<mark>**1 Marks Questions**</mark>

1. The next term of the AP in $1^2, 5^2, 7^2, 73$.......... is

(a) 97

(b) 92

(c) 99

(d) 95

Ans. (a) 97

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2. The 10ᵗʰ term of the AP in 2, 7, 12, ... is

(a) 45

(b) 47

(c) 48

(d) 50

Ans. (b) 47

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3. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of Radius R_1, then

(a) $R_1 + R_2 = R$

(b) $R_1 + R_2 > R$

(c) $R_1 + R_2 < R$

(d) None of these

Ans. a) $R_1 + R_2 = R$

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4. If the perimeter of a circle is equal to that of a square, then the ratio of their area is

(a) 22:7

(b) 14:11

(c) 7:22

(d) 11:14

Ans. (c) 7:22

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5. Area of a sector of angle p^0 of a circle with radius R is

(a) $\dfrac{P}{180} \times 2\pi R$

(b) $\dfrac{P}{180} \times \pi R^2$

(c) $\dfrac{P}{360} \times 2\pi R$

(d) $\dfrac{P}{720} \times 2\pi R^2$

Ans.

(d) $\dfrac{P}{720} \times 2\pi R^2$

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6. Area of the sector of angle $60°$ of a circle with radius 10cm is

(a) $52\dfrac{5}{21} cm^2$

(b) $52\dfrac{8}{21} cm^2$

(c) $52\dfrac{4}{21} cm^2$

(d) none of there

Ans. (b) $52\frac{8}{21}cm^2$

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7. 11ᵗʰ term of the AP $-3, -\frac{1}{2}, 2, \cdots$ **is**

(a) 28

(b) 22

(c) -38

(d) $-48\frac{1}{2}$

Ans. (b) 22

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8. If 17ᵗʰ term of an AP exceeds its 10ᵗʰ term by 7. The common difference is

(a) 2

(b) -1

(c) 3

(d) 1

Ans. (d) 1

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9. Which of the following list of no. form an AP?

(a) 2, 4, 8, 16 …

(b) 2, $\frac{5}{2}$ **, 3,** $\frac{7}{2}$ **,…**

(c) 0.2, 0.22, 0.222…

(d) 1, 3, 9, 27…

Ans. (b) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$,…

10. The nth term of the AP in 2, 5, 8... is

(a) 3n − 1

(b) 2n − 1

(c) 3n − 2

(d) 2n − 3

Ans. (a) 3n − 1

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11. If $a, (a-2)$ and 3a are in AP, then value of a is

(a) -3

(b) -2

(c) 3

(d) 2

Ans. (b) -2

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12. The sum of first n positive integers is given by

(a) $\dfrac{n(n-1)}{2}$;

(b) $\dfrac{n(2n+1)}{2}$;

(c) $\dfrac{n(n+1)}{2}$

(d) none of these

Ans. (c) $\dfrac{n(n+1)}{2}$

1. Find the missing variable from a, d, n and a_n, where a is the first term, d is the common difference and a_n is the nth term of AP.

(i) a= 7, d = 3, n = 8

(ii) a = −18, n= 10, a_n=0

(iii) d = −3, n = 18, a_n=−5

(iv) a = −18.9, d = 2.5, a_n=3.6

(v) a = 3.5, d = 0, n = 105

Ans. (i) a= 7, d = 3, n = 8

We need to find a_n here.

Using formula $a_n=a+(n−1)d$

Putting values of a, d and n,

$a_n=7+(8−1)3=7+(7)3=7+21=28$

(ii) a = −18, n= 10, a_n=0

We need to find d here.

Using formula $a_n=a+(n−1)d$

Putting values of a, a_n and n,

0 = −18 + (10 − 1)d

⇒ $0=−18+9d$

⇒ $18=9d$

⇒ $d=2$

(iii) d = −3, n = 18, a_n=−5

We need to find a here.

Using formula $a_n=a+(n−1)d$

Putting values of d, a_n and n,

−5 = a + (18 − 1)(−3)

⇒ $−5=a+(17)(−3)$

$\Rightarrow -5 = a - 51$

$\Rightarrow a = 46$

(iv) a = −18.9, d = 2.5, $a_n = 3.6$

We need to find n here.

Using formula $a_n = a + (n-1)d$

Putting values of d, a_n and a,

3.6 = −18.9 + (n − 1) (2.5)

$\Rightarrow 3.6 = -18.9 + 2.5n - 2.5$

$\Rightarrow 2.5n = 25$

$\Rightarrow n = 10$

(v) a = 3.5, d = 0, n = 105

We need to find a_n here.

Using formula $a_n = a + (n-1)d$

Putting values of d, n and a,

$a_n = 3.5 + (105 - 1)(0)$

$\Rightarrow a_n = 3.5 + 104 \times 0$

$\Rightarrow a_n = 3.5 + 0$

$\Rightarrow a_n = 3.5$

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2. Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10,7,4... is

(A) 97

(B) 77

(C) −77

(D) −87

(ii) 11th term of the AP: −3,−12,2...is

(A) 28

(B) 22

(C) −38

(D) $-48\frac{1}{2}$

Ans. (i) 10,7,4...

First term = a =10, Common difference = d = 7 − 10= 4 − 7= −3

And n = 30 {Because, we need to find 30ᵗʰterm}

$a_n=a+(n-1)d$

$\Rightarrow a_{30}=10+(30-1)(-3)=10-87=-77$

Therefore, the answer is (C).

(ii) −3,−½,2...

First term = a = −3, Common difference = d = −½ −(−3)=2−(−½)= $\frac{5}{2}$

And n = 11 (Because, we need to find 11ᵗʰterm)

$a_n=-3+(11-1)\frac{5}{2}=-3+25=22$

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3. Which term of the AP: 3, 8, 13, 18 ... is 78?

Ans. First term = a=3, Common difference = d = 8 − 3=13 − 8=5 and a_n=78

Using formula $a_n=a+(n-1)d$,to find nᵗʰ term of arithmetic progression,

$a_n=3+(n-1)5$,

$\Rightarrow 78=3+(n-1)5$

$\Rightarrow 75=5n-5$

$\Rightarrow 80=5n$

$\Rightarrow n=16$

It means 16ᵗʰterm of the given AP is equal to 78.

4. Find the number of terms in each of the following APs:

(i) 7,13,19....,205

(ii) 18, $15\frac{1}{2}$,13..., −47

Ans. (i) 7,13,19..., 205

First term = a =7, Common difference = d = 13 − 7= 19 − 13= 6

And a_n=205

Using formula a_n=a+(n−1)d, to find nth term of arithmetic progression,

205=7+(n−1)6=7+6n−6

⇒ 205=6n+1

⇒ 204=6n

⇒ n=34

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$,13..., −47

First term = a =18, Common difference = d = $15\frac{1}{2}-18=\frac{31}{2}-18=\frac{31-36}{2}=\frac{-5}{2}$

And a_n=−47

Using formula a_n=a+(n−1)d, to find nth term of arithmetic progression,

−47=18+(n−1)$\left(-\frac{5}{2}\right)$=36−$\frac{5}{2}$n+$\frac{5}{2}$

⇒ −94=36−5n+5

⇒ 5n=135

⇒ n=27

Therefore, there are 27 terms in the given arithmetic progression

5. Check whether −150 is a term of the AP: 11,8,5,2...

Ans. Let −150 is the nth of AP 11,8,5,2... which means that $a_n = -150$

Here, First term = a = 11, Common difference = d = 8 − 11 = −3

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

−150 = 11 + (n−1)(−3)

⇒ −150 = 11 − 3n + 3

⇒ 3n = 164

⇒ n = $\dfrac{164}{3}$

But, n cannot be in fraction.

Therefore, our supposition is wrong. −150 cannot be term in AP.

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6. An AP consists of 50 terms of which 3rdterm is 12 and the last term is 106. Find the 29thterm.

Ans. An AP consists of 50 terms and the 50thterm is equal to 106 and $a_3 = 12$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{50} = a + (50-1)d$ And $a_3 = a + (3-1)d$

⇒ 106 = a + 49d And 12 = a + 2d

These are equations consisting of two variables.

Using equation 106 = a + 49d, we get a = 106 − 49d

Putting value of a in the equation 12 = a + 2d,

12 = 106 − 49d + 2d

⇒ 47d = 94

⇒ d = 2

Putting value of d in the equation, a = 106 − 49d,

a = 106 − 49(2) = 106 − 98 = 8

Therefore, First term = a = 8 and Common difference = d = 2

To find 29th term, we use formula $a_n=a+(n-1)d$ which is used to find n^{th} term of arithmetic progression,

$a_{29}=8+(29-1)2=8+56=64$

Therefore, 29th term of AP is equal to 64

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7. How many multiples of 4 lie between 10 and 250?

Ans. The odd numbers between 0 and 50 are 1,3,5,7...49

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a = 1, Common difference = 3 − 1= 2, Last term = l=49

We do not know how many odd numbers are present between 0 and 50.

Therefore, we need to find n first.

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression, we get

49=1+(n−1)2

⇒ 49=1+2n−2

⇒ 50=2n

⇒ n=25

Applying formula, $S_n=\dfrac{n}{2}(a+l)$ to find sum of n terms of AP, we get

$S_{25}=\dfrac{25}{2}(1+49)=\dfrac{25}{2}\times 50$
$=25\times 25=625$

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8. Which term of the AP: 121, 117, 113, ...is its first negative term?

Ans. Given: 121, 117, 113, ...

Here $a=121, \quad d=117-121=4$

Now, $a_n=a+(n-1)d$

$=121+(n-1)(-4)=121-4n+4=125-4n$

For the first negative term, $a_n < 0$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow 125 < 4n$

$\Rightarrow \dfrac{125}{4} < n$

$\Rightarrow 31\dfrac{1}{4} < n$

n is an integer and $n > 31\dfrac{1}{4}.$

Hence, the first negative term is 32nd term

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9. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of sixteen terms of the AP.

Ans. Let the AP be $a - 4d, a - 3d, a - 2d, a - d, a, a + d, a + 2d, a + 3d, ...$

Then, $a_3 = a - 2d, a_7 = a + 2d$

$\Rightarrow a_3 + a_7 = a - 2d + a + 2d = 6$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$ ....(i)

Also $(a - 2d)(a + 2d) = 8$

$\Rightarrow a^2 - 4d^2 = 8$

$\Rightarrow 4d^2 = a^2 - 8$

$\Rightarrow 4d^2 = 3^2 - 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d^2 = \dfrac{1}{4}$

$\Rightarrow d = \pm\dfrac{1}{2}$

Taking $d = \dfrac{1}{2}$,

$$S_{16} = \frac{16}{2}\left[2 \times (a-4d)+(16-1)d\right]$$

$$= 8\left[2 \times \left(3-4 \times \frac{1}{2}\right)+15 \times \frac{1}{2}\right]$$

$$= 8\left[2+\frac{15}{2}\right] = 8 \times \frac{19}{2} = 76$$
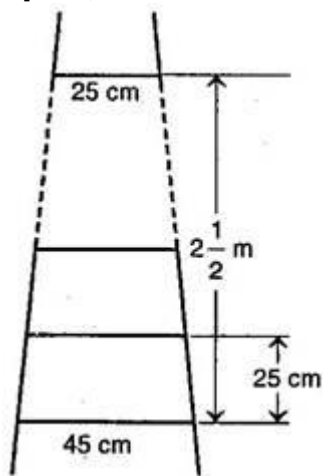
Taking $d = \dfrac{-1}{2}$,

$$S_{16} = \frac{16}{2}\left[2 \times (a-4d)+(16-1)d\right]$$

$$= 8\left[2 \times \left(3-4 \times \frac{-1}{2}\right)+15 \times \frac{-1}{2}\right]$$

$$= 8\left[\frac{20-15}{2}\right] = 8 \times \frac{5}{2} = 20$$

$\therefore$ S_{16} = 20 and 76

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10. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\dfrac{1}{2}$ m apart, what is the length of the wood required for the rungs?
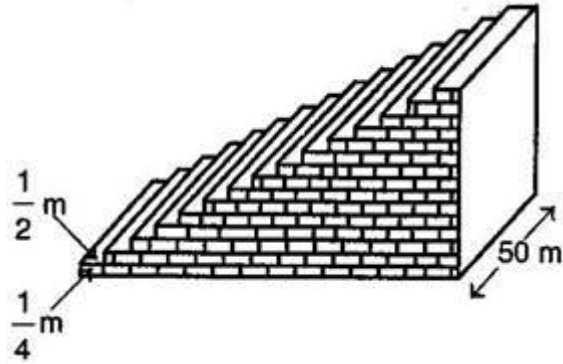

Ans. Number of rungs $(n) = \dfrac{2\dfrac{1}{2} \times 100}{25}$ = 10

The length of the wood required for rungs = sum of 10 rungs

$$= \frac{10}{2}[25+45]$$ = 5 x 70 = 350 cm

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11. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Ans. Here $a=1$ and $d=1$

$$\therefore S_{x-1} = \frac{x-1}{2}\left[2\times1+(x-1-1)\times1\right]$$

$$= \frac{x-1}{2}(2+x-2)$$

$$= \frac{(x-1)x}{2} = \frac{x^2-x}{2}$$

$$S_x = \frac{x}{2}\left[2\times1+(x-1)\times1\right]$$

$$= \frac{x}{2}(x+1) = \frac{x^2+x}{2}$$

$$S_{49} = \frac{49}{2}\left[2\times1+(49-1)\times1\right]$$

$$= \frac{49}{2}(2+48) = 49 \times 25$$

According to question,

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \frac{x^2 - x}{2} = 49 \times 25 - \frac{x^2 + x}{2}$$

$$\Rightarrow \frac{x^2 - x}{2} + \frac{x^2 + x}{2} = 49 \times 25$$

$$\Rightarrow \frac{x^2 - x + x^2 + x}{2} = 49 \times 25$$

$$\Rightarrow x^2 = 49 \times 25$$

$$\Rightarrow x = \pm 35$$

Since, x is a counting number, so negative value will be neglected.

$$\therefore x = 35$$

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12. Find the first term and the common difference $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \cdots$

Ans. $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \cdots$

$$a = \frac{1}{3}$$

$$d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$$

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13. Is $\sqrt{3}, \sqrt{6}, \sqrt{9}, \cdots$ **form an AP?**

Ans. $a_1 = \sqrt{3}, a_2 = \sqrt{6}, a_3 = \sqrt{9}$

$$d_1 = \sqrt{6} - \sqrt{3}$$
$$= \sqrt{3}(\sqrt{2} - 1)$$

$$d_2 = \sqrt{9} - \sqrt{6}$$
$$= 3 - \sqrt{6}$$

Since $d_1 \neq d_2$

Hence, it is not an AP.

14. Which is the next term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

Ans. $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots$

$$d = \sqrt{8} - \sqrt{2}$$
$$= 2\sqrt{2} - \sqrt{2}$$
$$= \sqrt{2}$$

$a_5 = a + (5-1)d$

$$= \sqrt{2} + 4 \times \sqrt{2} = 5\sqrt{2}$$

Next term is $5\sqrt{2} \text{ or } \sqrt{50}$

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15. Find the 11ᵗʰ term from the last term of the AP 10, 7, 4,………,-62.

Ans. $a = -62, \quad d = -(7-10) = 3$

$$a_{11} = a + 10d$$
$$= -62 + 10(3)$$
$$= -32$$

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16. If $x+1, 3x$ and $4x+2$ are in A.P, find the value of x.

Ans. Since $x+1,\ 3x$ and $4x+2$ are in AP

$$2(3x) = x+1+4x+2$$
$$\Rightarrow 6x = 5x+3$$

$$\Rightarrow 6x - 5x = 3$$
$$\Rightarrow x = 3$$

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17. Find the sum of first n odd natural numbers.

Ans. $1, 3, 5, 7, \ldots\ldots$

$$a = 1, d = 3 - 1 = 2$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 1 + (n-1)2\right]$$

$$= \frac{n}{2}\left[2 + 2n - 2\right]$$

$$= n^2$$

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18. Find the 12th term of the AP $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}...$

Ans. $a = \sqrt{2}, d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

$$a_{12} = a + 11d$$

$$= \sqrt{2} + 11\left(2\sqrt{2}\right)$$

$$= \sqrt{2} + 22\sqrt{2}$$

$$= 23\sqrt{2}$$

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19. Find the sum of first 11 terms of AP 2, 6, 10...

Ans. $2, 6, 10,...$

$$a = 2, d = 6 - 2 = 4$$

$$S_{11} = \frac{11}{2}\left[2 \times 2 + (11-1) \times 4\right]$$

$$= \frac{11}{2}\left[4 + 40\right]$$

$$= 11 \times 22 = 242$$

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20. Find the sum of first hundred even natural numbers divisible by 5.

Ans. Even natural no. divisible by 5 are 10, 20, 30...

$$a = 10, d = 10$$

$$n = 100$$

$$S_{100} = \frac{100}{2}\left[2(10) + (100-1).10\right]$$

$$= 50\left[20 + 99 \times 10\right]$$

$$= 50500$$

21. Find $a_{30} - a_{20}$ for the A.P $-9, -14, -19, -24, \ldots$

Ans.

$a = -9,$

$d = (-14) - (-9) = -14 + 9 = -5$

$a_{30} - a_{20} = a + 29d - a - 19d = 10d$

$\qquad = 10 \times (-5) = -50$

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22. Find the common difference and write the next two terms of the AP $1^2, 5^2, 7^2, 73, \ldots$

Ans. $1^2, 5^2, 7^2, 73, \ldots\ldots$

$\Rightarrow 1, 25, 49, 73, \ldots\ldots\ldots$

$d = a_2 - a_1 = 25 - 1 = 24$

$d = 49 - 25 = 24$

$d = 73 - 49 = 24$

Hence, it is AP.

$a_5 = 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

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23. Show that sequence defined by $a_n = 3 + 2n$ is an AP.

Ans. $a_n = 3 + 2n$

So $a_1 = 5, \ a_2 = 7, \ a_3 = 9, \ a_4 = 11$

$7 - 5 = 9 - 7 = 11 - 9 = 2$

Hence, it is AP.

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24. The first term of an AP is -7 and common difference 5. Find its general term.

Ans. $a = -7, \ d = 5$

$$a_n = a + (n-1)d$$
$$= -7 + (n-1)(5)$$
$$= -7 + 5n - 5$$
$$\therefore \qquad a_n = 5n - 12$$

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25. How many terms are there in A.P? $18, 15\frac{1}{2}, 13, \ldots\ldots, -47$

Ans. $a = 18, \quad d = \frac{31}{2} - \frac{18}{1} = \frac{-5}{2}$

$$a_n = -47$$
$$a_n = a + (n-1)d$$
$$-47 = 18 + (n-1)\left(\frac{-5}{2}\right)$$
$$\Rightarrow \qquad -47 - 18 = \frac{-5}{2}n + \frac{5}{2}$$
$$\Rightarrow \qquad n = 27$$

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26. In an AP, the sum of first n terms is $\dfrac{3n^2}{2} + \dfrac{13}{2}n.$ find its 2nd term.

Ans. $S_n = \dfrac{3n^2}{2} + \dfrac{13}{2}n$

Put $n = 1, 2, 3 \ldots$

$$S_1 = \frac{16}{2} = 8$$
$$S_2 = 19$$
$$a_1 = s_1 = 8$$

$$a_2 = S_2 - S_1 = 19 - 8 = 11$$

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27. Show that the progression $4, 7\frac{1}{4}, 10\frac{1}{2}, 13\frac{3}{4}, 17, \ldots\ldots$ **is an AP.**

Ans. $\dfrac{29}{4} - \dfrac{4}{1} = \dfrac{29-16}{4} = \dfrac{13}{4}$

And $\dfrac{21}{2} - \dfrac{29}{4} = \dfrac{13}{4}$

And $\dfrac{17}{1} - \dfrac{55}{4} = \dfrac{13}{4}$

Hence, it is an AP.

1. Write first four terms of the AP, when the first term a and common difference d are given as follows:

(i) a = 10, d=10

(ii) a= -2, d=0

(iii) a=4, d=-3

(iv) a=-1, d= ½

(v) a=-1.25, d=-0.25

Ans. (i) First term = a =10, d=10

Second term = a+d = 10 + 10 = 20

Third term = second term + d = 20 + 10 = 30

Fourth term = third term + d = 30 + 10 = 40

Therefore, first four terms are: 10, 20, 30, 40

(ii) First term = a = −2 , d=0

Second term = a+d = −2 + 0 = −2

Third term = second term + d = −2 + 0 = −2

Fourth term = third term + d = −2 + 0 = −2

Therefore, first four terms are:−2, −2, −2, −2

(iii) First term = a = 4, d=−3

Second term = a+d = 4 − 3= 1

Third term = second term + d = 1 − 3= −2

Fourth term = third term + d = −2 − 3= −5

Therefore, first four terms are: 4, 1, −2, −5

(iv) First term = a = −1, d= ½

Second term = a+d = −1+ ½ =−½

Third term = second term + d = −½ + ½ = 0

Fourth term = third term + d = 0 + ½ = ½

Therefore, first four terms are:−1, −½, 0, ½

(v) First term = a =−1.25, d = −0.25

Second term = a+d = −1.25 − 0.25= −1.50

Third term = second term + d = −1.50 − 0.25 = −1.75

Fourth term = third term + d = −1.75 − 0.25 = −2.00

Therefore, first four terms are: −1.25, −1.50, −1.75, −2.00

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2. Find the 31st term of an AP whose 11th term is 38 and 16th term is 73.

Ans. Here a_{11}=38 and a_{16}=73

Using formula a_n=a+(n−1)d,to find nth term of arithmetic progression,

38=a+(11−1)(d) And 73=a+(16−1)(d)

⇒ 38=a+10d And 73=a+15d

These are equations consisting of two variables.

We have, 38=a+10d

⇒ a=38−10d

Let us put value of a in equation (73=a+15d),

73=38−10d+15d

⇒ 35=5d

Therefore, Common difference =d=7

Putting value of d in equation 38=a+10d,

$38 = a + 70$

$\Rightarrow a = -32$

Therefore, common difference = d = 7 and First term = a = −32

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{31} = -32 + (31-1)(7) = -32 + 210 = 178$

Therefore, 31st term of AP is 178.

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3. If the third and the ninth terms of an AP are 4 and −8 respectively, which term of this AP is zero?

Ans. It is given that 3rd and 9th term of AP are 4 and −8 respectively.

It means $a_3 = 4$ and $a_9 = -8$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

4 = a + (3 − 1)d And, −8 = a + (9 − 1)d

$\Rightarrow 4 = a + 2d$ And, $-8 = a + 8d$

These are equations in two variables.

Using equation $4 = a + 2d$, we can say that $a = 4 - 2d$

Putting value of a in other equation $-8 = a + 8d$,

$-8 = 4 - 2d + 8d$

$\Rightarrow -12 = 6d$

$\Rightarrow d = -2$

Putting value of d in equation $-8 = a + 8d$,

$-8 = a + 8(-2)$

$\Rightarrow -8 = a - 16$

$\Rightarrow a = 8$

Therefore, first term $= a = 8$ and Common Difference $= d = -2$

We want to know which term is equal to zero.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$0 = 8 + (n-1)(-2)$

$\Rightarrow 0 = 8 - 2n + 2$

$\Rightarrow 0 = 10 - 2n$

$\Rightarrow 2n = 10$

$\Rightarrow n = 5$

Therefore, 5th term is equal to 0.

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4. Two AP's have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms.

Ans. Let first term of 1st AP = a

Let first term of 2nd AP = a'

It is given that their common difference is same.

Let their common difference be d.

It is given that difference between their 100th terms is 100.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a + (100-1)d - [a' + (100-1)d] = a + 99d - a' - 99d = 100$

$\Rightarrow a - a' = 100 \ldots (1)$

We want to find difference between their 1000th terms which means we want to calculate:

$a + (1000-1)d - [a' + (1000-1)d] = a + 999d - a' - 999d = a - a'$

Putting equation (1) in the above equation,

$a + (1000-1)d - [a' + (1000-1)d] = a + 999d - a' + 999d = a - a' = 100$

Therefore, difference between their 1000th terms would be equal to 100.

---

5. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105,112,119..., 994

Let 994 is the nth term of AP.

We need to find n here.

First term = a = 105, Common difference = d = 112 − 105= 7

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$994 = 105 + (n-1)(7)$

$\Rightarrow 994 = 105 + 7n - 7$

$\Rightarrow 896 = 7n$

$\Rightarrow n = 128$

It means 994 is the 128th term of AP.

Therefore, there are 128 terms in AP.

___

6. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250
Penalty for third day = Rs 300
It is given that penalty for each succeeding day is Rs 50 more than the preceding day.
It makes it an arithmetic progression because the difference between consecutive terms is constant.
We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.
So, we have an AP of the form 200, 250, 300, 350 ... 30 terms
First term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$S_n = \frac{30}{2}[400 + (30-1)50]$

$\Rightarrow S_n = 15(400 + 29 \times 50)$
$\Rightarrow S_n = 15(400 + 1450) = 27750$
Therefore, penalty for 30 days is Rs. 27750.

___

7. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.
The number of trees planted by class I = number of sections × 1 = 3 × 1 = 3
The number of trees planted by class II = number of sections × 2 = 3 × 2 = 6
The number of trees planted by class III = number of sections × 3 = 3 × 3 = 9
Therefore, we have sequence of the form 3, 6, 9 ... 12 terms
To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 ... 12 terms.
First term = a = 3, Common difference = d= 6 − 3= 3 and n = 12

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_{12} = \frac{12}{2}[6+(12-1)3] = 6(6+33) = 6\times39 = 234$

---

8. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, ... (in m²) are

$\frac{1}{4}\times\frac{1}{2}\times50, \left(2\times\frac{1}{4}\right)\times\frac{1}{2}\times50, \left(3\times\frac{1}{4}\right)\times\frac{1}{2}\times50,....$

$\Rightarrow \frac{50}{8}, 2\times\frac{50}{8}, 3\times\frac{50}{8},.........$

∴ Total volume of concrete required = $\frac{50}{8}+2\times\frac{50}{8}+3\times\frac{50}{8}+........$

$= \frac{50}{8}[1+2+3+.......]$

$= \frac{50}{8}\times\frac{15}{2}[2\times1+(15-1)\times1]$ $[\because n=15]$

$= \frac{50}{8}\times\frac{15}{2}\times16$

= 750 m³

---

9. For what value of n are the nth term of the following two AP's are same 13, 19, 25,…. and 69, 68, 67 …

Ans. nth term of 13, 19, 25 ,…………. = nth term of 69, 68, 67,………
13+(n − 1) × 6 = 69 + (n − 1) (-1)
Therefore, n = 9

---

10. Check whether 301 is a term of the list of numbers 5, 11, 17, 32,……..?

Ans. $d = 11 - 5 = 6$
$a = 5$
$a_n = a + (n-1)d$
$\Rightarrow 301 = 5 + (n-1)d$
$\Rightarrow n = 151$
So, 301 is not a term of the given list.

---

11. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Ans.
$a_3 = 16$
$\Rightarrow a + 2d = 16 \ldots (i)$
$a_7 = a_5 + 12$
$\Rightarrow a + 6d = a + 4d + 12$
$\Rightarrow 2d = 12$
$\Rightarrow d = 6$
Put the valve of d in eq. (i)
$a + 2 \times 6 = 16$
$\Rightarrow a = 16 - 12$
$\Rightarrow a = 4$
4,10,16…

---

12. Find the sum of AP in $-5 + (-8) + (-11) + \ldots\ldots + (-230)$

Ans. $a = -5$

$d = -8 - (5)$

$= -8 + 5 = -3$

$a_n = -230$

$a_n = a + (n-1)d$

$\Rightarrow -230 = -5 + (n-1)(-3)$

$\Rightarrow -230 = -5 - 3n + 3$

$\Rightarrow -230 + 2 = -3n$

$\Rightarrow n = 76$

$S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

$S_{76} = \dfrac{76}{2}\left[2 \times (-5) + (76-1)(-3)\right]$

$= 38\left[-10 + 75 \times (-3)\right]$

$= 38\left[-10 - 225\right]$

$= 38 \times (-235)$

$= -8930$

---

13. In an AP, $a_n = 4, d = 2, S_n = -14$ find n and a.

Ans. $a_n = a + (n-1)d$

$\Rightarrow 4 = a + (n-1).(2)$

$\Rightarrow a + 2n = 6 \dots (i)$

$S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\Rightarrow -14 = \dfrac{n}{2}(a+4)$

$\Rightarrow -28 = n[6 - 2n + 4] \ [\because a = 6 - 2n]$

$\Rightarrow n^2 - 5n - 14 = 0$

$\Rightarrow n = 7, \quad n = -2$

$a = -8$

---

14. Find $a_{30} - a_{20}$ for the AP in -9, -14, -19, -24…

Ans. $a = -9$

$d = -14 - (-9) = -14 + 9 = -5$

$a_{30} - a_{20} = (a + 29d) - (a + 19d)$

$= 10d = 10 \times (-5) = -50$

---

15. Find the sum to n term of the AP in 5, 2, -1, -4, -7……….

Ans. $a = 5, d = 2 - 5 = -3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2 \times 5 + (n-1)(-3)\right]$$

$$= \frac{n}{2}\left[10 - 3n + 3\right]$$

$$= \frac{n}{2}\left[13 - 3n\right]$$

---

16. Find the sum of first 24 terms of the list of no. whose nth term is given by $a_n = 3 + 2n$

Ans. $a_n = 3 + 2n$

Put $n = 1, 2, 3, ...$

$a_1 = 5, \ a_2 = 7, \ a_3 = 9...$

$a = 5, \ d = 7 - 5 = 2$

$$S_{24} = \frac{24}{2}\left[2 \times 5 + (24 - 1) \times 2\right]$$

$$= 12\left[10 + 46\right] = 672$$

==**4 Marks Questions**==

1. For the following APs, write the first term and the common difference.

(i) 3, 1, −1, −3 …

(ii) −5, −1, 3, 7...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3} ...$

(iv) 0.6, 1.7, 2.8, 3.9...

Ans. (i) 3, 1, −1, −3...
First term = a= 3,
Common difference (d) = Second term − firstterm = Third term − second term and so on
Therefore, Common difference (d) = 1 − 3= −2

(ii) −5, −1, 3, 7...
First term = a= −5
Common difference (d) = Second term −Firstterm

= Third term − Second term and so on
Therefore, Common difference (d) = −1 − (−5)= −1+5 =4

(iii) $\dfrac{1}{3}, \dfrac{5}{3}, \dfrac{9}{3}, \dfrac{13}{3} \ldots$

First term = a= $\dfrac{1}{3}$

Common difference (d) = Second term −First term
= Third term −Second term and so on

Therefore, Common difference (d) = $\dfrac{5}{3} - \dfrac{1}{3} = \dfrac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9...
First term = a= 0.6
Common difference (d) = Second term −Firstterm
= Third term −Secondterm and so on
Therefore, Common difference (d) = 1.7−0.6=1.1

---

2. The 17th term of an AP exceeds its 10th term by 7. Find the common difference

Ans. (i) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 3 + 4n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 3 + 4 = 7$ $a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$ $a_4 = 3 + 4(4) = 3 + 16 = 19$

So, the sequence is of the form 7,11,15,19...

Let us check difference between consecutive terms of this sequence.

11 − 7= 4, 15 − 11= 4, 19 − 15= 4

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence 7,11,15,19...

First term = a =7 and Common difference = d = 4

Applying formula, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$S_{15} = \dfrac{15}{2}\left[14 + (15-1)4\right] = \dfrac{15}{2}(14 + 56) = \dfrac{15}{2} \times 70 = 15 \times 35 = 525$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 9 - 5n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 9 - 5 = 4 \quad a_2 = 9 - 5(2) = 9 - 10 = -1$

$a_3 = 9 - 5(3) = 9 - 15 = -6 \quad a_4 = 9 - 5(4) = 9 - 20 = -11$

So, the sequence is of the form $4, -1, -6, -11 \ldots$

Let us check difference between consecutive terms of this sequence.

$-1 - (4) = -5, \ -6 - (-1) = -6 + 1 = -5, \ -11 - (-6) = -11 + 6 = -5$

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 \ldots a_n$ form an AP.

We have sequence $4, -1, -6, -11 \ldots$

First term = a = 4 and Common difference = d = -5

Applying formula, $S_n = \dfrac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP , we get

$S_{15} = \dfrac{15}{2}[8 + (15-1)(-5)] = \dfrac{15}{2}(8 - 70) = \dfrac{15}{2} \times (-62) = 15 \times (-31) = -465$

Therefore, sum of first 15 terms of AP is equal to -465.

---

3. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is R.s 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize = Rs (a−20)

Let value of third prize = Rs (a−40)

So, we have sequence of the form:

a, a−20, a−40, a − 60...

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a, Common difference = d = (a − 20) − a= −20

n = 7 (Because there are total of seven prizes)

S_7 = Rs 700 {given}

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_7 = \frac{7}{2}[2a+(7-1)(-20)]$ $\Rightarrow$ $700 = \frac{7}{2}[2a-120]$

$\Rightarrow 200=2a-120$

$\Rightarrow 320=2a$

$\Rightarrow a=160$

Therefore, value of first prize = Rs 160

Value of second prize = 160 − 20= Rs 140

Value of third prize = 140 − 20= Rs 120

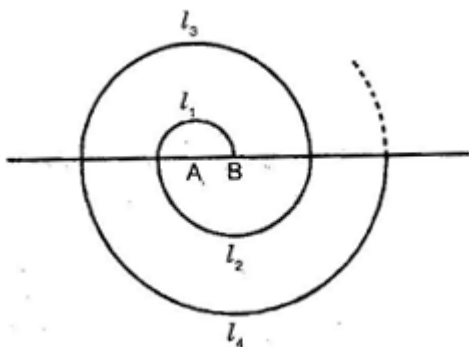Value of fourth prize = 120 − 20 = Rs 100

Value of fifth prize = 100 − 20 = Rs 80

Value of sixth prize = 80 − 20 = Rs 60

Value of seventh prize = 60 − 20 = Rs 40

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4. A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircle.

Ans. Length of semi−circle = $\dfrac{\text{Circumference of circle}}{2} = \dfrac{2\pi r}{2} = \pi r$

Length of semicircle of radii 0.5 cm = π (0.5) cm

Length of semicircle of radii 1.0 cm = π (1.0) cm

Length of semicircle of radii 1.5 cm = π (1.5) cm

Therefore, we have sequence of the form:

π (0.5), π (1.0), π (1.5) ... 13 terms {There are total of thirteen semicircles}.

To find total length of the spiral, we need to find sum of the sequence π (0.5), π (1.0), π (1.5) ... 13 terms

Total length of spiral = π (0.5)+ π (1.0)+ π (1.5) ... 13 terms

⇒ Total length of spiral = π (0.5+1.0+1.5)... 13 terms … (1)

Sequence 0.5, 1.0, 1.5 ...13 terms is an arithmetic progression.

Let us find the sum of this sequence.

First term = a= 0.5, Common difference = 1.0 − 0.5 = 0.5 and n = 13

Applying formula, $S_n = \dfrac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$S_{13} = \dfrac{13}{2}[1+(13-1)0.5] = 6.5(1+6) = 6.5 \times 7 = 45.5$

Therefore, 0.5 + 1.0 + 1.5 + 2.0 ...13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral = π (0.5+1.5+2.0+ ...13 terms) = π (45.5) = 143 cm

---

5. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?

Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = a = 20, Common difference = d = 19 − 20 = −1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$200 = \frac{n}{2}[40+(n-1)(-1)]$ ⇒ 400=n(40 − n+1)

⇒ 400=40n−n²+n

⇒ n²−41n+400=0

It is a quadratic equation, we can factorize to solve the equation.

⇒ n²−25n−16n+400=0

⇒ n(n−25) − 16(n−25)=0

⇒ (n−25)(n−16)

⇒ n=25,16

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a+l)$ to find sum of n terms of AP, we get

200= 8 (20+*l*)

⇒ 200 = 160 + 8*l*

⇒ 40 = 8*l*

⇒ *l*= 5

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

---

6. In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
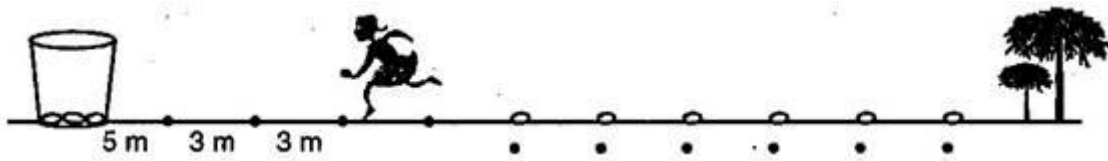
Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pickup first potato and put it in bucket = 5 × 2 = 10 meters

The distance of Second potato from the starting point = 5 + 3 = 8 meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pickup 2nd potato and put it in bucket = 8 × 2 = 16 meters

The distance of third potato from the starting point = 8 + 3 = 11 meters

Therefore, the distance covered by competitor to pickup 3rd potato and put it in bucket = 11 × 2 = 22 meters

Therefore, we have a sequence of the form 10, 16, 22...10 terms

(There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find :

10 + 16 + 22 + ... 10 terms

First term = a = 10, Common difference = d =16 − 10= 6

n = 10 {There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP, we get

$$S_{n10} = \frac{10}{2}[20 + (10-1)6] = 5(20+54) = 5 \times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.

7. Which term of the sequence $20, \ 19\frac{1}{4}, \ 18\frac{1}{2}, \ 17\frac{3}{4}, \ldots$ **is the first negative term?**

Ans. For first negative term

$a_n < 0$

$20 + (n-1).\left(\dfrac{-3}{4}\right) < 0 \quad \left[\because d = \dfrac{77}{4} - 20 = \dfrac{-3}{4}\right]$

$\Rightarrow \dfrac{20}{1} - \dfrac{3}{4}n + \dfrac{3}{4} < 0$

$\Rightarrow \dfrac{80 - 3n + 3}{4} < 0$

$\Rightarrow 83 - 3n < 0$

$\Rightarrow -3n < -83$

$\Rightarrow n > \dfrac{83}{3}$

28ᵗʰ term is first negative term.

---

8. The pᵗʰ term of an AP is q and qᵗʰ term is p. Find its $(p+q)^{th}$ **term.**

Ans. $q = a + (p-1)d \ldots (i)$

$p = a + (q-1)d \ldots (ii)$

$q - p = (p - 1 - q + 1)d$

$\dfrac{q-p}{p-q} = d$

$\Rightarrow d = -1$

Put the value of d in eq (i)

$q = a + (p-1)(-1)$

$\Rightarrow q = a - p + 1$

$\Rightarrow a = q + p - 1$

$a_{p+q} = a + (p + q - 1)d$

$= (q + p - 1) + (p + q - 1)(-1)$

$= q + p - 1 - p - q + 1$

$= 0$

---

9. If m times the mᵗʰ term of an A.P is equal to n times its nᵗʰ term, show that the $(m+n)^{th}$ **term of the AP is zero.**

Ans. $ma_m = n\,a_n$

$m\left[a+(m-1)d\right]=n\left[a+(n-1)d\right]$

$\Rightarrow ma+m^2d-md=na+n^2d-nd$

$\Rightarrow a(m-n)+(m^2-n^2)d-md+nd=0$

$\Rightarrow a(m-n)+(m-n)(m+n)d-(m-n)d=0$

$\Rightarrow (m-n)\left[a+(m+n-1)d\right]=0$

$\Rightarrow a+(m+n-1)d=0$

$\Rightarrow a_{m+n}=0$

Hence proved.

---

10. The sum of the 4ᵗʰ and 8ᵗʰ terms of an AP is 24 and the sum of the 6ᵗʰ and 10ᵗʰ terms is 44. Find the first three terms of the AP.

Ans. $a_4+a_8=24$ (Given)

$\Rightarrow a+3d+a+7d=24$

$\Rightarrow 2a+10d=24$

$\Rightarrow a+5d=12.......(i)$

$a_6+a_{10}=44$

$\Rightarrow a+5d+a+9d=44$

$\Rightarrow 2a+14d=44$

$\Rightarrow a+7d=22.........(ii)$

On solving equations (i) and (ii)

$d=5,\ a=-13$

First three terms are -13,-8,-3.

---

11. If the sum of n terms of an AP is $3n^2+5n$ and its mᵗʰ term is 164, find the value of m.

Ans. $Sn=3n^2+5n$

Put $n=1,2,3,....$

$S_1=8$

$S_2=3\times4+10=22$

$a_1=S_1=8$

$a_2 = S_2 - S_1$

$= 22 - 8 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

$a_m = 164$

$\Rightarrow a + (m-1)d = 164$

$\Rightarrow 8 + (m-1)(6) = 164$

$\Rightarrow 8 + 6m - 6 = 164$

$\Rightarrow 6m = 164 - 2$

$\Rightarrow 6m = 162$

$\Rightarrow m = \dfrac{162}{6} = 27$

---

12. If the sum of three numbers in AP, be 24 and their product is 440, find the numbers.

Ans. Let no. be a − d, a, a + d

$(a-d) + a + (a+d) = 24$ (Given)

$\Rightarrow a = 8$

$(a-d)(a)(a+d) = 440$

$\Rightarrow (8-d).8.(8+d) = 440$

$\Rightarrow 8^2 - d^2 = 55$

$\Rightarrow d = \pm 3$

a = 8	a = 8
d = +3	d = −3
Then AP	Then AP
5, 8, 11,...	11, 8, 5,....

.

---

13. If a^2, b^2, c^2 are in AP, then prove that $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in AP.

Ans. Given a^2, b^2, c^2 are in AP

Then $2b^2 = a^2 + c^2 \dots\dots(i)$

If $\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in AP then

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{(b+c)-(c+a)}{(c+a)(b+c)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$$

$$\frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$2b^2 = c^2 + a^2 \dots\dots(ii)$$

From (i) and (ii),

Hence proved.

---

14. If S_1, S_2, S_3 be the sum of n, 2n and 3n terms respectively of an AP, prove that $S_3 = 3(S_2 - S_1)$

Ans. $S_1 = \frac{n}{2}\left[2a + (n-1)d\right]$

$$S_2 = \frac{2n}{2}\left[2a + (2n-1)d\right]$$

$$S_3 = \frac{3n}{2}\left[2a + (3n-1)d\right]$$

R.H.S $= 3(S_2 - S_1)$

$$= 3\left[\frac{2n}{2}\left(2a + (2n-1)d\right) - \frac{n}{2}\left(2a + (n-1)d\right)\right]$$

$$= 3\left[\frac{n}{2}\left[4a + 4nd - 2d - 2a - nd + d\right]\right]$$

$$= 3\left[\frac{n}{2}\left(2a + 3nd - d\right)\right]$$

$$= \frac{3n}{2}\left[2a + (3n-1)d\right] = S_3$$

---

15. The ratio of the sums of m and n terms of an AP is $m^2 : n^2$, show that the ratio of the mth and n^{th} term is $(2m-1):(2n-1)$.

Ans. $\dfrac{S_m}{S_n} = \dfrac{\frac{m}{2}\left[2a + (m-1)d\right]}{\frac{n}{2}\left[2a + (n-1)d\right]}$

$$\Rightarrow \frac{m^2}{n^2} = \frac{m}{n}\left[\frac{2a + (m-1)d}{2a + (n-1)d}\right]$$

$$\Rightarrow \frac{m}{n} = \frac{2a + md - d}{2a + nd - d}$$

$$\Rightarrow 2am + mnd - md = 2an + mnd - nd$$

$$\Rightarrow 2am - 2an - md + nd = 0$$

$$\Rightarrow 2a(m-n) - (m-n)d = 0$$

$$\Rightarrow (m-n)(2a-d) = 0$$

$$\Rightarrow 2a = d$$

According to question,

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$$

$$= \frac{a+(m-1)2a}{a+(n+1)2a}$$

$$= \frac{a[1+2m-2]}{a[1+2n-2]}$$

$$= \frac{2m-1}{2n-1}$$

Hence Proved

---

16. If the sum of first p terms of an AP is the same as the sum of its first q terms, show that the sum of the first $(p+q)$ term is zero.

Ans. $S_p = S_q$

$$\Rightarrow \frac{p}{2}[2a+(p-1)d] = \frac{q}{2}[2a+(q-1)d]$$

$$\Rightarrow p[2a+pd-d] = q[2a+qd-d]$$

$$\Rightarrow 2ap + p^2d - pd = 2aq + q^2d - qd$$

$$\Rightarrow 2a(p-q) + (p^2-q^2)d - (p-q)d = 0$$

$$\Rightarrow 2a(p-q) + (p-q)(p-q)d - (p-q)d = 0$$

$$\Rightarrow (p-q)[2a+(p+q-1)d] = 0$$

$$\Rightarrow S_{p+q} = 0$$

---

17. For the A.P $a_1, a_2, a_3, \ldots$ if $\frac{a_4}{a_7} = \frac{2}{3}$, find $\frac{a_6}{a_8}$.

Ans. $\dfrac{a_4}{a_7} = \dfrac{2}{3}$ (Given)

$\Rightarrow \dfrac{a+3d}{a+6d} = \dfrac{2}{3}$

$\Rightarrow 3a+9d = 2a+12d$

$\Rightarrow a = 12d-9d$

$\Rightarrow a = 3d$

$\dfrac{a_6}{a_7} = \dfrac{a+5d}{a+6d}$

$= \dfrac{3d+5d}{3d+6d} = \dfrac{8d}{9d} = 8:9$

---

18. In an AP p^{th}, q^{th} and r^{th} terms are respectively a, b and c. Prove that $p(b-c)+q(c-a)+r(a-b)=0$

Ans. $A+(p-1)D = a$..........(i)

$A+(q-1)D = b$...........(ii)

$A+(r-1)D = c$...........(iii)

$(ii)-(iii)$

$b-c = (q-1)D-(r-1)D$

$\Rightarrow b-c = D(q-r)$

$\Rightarrow p(b-c) = p\ D(q-r)$.........(iv)

Similarly,

$q(c-a) = q\ D(r-p)$...........(v)

$r(a-b) = r\ D(p-q)$............(vi)

Adding (iv), (v) and (vi)

$p(b-c)+q(c-a)+r(a-b)=0$

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19. If $(p+1)^{th}$ term of an A.P is twice the $(q+1)^{th}$ term, show that $(3p+1)^{th}$ term is twice the $(p+q+1)^{th}$ term.

Ans. $a_{p+1} = 2a_{q+1}$

$\Rightarrow a + (p+1-1)d = 2\left[a + (q+1-1)d\right]$

$\Rightarrow a + pd = 2\left[a + qd\right]$

$\Rightarrow a + pd = 2a + 2qd$

$\Rightarrow pd - 2qd = a$

$\Rightarrow (p - 2q)d = a$

$\dfrac{a_{3p+1}}{a_{p+q+1}} = \dfrac{a + (3p+1-1)d}{a + (p+q+1-1)d}$

$= \dfrac{(p-2q)d + 3pd}{p - 2q + (p+q)d} = 2$

---

20. The sum of four numbers in AP is 50 and the greatest number four times the least. Find the numbers.

Ans. Let no. be $(a-3d),(a-d),(a+d),(a+3d)$

$(a-3d) + (a-d) + (a+d) + (a+3d) = 50$

$\Rightarrow 4a = 50$

$\Rightarrow a = \dfrac{50}{4} = \dfrac{25}{2}$

According to question,

$(a+3d) = 4 \times (a-3d)$

$\Rightarrow a + 3d = 4a - 12d$

$\Rightarrow -3a = -15d$

$\Rightarrow a = 5d$

$\Rightarrow \dfrac{25}{2} = 5d$

$\Rightarrow \dfrac{5}{2} = d$

Numbers be 5, 10, 15, 20

---

21. Find the sum of all integers between 84 and 719 which are multiples of 5.

Ans. 85, 90, 95, ... 715

$a = 85,\ d = 5,\ a_n = 715$

$a + (n-1)d = a_n$

$\Rightarrow 85 + (n-1).5 = 715$

$\Rightarrow n = 127$

$S_{127} = \dfrac{127}{2}(85 + 715)$

$= 50800$

22. If m^{th} term of an A.P is $\dfrac{1}{n}$ and the nth term is $\dfrac{1}{m}$, show that the sum of mn terms is $\dfrac{1}{2}(mn+1)$.

Ans. $\dfrac{1}{n} = a + (m+1)d \dots (i)$

$\dfrac{1}{m} = a + (n+1)d \dots (ii)$

On solving (i) and (ii),

$a = \dfrac{1}{mn}, \quad d = \dfrac{1}{mn}$

$S_{mn} = \dfrac{mn}{2}\left[2a + (mn-1)d\right]$

$= \dfrac{mn}{2}\left[2 \cdot \dfrac{1}{mn} + (mn-1) \cdot \dfrac{1}{mn}\right]$

$= \dfrac{1}{2}(mn+1)$

---

23. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes 14th of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

(iv) The amount of money in the account every year, when Rs 10,000 is deposited at compound Interest at 8% per annum.

Ans. (i) Taxi fare for 1st km = Rs 15, Taxi fare after 2 km = 15+8 = Rs 23

Taxi fare after 3 km = 23+8 = Rs 31

Taxi fare after 4 km = 31 +8 = Rs 39

Therefore, the sequence is 15, 23, 31, 39...

It is an arithmetic progression because difference between any two consecutive terms is equal which is 8.(23 − 15=8, 31 − 23=8, 39 − 31=8,...)

(ii) Let amount of air initially present in a cylinder = V

Amount of air left after pumping out air by vacuum pump = $V - \dfrac{V}{4} = \dfrac{4V - V}{4} = \dfrac{3V}{4}$

Amount of air left when vacuum pump again pumps out air

$= \dfrac{3}{4}V - \left(\dfrac{1}{4} \times \dfrac{3}{4}V\right) = \dfrac{3}{4}V - \dfrac{3}{16}V = \dfrac{12V - 3V}{16} = \dfrac{9}{16}V$

So, the sequence we get is like $V, \dfrac{3}{4}V, \dfrac{9}{16}V \ldots$

Checking for difference between consecutive terms ...

$\dfrac{3}{4}V - V = -\dfrac{V}{4}, \; \dfrac{9}{16}V - \dfrac{3}{4}V = \dfrac{9V - 12V}{16} = \dfrac{-3V}{16}$

Difference between consecutive terms is not equal.

Therefore, it is not an arithmetic progression.

(iii) Cost of digging 1 meter of well = Rs 150

Cost of digging 2 meters of well = 150+50=Rs 200

Cost of digging 3 meters of well = 200+50 = Rs 250

Therefore, we get a sequence of the form 150, 200, 250...

It is an arithmetic progression because difference between any two consecutive terms is equal.(200 − 150=250 − 200= 50...)

Here, difference between any two consecutive terms which is also called common difference is equal to 50.

(iv) Amount in bank after Ist year = $10000\left(1 + \dfrac{8}{100}\right)$ ... (1)

Amount in bank after two years = $10000\left(1 + \dfrac{8}{100}\right)^2$ ... (2)

Amount in bank after three years = $10000\left(1 + \dfrac{8}{100}\right)^3$ ... (3)

Amount in bank after four years = $10000\left(1 + \dfrac{8}{100}\right)^4$ ... (4)

It is not an arithmetic progression because (2)−(1)≠(3)−(2)

(Difference between consecutive terms is not equal)

Therefore, it is not an Arithmetic Progression.

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24. In the following AP's find the missing terms:

(i) 2,__ , 26

(ii) __, 13, __, 3

(iii) 5, __, __, $9\frac{1}{2}$

(iv) −4. __, __, __, __, 6

(v) __, 38, __, __, __, −22

Ans. (i) 2, __ , 26

We know that difference between consecutive terms is equal in any A.P.

Let the missing term be x.

$x-2=26-x$

$\Rightarrow 2x=28 \Rightarrow x=14$

Therefore, missing term is 14.

(ii) __, 13, __, 3

Let missing terms be x and y.

The sequence becomes x, 13, y, 3

We know that difference between consecutive terms is constant in anyA.P.

$y-13=3-y$

$\Rightarrow 2y=16$

$\Rightarrow y=8$

And $13-x=y-13$

$\Rightarrow x+y=26$

But, we have $y=8$,

$\Rightarrow x+8=26$

$\Rightarrow x=18$

Therefore, missing terms are 18 and 8.

(iii) $5, \underline{\ \ }, \underline{\ \ }, 9\frac{1}{2}$

Here, first term = $a=5$ And, 4thterm $=a_4= 9\frac{1}{2}$

Using formula $a_n=a+(n-1)d$, to find nthterm of arithmetic progression,

$a_4=5+(4-1)d \Rightarrow \frac{19}{2} =5+3d$

$\Rightarrow 19 = 2(5+3d)$

$\Rightarrow 19 = 10 + 6d$
$\Rightarrow 6d = 19 - 10$

$\Rightarrow 6d = 9 \Rightarrow d = \frac{3}{2}$

Therefore, we get common difference = d = $\frac{3}{2}$

Second term = a+d = $5+\frac{3}{2}=\frac{13}{2}$

Third term = second term + d = $\frac{13}{2}+\frac{3}{2}=\frac{16}{2}=8$

Therefore, missing terms are $\frac{13}{2}$ and 8

(iv) $-4. \underline{\ \ }, \underline{\ \ }, \underline{\ \ }, \underline{\ \ }, 6$

Here, First term = a = −4 and 6th term = a_6 = 6

Using formula $a_n=a+(n-1)d$, to find nth term of arithmetic progression,

$a_6=-4+(6-1)d \Rightarrow 6=-4+5d$

$\Rightarrow 5d=10 \Rightarrow d=2$

Therefore, common difference = d = 2

Second term = first term + d = a+ d = −4 + 2 = −2

Third term = second term + d = −2 + 2 = 0

Fourth term = third term + d = 0 + 2 = 2

Fifth term = fourth term + d = 2 + 2 = 4

Therefore, missing terms are −2, 0, 2 and 4.

(v) __, 38, __, __, __, −22

We are given 2nd and 6th term.

Using formula $a_n = a+(n-1)d$, to find n^{th} term of arithmetic progression,

$a_2 = a+(2-1)d$ And $a_6 = a+(6-1)d$

$\Rightarrow 38 = a+d$ And $-22 = a+5d$

These are equations in two variables, we can solve them using any method.

Using equation ($38 = a+d$), we can say that $a = 38-d$.

Putting value of a in equation ($-22 = a+5d$),

$-22 = 38 - d+5d$

$\Rightarrow 4d = -60$

$\Rightarrow d = -15$

Using this value of d and putting this in equation $38 = a+d$,

$38 = a-15$

$\Rightarrow a = 53$

Therefore, we get $a = 53$ and $d = -15$

First term = a = 53

Third term = second term + d = 38 − 15= 23

Fourth term = third term + d = 23 − 15= 8

Fifth term = fourth term + d = 8 − 15= −7

Therefore, missing terms are 53, 23, 8 and −7.

25. The given figure depicts a racing track whose left and right ends are semi-circular. The difference between the two inner parallel line segments is 60m and they are each 106 m long. If the track is 10m wide, find:

(i) The distance around the track along its inner edge,

(ii) The area of the track

Ans. (i) The distance around the track along the inner edge

$$= 106 + 106 + (\pi \times 30 + \pi \times 30)$$

$$= 212 + \frac{22}{7} \times 60 = 212 + \frac{1320}{7}$$

$$= \frac{2804}{7} m$$

(ii) The area of the track $= 106 \times 80 - 106 \times 60 + 2.\frac{1}{2}\pi\left[40^2 - 30^2\right]$

$$= 106 \times 20 + \pi(70)(10)$$

$$= 2120 + 700 \times \frac{22}{7} = 2120 + 2200$$

$$= 4320 m^2$$

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26. Which of the following are APs? If they form an AP, find the common difference d and write threemore terms.

(i) 2,4,8,16...

(ii) 2, $\frac{5}{2}$, 3, $\frac{7}{2}$ . ..

(iii) −1.2,−3.2,−5.2,−7.2…

(iv) −10,−6,−2,2...

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}...$

(vi) 0.2,0.22,0.222,0.2222...

(vii) 0,−4,−8,−12...

(viii) $-\dfrac{1}{2}, -\dfrac{1}{2}, -\dfrac{1}{2}, -\dfrac{1}{2} \ldots$

(ix) 1,3,9,27...

(x) *a,2a,3a,4a...*

(xi) *a,a²,a³,a⁴...*

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

(xiv) $1^2, 3^2, 5^2, 7^2 \ldots$

(xv) $1^2, 5^2, 7^2, 73 \ldots$

Ans. (i) 2,4,8,16...

It is not an AP because difference between consecutive terms is not equal.

As $4 - 2 \neq 8 - 4$

(ii) $2, \dfrac{5}{2}, 3, \dfrac{7}{2} \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow \dfrac{5}{2} - 2 = 3 - \dfrac{5}{2} = \dfrac{1}{2}$

Common difference (d) = ½

Fifth term = $\dfrac{7}{2} + \dfrac{1}{2} = 4$ Sixth term = 4 + ½ = $\dfrac{9}{2}$

Seventh term = $\dfrac{9}{2} + \dfrac{1}{2} = 5$

Therefore, next three terms are 4, $\dfrac{9}{2}$ and 5.

(iii) −1.2,−3.2,−5.2,−7.2...

It is an AP because difference between consecutive terms is equal.

$\Rightarrow$ −3.2−(−1.2)=−5.2−(−3.2)=−7.2−(−5.2)=−2

Common difference (d) = −2

Fifth term = −7.2 − 2=−9.2

Sixth term = −9.2 − 2=−11.2
Seventh term = −11.2 − 2=−13.2

Therefore, next three terms are −9.2,−11.2 and −13.2

(iv) −10,−6,−2,2...

It is an AP because difference between consecutive terms is equal.

⟹ −6−(−10) =−2−(−6) =2−(−2)=4

Common difference (d) = 4

Fifth term = 2 + 4 = 6 Sixth term = 6 + 4 = 10

Seventh term = 10 + 4 = 14

Therefore, next three terms are 6,10 and 14

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}...$

It is an AP because difference between consecutive terms is equal.

⟹ $3+\sqrt{2}-3=\sqrt{2}, 3+2\sqrt{2}-(3+\sqrt{2})=3+2\sqrt{2}-3-\sqrt{2}=\sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $3+3\sqrt{2}+\sqrt{2}=3+4\sqrt{2}$

Sixth term = $3+4\sqrt{2}+\sqrt{2}=3+5\sqrt{2}$

Seventh term = $3+5\sqrt{2}+\sqrt{2}=3+6\sqrt{2}$

Therefore, next three terms are $\left(3+4\sqrt{2}\right),\left(3+5\sqrt{2}\right),\left(3+6\sqrt{2}\right)$

(vi) 0.2,0.22,0.222,0.2222...

It is not an AP because difference between consecutive terms is not equal.

⟹0.22−0.2≠0.222−0.22

(vii) 0,−4,−8,−12...

It is an AP because difference between consecutive terms is equal.

⟹ −4 − 0=−8−(−4)=−12−(−8)=−4

Common difference (d) = −4

Fifth term = −12 − 4=−16 Sixth term = −16 − 4=−20

Seventh term = −20 − 4=−24

Therefore, next three terms are −16, −20 and −24

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

Common difference (d) = 0

Fifth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$ Sixth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Seventh term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2}, -\frac{1}{2} \, and -\frac{1}{2}$

(ix) 1,3,9,27...

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3 - 1 \neq 9 - 3$

(x) $a, 2a, 3a, 4a \ldots$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2a - a = 3a - 2a = 4a - 3a = a$

Common difference (d) = a

Fifth term = $4a + a = 5a$ Sixth term = $5a + a = 6a$

Seventh term = $6a + a = 7a$

Therefore, next three terms are 5a, 6a and 7a

(xi) $a, a^2, a^3, a^4 \ldots$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow a^2 - a \neq a^3 - a^2$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots \Rightarrow \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

Common difference (d) = $\sqrt{2}$

Fifth term = $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$ Sixth term = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

Seventh term = $6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}...$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow \sqrt{6} - \sqrt{3} \neq \sqrt{9} - \sqrt{6}$

(xiv) $1^2, 3^2, 5^2, 7^2...$

It is not an AP because difference between consecutive terms is not equal.

$\Rightarrow 3^2 - 1^2 \neq 5^2 - 3^2$

(xv) $1^2, 5^2, 7^2, 73... \Rightarrow 1, 25, 49, 73...$

It is an AP because difference between consecutive terms is equal.

$\Rightarrow 5^2 - 1^2 = 7^2 - 5^2 = 73 - 7^2 = 24$

Common difference (d) = 24

Fifth term = 73+24=97 Sixth term = 97+24=121

Seventh term = 121 + 24 = 145

Therefore, next three terms are 97,121 and 145