt{3} = \frac{50}{\sqrt{3}}$

$\Rightarrow h = \frac{50}{3} = 16\frac{2}{3}$ m

---

2. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.



Ans. In right triangle ABC,

$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{AB}{BC}$

$\Rightarrow$ AB = $BC\sqrt{3}$ m ……….(i)

In right triangle ABD,

$\tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + CD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BC + 20}$

$\Rightarrow$ AB = $\frac{BC + 20}{\sqrt{3}}$ m ……….(ii)

From eq. (i) and (ii),

$BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$

$\Rightarrow$ 3BC = BC + 20

$\Rightarrow$ BC = 10 m

From eq. (i), AB = $10\sqrt{3}$ m

Hence height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

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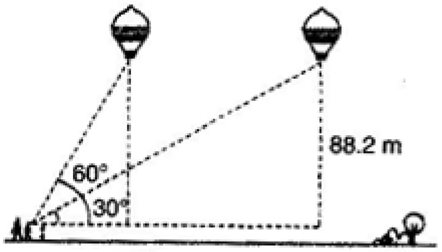
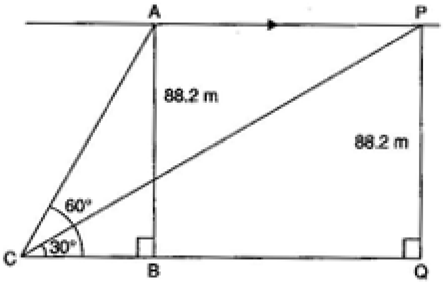
3. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any distant is 60°. After some time, the angle of elevation reduces to 30° (see figure). Find the distance travelled by the balloon during the interval.



Ans. In right triangle ABC,



$\tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{88.2}{BC}$

$\Rightarrow$ BC = $\frac{88.2}{\sqrt{3}}$ m

In right triangle PQC,

$\tan 30^\circ = \frac{PQ}{CQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{PQ}{CB + BQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{88.2}{\frac{88.2}{\sqrt{3}} + BQ}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{88.2\sqrt{3}}{88.2 + BQ\sqrt{3}}$

$\Rightarrow 88.2 + BQ\sqrt{3} = 264.6$

$\Rightarrow BQ\sqrt{3} = 264.6 - 88.2$ = 176.4

$\Rightarrow$ BQ = $\frac{176.4}{\sqrt{3}}$ = $\frac{176.4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$

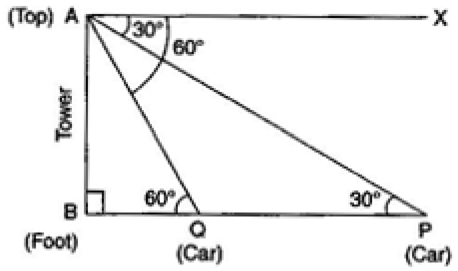
= $58.8\sqrt{3}$ = $\frac{588\sqrt{3}}{10}$ = $\frac{294\sqrt{3}}{5}$ m

Hence the distance travelled by the balloon during the interval is $\frac{294\sqrt{3}}{5}$ m.

---

4. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower

with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.



Ans. In right triangle ABP,

$$\tan 30^\circ = \frac{AB}{BP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BP}$$

$\Rightarrow$ BP = AB$\sqrt{3}$ ……….(i)

In right triangle ABQ,

$$\tan 60^\circ = \frac{AB}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BQ}$$

$\Rightarrow$ BQ = $\frac{AB}{\sqrt{3}}$ ……….(ii)

$\because$ PQ = BP – BQ

$\therefore$ PQ = AB$\sqrt{3} - \frac{AB}{\sqrt{3}} = \frac{3AB - AB}{\sqrt{3}} = \frac{2AB}{\sqrt{3}}$ = 2BQ [From eq. (ii)]

$\Rightarrow$ BQ = $\frac{1}{2}$ PQ

$\because$ Time taken by the car to travel a distance PQ = 6 seconds.

$\therefore$ Time taken by the car to travel a distance BQ, i.e. $\frac{1}{2}$ PQ = $\frac{1}{2}$ x 6 = 3 seconds.

Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

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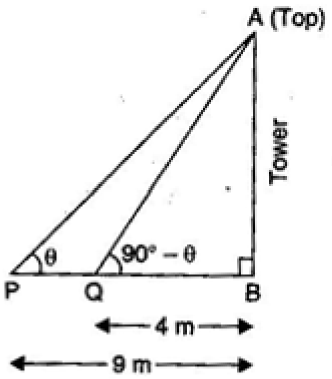
5. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.



Ans. Let ∠ APB = θ

Then, ∠ AQB = $(90^\circ - \theta)$

[∠ APB and ∠ AQB are complementary]

In right triangle ABP,

$$\tan\theta = \frac{AB}{PB}$$

$\Rightarrow \tan\theta = \frac{AB}{9}$ ....(i)

In right triangle ABQ,

$$\tan(90^\circ - \theta) = \frac{AB}{QB}$$

$\Rightarrow \cot\theta = \frac{AB}{4}$ ....(ii)

Multiplying eq. (i) and eq. (ii),

$$\frac{AB}{9}.\frac{AB}{4} = \tan\theta.\cot\theta$$

$\Rightarrow \frac{AB^2}{36} = 1$

$\Rightarrow AB^2 = 36$

$\Rightarrow$ AB = 6 m

Hence, the height of the tower is 6 m.

Proved.

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