

CBSE Class 10 Mathematics
Important Questions
Chapter 10
Circles

1 Marks Questions

1. How many tangents can a circle have?

Ans. A circle can have infinitely many tangents since there are infinitely many points on the circumference of the circle and at each point of it, it has a unique tangent.

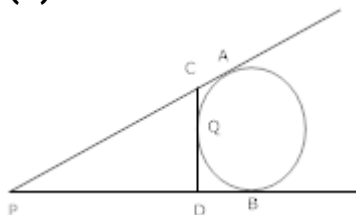
2. The perimeter of a sector of a circle of radius 8 cm is 25m, what is area of sector?

- (a) $50cm^2$
- (b) $42cm^2$
- (c) $52cm^2$
- (d) none of these

Ans. (a) $50cm^2$

3. In figure given below PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PA = 10cm and DQ = 2cm. What is length of PC?

- (a) 8 cm
- (b) 7 cm
- (c) 4 cm
- (d) none of these



Ans. (a) 8 cm

4. Tangent of circle intersect the circle

- (a) Only one point
- (b) Two points
- (c) Three points
- (d) None of these

Ans. (a) Only one point

5. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25cm. The radius of the circle is

- (a) 7 cm
- (b) 12 cm
- (c) 15 cm
- (d) 24.5 cm

Ans. (a) 7 cm

6. How many tangents can a circle have?

- (a) 1
- (b) 2
- (c) 0
- (d) infinite

Ans. d) infinite

7. If PA and PB are tangents from a point P lying outside the circle such that PA = 10 cm and $\angle APB = 60^\circ$. Find length of chord AB.

- (a) 10 cm
- (b) 20 cm
- (c) 30 cm
- (d) 40 cm

Ans. (a) 10cm

8. A tangent PQ at a point P to a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 13cm, then length of PQ is

- (a) 11 cm
- (b) 12 cm
- (c) 10 cm
- (d) None of these

Ans. (b) 12 cm

9. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to

- (a) 50°
- (b) 60°
- (c) 70°
- (d) 80°

Ans. (a) 50°

10. How many tangents can a circle have?

- (a) 1
- (b) 2
- (c) 0
- (d) infinite

Ans. (d) infinite

11. If PA and PB are tangents from a point P lying outside the circle such that PA = 10 cm and $\angle APB = 60^\circ$. Find length of chord AB.

- (a) 10 cm

(b) 20 cm

(c) 30 cm

(d) 40 cm

Ans. (a) 10 cm

12. A tangent PQ at a point P to a circle of radius 5 cm meets a line through the centre O at a point Q, so that OQ = 13cm, then length of PQ is

(a) 11 cm

(b) 12 cm

(c) 10 cm

(d) None of these

Ans. (b) 12 cm

13. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to

(a) 50°

(b) 60°

(c) 70°

(d) 80°

Ans. (a) 50°

14. The length of tangent drawn to a circle with radius 3 cm from a point 5 cm from the centre of the circle is

(a) 6 cm

(b) 8 cm

(c) 4 cm

(d) 7 cm

Ans. (c) 4 cm

15. A circle touches all the four sides of a quadrilateral ABCD whose sides $AB = 6$ cm, $BC = 7$ cm, $CD = 4$ cm, then $AD =$ ____

(a) 2 cm

(b) 3 cm

(c) 5 cm

(d) 6 cm

Ans. (b) 3 cm

16. If a point lies on a circle, then what will be the number of tangents drawn from that point to the circle?

(a) 1

(b) 2

(c) 3

(d) infinite

Ans. (a) 1

17. A quadrilateral ABCD is drawn to circumscribe a circle IF $AB = 4$ cm, $CD = 7$ cm, $BC = 3$ cm, then length of AD is

(a) 7 cm

(b) 2 cm

(c) 8 cm

(d) none of these

Ans. (c) 8 cm

18. A tangent PQ at point P of a circle of radius 12 cm meets a line through the centre O to a point Q so that $OQ = 20$ cm, then length of PQ is

(a) 14 cm

(b) 15 cm

(c) 16 cm

(d) 10 cm

Ans. (d) 10 cm

19. A line intersecting a circle in two points is called

(a) tangent

(b) secant

(c) diameter

(d) none of these

Ans. (b) secant

20. The length of tangent from a point A at a distance of 5 cm from the centre of the circle is 4 cm. What will be the radius of circle?

(a) 1 cm

(b) 2 cm

(c) 3 cm

(d) none of these

Ans. (c) 3 cm

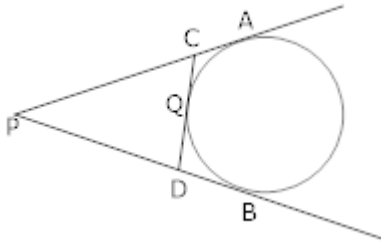
21. In the figure given below, PA and PB are tangents to the circle drawn from an external point P. CD is a third tangent touching the circle at Q. If PB = 12 cm and CQ = 3 cm, what is the length of PC?

(a) 9 cm

(b) 10 cm

(c) 1 cm

(d) 13 cm



Ans. (a) 9 cm

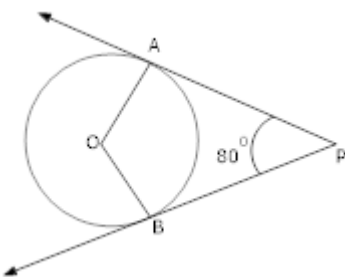
22. The tangent of a circle makes angle with radius at point of contact

- (a) 60°
- (b) 30°
- (c) 90°
- (d) none of these

Ans. (c) 90°

23. If tangent PA and PB from a point P to a circle with centre O are inclined to each other at an angle of 80° , then what is the value of $\angle POA$?

- (a) 30°
- (b) 50°
- (c) 70°
- (d) 90°



Ans. (b) 50°

2 Marks Questions

1. Fill in the blanks:

(i) A tangent to a circle intersects it in _____ point(s).

(ii) A line intersecting a circle in two points is called a _____.

(iii) A circle can have _____ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called _____.

Ans. (i) A tangent to a circle intersects it in one point.

(ii) A line intersecting a circle in two points is called a secant.

(iii) A circle can have two parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called point of contact.

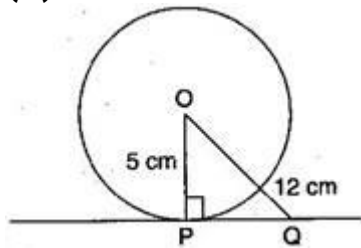
2. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm



Ans. (D) \because PQ is the tangent and OP is the radius through the point of contact.

$\therefore \angle OPQ = 90^\circ$ [The tangent at any point of a circle is \perp to the radius through the point of contact]

\therefore In right triangle OPQ,

$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

$$\Rightarrow (12)^2 = (5)^2 + PQ^2$$

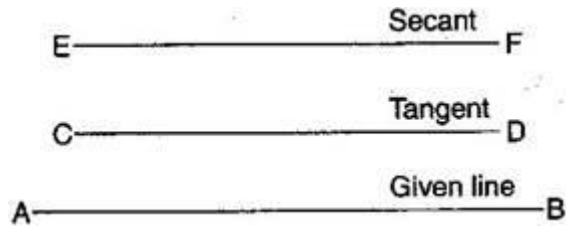
$$\Rightarrow 144 = 25 + PQ^2$$

$$\Rightarrow PQ^2 = 144 - 25 = 119$$

$$\Rightarrow PQ = \sqrt{119} \text{ cm}$$

3. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Ans.



4. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is:

(A) 7 cm

(B) 12 cm

(C) 15 cm

(D) 24.5 cm

Ans. (A) $\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is \perp to the radius through the point of contact]

\therefore In right triangle OPQ,

$OQ^2 = OP^2 + PQ^2$ [By Pythagoras theorem]

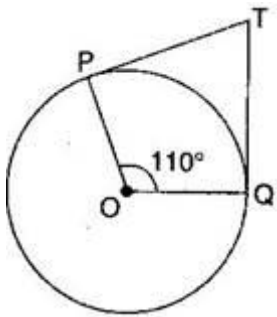
$$\Rightarrow (25)^2 = OP^2 + (24)^2$$

$$\Rightarrow 625 = OP^2 + 576$$

$$\Rightarrow OP^2 = 625 - 576 = 49$$

$$\Rightarrow OP = 7 \text{ cm}$$

5. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to:



(A) 60°

(B) 70°

(C) 80°

(D) 90°

Ans. (B) $\angle POQ = 110^\circ$, $\angle OPT = 90^\circ$ and $\angle OQT = 90^\circ$

[The tangent at any point of a circle is \perp to the radius through the point of contact]

In quadrilateral OPTQ,

$$\angle POQ + \angle OPT + \angle OQT + \angle PTQ = 360^\circ$$

[Angle sum property of quadrilateral]

$$\Rightarrow 110^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow 290^\circ + \angle PTQ = 360^\circ$$

$$\Rightarrow \angle PTQ = 70^\circ$$

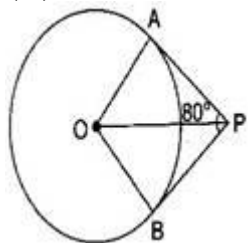
6. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80° , then $\angle POA$ is equal to:

(A) 50°

(B) 60°

(C) 70°

(D) 80°



Ans. (A) $\because \angle OPQ = 90^\circ$

[The tangent at any point of a circle is \perp to the radius through the point of contact]

$\angle OPA = \frac{1}{2} \angle BPA$ [Centre lies on the bisector of the angle between the two tangents]

In $\triangle OPA$,

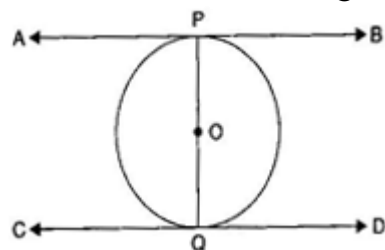
$\angle OAP + \angle OPA + \angle POA = 180^\circ$ [Angle sum property of a triangle]

$\Rightarrow 90^\circ + 40^\circ + \angle POA = 180^\circ$

$\Rightarrow 130^\circ + \angle POA = 180^\circ$

$\Rightarrow \angle POA = 50^\circ$

7. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.



Ans. Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove: $AB \parallel CD$

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$\therefore \angle OPA = 90^\circ$ (i)

[The tangent at any point of a circle is \perp to the radius through the point of contact]

\because CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$\therefore \angle OQD = 90^\circ$ (ii)

[The tangent at any point of a circle is \perp to the radius through the point of contact]

From eq. (i) and (ii), $\angle OPA = \angle OQD$

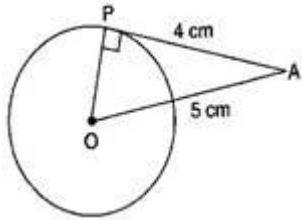
But these form a pair of equal alternate angles also,

$\therefore AB \parallel CD$

8. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Ans. We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact and the radius essentially passes through the centre of the circle, therefore the perpendicular at the point of contact to the tangent to a circle passes through the centre.

9. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.



Ans. We know that the tangent at any point of a circle is \perp to the radius through the point of contact.

$$\therefore \angle OPA = 90^\circ$$

$$\therefore OA^2 = OP^2 + AP^2 \text{ [By Pythagoras theorem]}$$

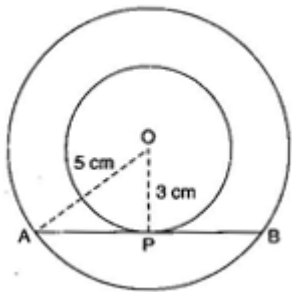
$$\Rightarrow (5)^2 = (OP)^2 + (4)^2$$

$$\Rightarrow 25 = (OP)^2 + 16$$

$$\Rightarrow OP^2 = 9$$

$$\Rightarrow OP = 3 \text{ cm}$$

10. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.



Ans. Let O be the common centre of the two concentric circles.

Let AB be a chord of the larger circle which touches the smaller circle at P.

Join OP and OA.

Then, $\angle OPA = 90^\circ$

[The tangent at any point of a circle is \perp to the radius through the point of contact

$\therefore OA^2 = OP^2 + AP^2$ [By Pythagoras theorem]

$$\Rightarrow (5)^2 = (3)^2 + AP^2$$

$$\Rightarrow 25 = 9 + AP^2$$

$$\Rightarrow AP^2 = 16$$

$$\Rightarrow AP = 4 \text{ cm}$$

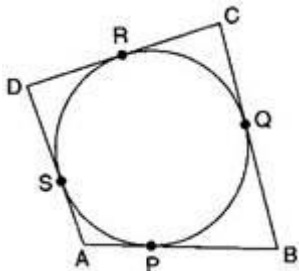
Since the perpendicular from the centre of a circle to a chord bisects the chord, therefore

$$AP = BP = 4 \text{ cm}$$

$$\Rightarrow AB = AP + BP = AP + AP = 2AP = 2 \times 4 = 8 \text{ cm}$$

11. A quadrilateral ABCD is drawn to circumscribe a circle (see figure). Prove that:

$$AB + CD = AD + BC$$



Ans. We know that the tangents from an external point to a circle are equal.

$$\therefore AP = AS \dots\dots\dots(i)$$

$$BP = BQ \dots\dots\dots(ii)$$

$$CR = CQ \dots\dots(iii)$$

$$DR = DS \dots\dots(iv)$$

On adding eq. (i), (ii), (iii) and (iv), we get

$$(AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS)$$

$$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

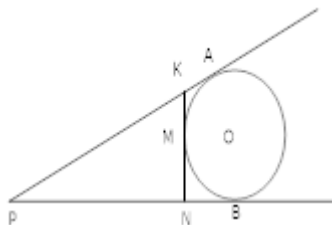
12. In two concentric circles prove that all chords of the outer circle which touch the inner circle are of equal length.

Ans. AB and CD are two chords of the circle which touch the inner circle at M and N.

Respectively $\therefore OM = ON$

$$\Rightarrow AB = CD [\because AB \text{ and } CD \text{ are two chords of larger circle}]$$

13. PA and PB are tangents from P to the circle with centre O. At the point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN=AK+BN$.



Ans. We know that the lengths of the tangents drawn from an external point to a circle are equal.

$$\therefore PA = PB \dots\dots(i)$$

$$KA = KM \dots\dots(ii)$$

$$NB = NM \dots\dots(iii)$$

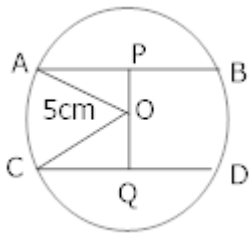
(ii) + (iii)

$$KA + NB = KM + NM$$

$$\Rightarrow AK + BN = KM + MN$$

$$\Rightarrow AK + BN = KN$$

14. In the given figure, O is the centre of the circle with radius 5 cm and $AB \parallel CD$. If $AB = 6$ cm, find OP.



Ans. $\because OP \perp AB$

$\therefore OP$ bisects AB

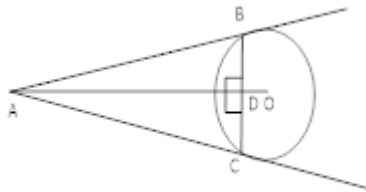
$$\therefore AP = \frac{1}{2} AB = \frac{1}{2} \times 6 = 3 \text{ cm}$$

From right $\triangle OAP$, $OA^2 = OP^2 + AP^2$

$$\Rightarrow 5^2 = OP^2 + 3^2$$

$$\Rightarrow OP = 4 \text{ cm}$$

15. Prove that the tangents at the end of a chord of a circle make equal angles with the chord.



Ans. In $\triangle ADB$ and $\triangle ADC$,

$$BD = DC$$

$$\text{And } \angle ADB = \angle ADC = 90^\circ$$

$$AD = AD \text{ [Common]}$$

$$\therefore \triangle ADB \cong \triangle ADC \text{ [SAS]}$$

$$\therefore \angle ABD = \angle ACD \text{ [By CPCT]}$$

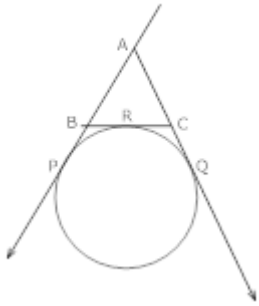
16. Find the locus of the centre of circles which touch a given line at a given point.

Ans. Let APB be the given line and let a circle with centre O touch APB at P . Then $\angle OPB = 90^\circ$, let there be another circle with centre O' which touches the line APB at P .

$$\text{Thus, } \angle O'PB = 90^\circ$$

$$\therefore \angle OPB = \angle O'PB = 90^\circ$$

17. In the given figure, find the perimeter of $\triangle ABC$, if $AP = 10$ cm.



Ans. \because BC touches the circle at R

\because Tangents drawn from external point to the circle are equal.

$\therefore AP = AQ, BR = BP$

And $CR = CQ$

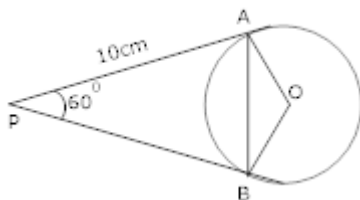
\therefore Perimeter of $\triangle ABC = AB + BC + AC$

$= AB + (BR + RC) + AC$

$= AB + BP + CQ + AC$

$= AP + AQ = 2AP = 2 \times 10 = 20\text{cm}$

18. If PA and PB are tangents drawn from external point P such that $PA = 10\text{cm}$ and $\angle APB = 60^\circ$, find the length of chord AB.



Ans. $\because \angle APB = 60^\circ$

$\angle AOB = 120^\circ$ [O is centre of circle]

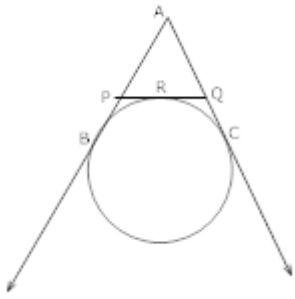
$\angle OAB = \angle OBA = 30^\circ$

$\therefore \angle PAB = 60^\circ, \angle PBA = 60^\circ$

$\therefore \triangle PAB$ is equilateral triangle

$\therefore AB = PA = 10\text{cm}$

19. If AB, AC and PQ are tangents in the given figure and $AB = 25\text{cm}$, find the perimeter of $\triangle APQ$.



Ans. Perimeter of $\triangle APQ = AP + AQ + PQ$

$$\begin{aligned}
 &= AP + AQ + PX + XQ \\
 &= (AP + PB) + (AQ + QC) \\
 &= AB + AC \\
 &= 2AB = 2 \times 25 = 50 \text{ cm}
 \end{aligned}$$

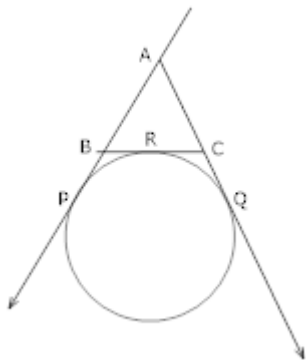
20. Find the locus of the centre of circles which touch a given line at a given point.

Ans. Let APB be the given line and let a circle with centre O touch APB at P. Then $\angle OPB = 90^\circ$, let there be another circle with centre O' which touches the line APB at P.

Thus, $\angle O'PB = 90^\circ$

$$\therefore \angle OPB = \angle O'PB = 90^\circ$$

21. In the given figure, find the perimeter of $\triangle ABC$, if AP = 10 cm.



Ans. \because BC touches the circle at R

\because Tangents drawn from external point to the circle are equal.

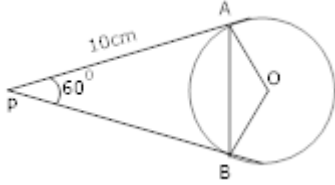
$$\therefore AP = AQ, BR = BP$$

And $CR = CQ$

$$\therefore \text{Perimeter of } \triangle ABC = AB + BC + AC$$

$$\begin{aligned}
&= AB + (BR + RC) + AC \\
&= AB + BP + CQ + AC \\
&= AP + AQ = 2AP = 2 \times 10 = 20\text{cm}
\end{aligned}$$

22. If PA and PB are tangents drawn from external point P such that PA = 10 cm and $\angle APB = 60^\circ$, find the length of chord AB.



Ans. $\because \angle APB = 60^\circ$

$$\angle AOB = 120^\circ \text{ [O is centre of circle]}$$

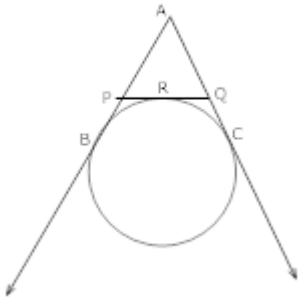
$$\angle OAB = \angle OBA = 30^\circ$$

$$\therefore \angle PAB = 60^\circ, \angle PBA = 60^\circ$$

$\therefore \triangle PAB$ is equilateral triangle

$$\therefore AB = PA = 10\text{cm}$$

23. If AB, AC and PQ are tangents in the given figure and AB = 25cm, find the perimeter of $\triangle APQ$.



Ans. Perimeter of $\triangle APQ = AP + AQ + PQ$

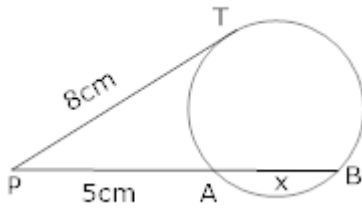
$$= AP + AQ + PX + XQ$$

$$= (AP + PB) + (AQ + QC)$$

$$= AB + AC$$

$$= 2AB = 2 \times 25 = 50\text{cm}$$

length x.



∴

Ans. PT is tangent to a circle and PAB is a secant.

$$\therefore PA \cdot PB = PT^2$$

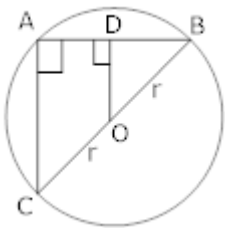
$$\Rightarrow 5(5+x) = 8^2$$

$$\Rightarrow 25 + 5x = 64$$

$$\Rightarrow x = \frac{39}{5} = 7.8 \text{ cm}$$

25. In the given figure, OD is perpendicular to the chord AB of a circle whose centre is O. If

BC is a diameter, find $\frac{CA}{OD}$.



Ans. Since BC is a diameter

$$\therefore \angle CAB = 90^\circ$$

Also $OD \perp AB$

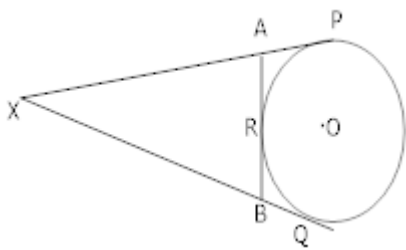
$$\therefore \angle ODB = 90^\circ \quad \Delta ACB \sim \Delta DOB$$

$$[\because \angle CAB = \angle ODB = 90^\circ]$$

$$\angle ABC = \angle DBO \text{ [Common]}$$

$$\therefore \frac{CA}{OD} = \frac{CB}{OB} = \frac{2r}{r} = 2$$

26. In the given figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle such that ARB is a tangent to the circle prove that $XA + AR = XB + BR$.



Ans. In the given figure, XP and XQ are tangents from external point

$$\therefore XP = XQ \dots\dots(i)$$

$$AR = AP \dots\dots(ii)$$

$$BR = BQ \dots\dots(iii)$$

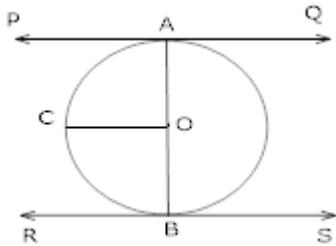
[\because Length of tangents are equal from external point]

$$XP = XQ$$

$$XA + AP = XB + BQ \text{ [By (ii) and (iii)]}$$

$$XA + AR = XB + BR \text{ [By (ii) and (iii)]}$$

27. Prove that the segment joining the points of contact of two parallel tangents, passes through the centre.

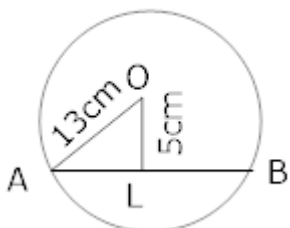


Ans. Given two parallel tangents PQ and RS of a circle with centre O

Draw line $OC \parallel RS$.

$$\text{i.e., } \angle PAO + \angle COA = 180^\circ$$

28. In figure, if $OL = 5 \text{ cm}$, $OA = 13 \text{ cm}$, then length of AB is



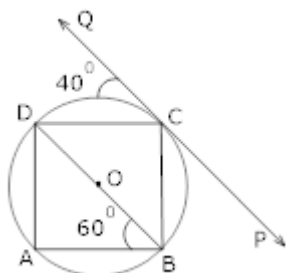
$$\text{Ans. } AB = 2AL = 2\sqrt{OA^2 - OL^2}$$

$$= 2\sqrt{13^2 - 5^2}$$

$$= 2\sqrt{169-25} = 2\sqrt{144}$$

$$= 2 \times 12 = 24 \text{ cm}$$

29. In the given figure, ABCD is a cyclic quadrilateral and PQ is a tangent to the circle at C. If BD is a diameter, $\angle OCQ = 40^\circ$ and $\angle ABD = 60^\circ$, find $\angle BCP$.

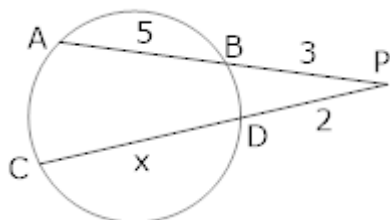


Ans. \because BD is a diameter

$$\therefore \angle BCD = 90^\circ \text{ [Angle in the semi-circle]}$$

$$\therefore \angle BCP = 180^\circ - 90^\circ - 40^\circ = 50^\circ$$

30. Two chords AB and CD of a circle intersect each other at P outside the circle. If AB = 5 cm, BP = 3 cm and PD = 2cm, find CD.



Ans. \because Two chords AB and CD of a circle intersect each other at P

$$\therefore PA \times PB = PC \times PD \text{ [length of tangent from P]}$$

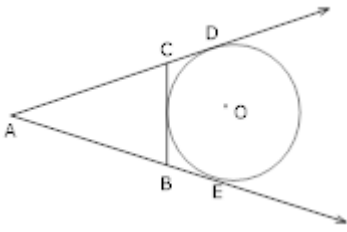
$$\Rightarrow (AB + PB) \times PB = (PD + PC) PD$$

$$\Rightarrow (5 + 3)(3) = (2 + x)^2$$

$$\Rightarrow 24 = (2 + x)^2$$

$$\Rightarrow x = 10 \Rightarrow CD = 10 \text{ cm}$$

31. In the adjoining figure, if AD, AE and BC are tangents to the circle at D, E and F respectively, then prove that $2 AD = AB + BC + CA$.



Ans. $CD = CF$, $BE = BF$

$$\Rightarrow CD + BE = CF + BF = BC$$

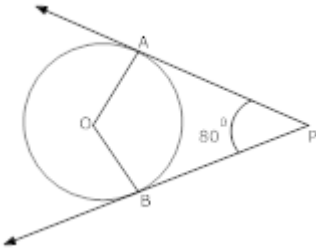
$$\text{Now } AD = AC + CD = AC + CF$$

$$AE = AB + BE = AB + BF$$

$$\therefore AD + AE = AB + AC + BC$$

$$\Rightarrow 2AD = AB + BC + AC$$

32. In figure, PA and PB are tangents from P to the circle with centre O. R is a point on the circle, prove that $PC + CR = PD + DR$.



Ans. Since length of tangents from an external point to a circle are equal in length

$$\therefore PA = PB$$

$$CA = CR \dots(i)$$

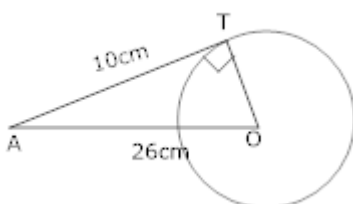
And $DB = DR$

Now $PA = PB$

$$\Rightarrow PC + CA = PD + DB$$

$$\Rightarrow PC + CR = PD + DR \text{ [By (i)]}$$

33. The length of tangents from a point A at distance of 26 cm from the centre of the circle is 10cm, what will be the radius of the circle?



Ans. Since tangents to a circle is perpendicular to radius through the point of contact

$$\therefore \angle OTA = 90^\circ$$

In right $\triangle OTA = 90^\circ$, we have

$$OA^2 = OT^2 + AT^2$$

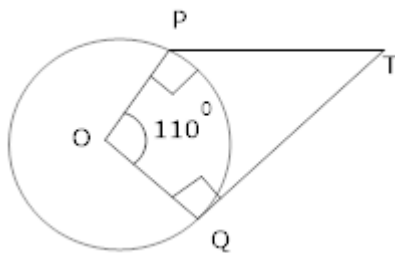
$$\Rightarrow (26)^2 = OT^2 + (10)^2$$

$$\Rightarrow OT^2 = 676 - 100$$

$$\Rightarrow OT^2 = 576$$

$$\Rightarrow OT = 24$$

34. In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then find $\angle PTO$.

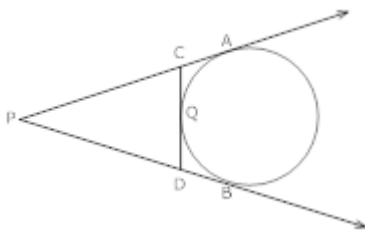


Ans. Since $\angle POQ + \angle PTO = 180^\circ$ [$\because \angle OPT = 90^\circ, \angle OQT = 90^\circ$]

$$\Rightarrow 110^\circ + \angle PTO = 180^\circ$$

$$\Rightarrow \angle PTO = 180^\circ - 110^\circ = 70^\circ$$

35. In the figure, given below PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PB = 10 cm and CQ = 2 cm, what is the length of PC?



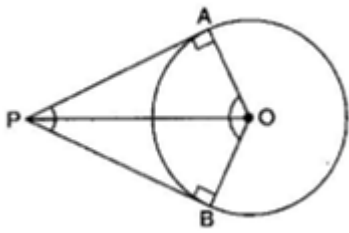
Ans. $PA = PB = 10$ cm

$$CQ = CA = 2$$
 cm

$$PC = PA - CA = 10 - 2 = 8$$
 cm

3 Marks Questions

1. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.



Ans. $\angle OPA = 90^\circ$ (i)

$\angle OCA = 90^\circ$ (ii)

[Tangent at any point of a circle is \perp to the radius through the point of contact]

\therefore OAPB is quadrilateral.

$\therefore \angle APB + \angle AOB + \angle OAP + \angle OBP = 360^\circ$

[Angle sum property of a quadrilateral]

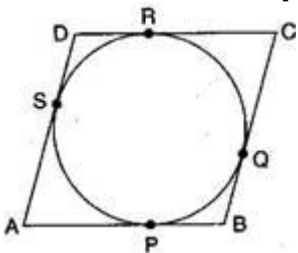
$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$

[From eq. (i) & (ii)]

$\Rightarrow \angle APB + \angle AOB = 180^\circ$

$\therefore \angle APB$ and $\angle AOB$ are supplementary.

2. Prove that the parallelogram circumscribing a circle is a rhombus.



Ans. Given: ABCD is a parallelogram circumscribing a circle.

To Prove: ABCD is a rhombus.

Proof: Since, the tangents from an external point to a circle are equal.

$\therefore AP = AS$ (i)

$BP = BQ$ (ii)

$CR = CQ$ (iii)

$DR = DS$ (iv)

On adding eq. (i), (ii), (iii) and (iv), we get

$$(AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS)$$

$$\Rightarrow AB + CD = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

$$\Rightarrow AB + AB = AD + AD \text{ [Opposite sides of } \parallel \text{ gm are equal]}$$

$$\Rightarrow 2AB = 2AD$$

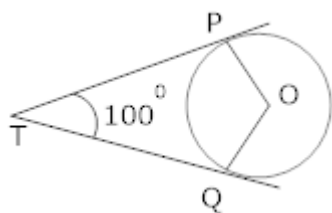
$$\Rightarrow AB = AD$$

But $AB = CD$ and $AD = BC$ [Opposite sides of \parallel gm]

$$\therefore AB = BC = CD = AD$$

\therefore Parallelogram ABCD is a rhombus.

3. Two tangents TP and TQ are drawn from an external point T with centre O as shown in figure. If they are inclined to each other at an angle of 100° , then what is the value of $\angle POQ$?



Ans. $\therefore TP$ and TQ are tangents and O is the centre of the circle

$$\therefore OP \perp PT, OQ \perp QT$$

$$\therefore \angle TPO + \angle TQO = 180^\circ$$

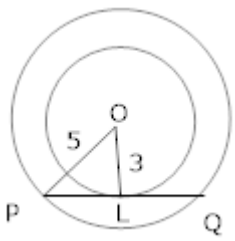
\therefore Quadrilateral OPTQ is cyclic.

$$\therefore \angle PTQ + \angle POQ = 180^\circ$$

$$\therefore 100^\circ + \angle POQ = 180^\circ$$

$$\therefore \angle POQ = 180^\circ - 100^\circ = 80^\circ$$

4. Two concentric circles are of radii 5 cm and 3 cm, find the length of the chord of the larger circle which touches the smaller circle.



Ans. \because PQ is the chord of the larger circle which touches the smaller circle at the point L. Since PQ is tangent at the point L to the smaller circle with centre O.

$$\therefore OL \perp PQ$$

\because PQ is a chord of the bigger circle and $OL \perp PQ$

$\therefore OL$ bisects PQ

$$\therefore PQ = 2PL$$

$$\text{In } \triangle OPL, PL = \sqrt{OP^2 - OL^2}$$

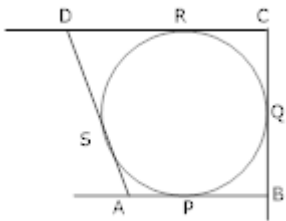
$$= \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9} = 4$$

$$\therefore \text{Chord } PQ = 2PL = 8 \text{ cm}$$

$$\therefore \text{Length of chord } PQ = 8 \text{ cm}$$

5. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB+CD=AD+BC$.



Ans. \because AP, AS are tangents from a point A (Outside the circle) to the circle.

$$\therefore AP = AS$$

Similarly, $BP = BQ$

$$CQ = CR$$

$$DR = DS$$

$$\text{Now } AB + CD = AP + PB + CR + RD$$

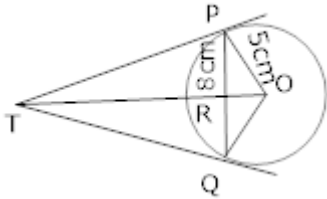
$$= AS + BQ + CQ + DS$$

$$= (AS + DS) + (BQ + CQ)$$

$$= AD + BC$$

$$\backslash AB + CD = AD + BC$$

6. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at point T. Find the length TP.



Ans. Join OT.

$$TP = TQ \text{ [tangents from T upon the circle]}$$

$$\therefore OT \perp PQ$$

And OT bisects PQ

$$\therefore PR = RQ = 4 \text{ cm}$$

$$\text{Now } OR = \sqrt{OP^2 - PR^2}$$

$$OR = \sqrt{5^2 - 4^2} = 3 \text{ cm}$$

$$\text{Now } \angle TPR + \angle RPO = 90^\circ \text{ [}\because \angle TPO = 90^\circ\text{]}$$

$$= \angle TPR + \angle PTR$$

$$\therefore \angle RPO = \angle PTR$$

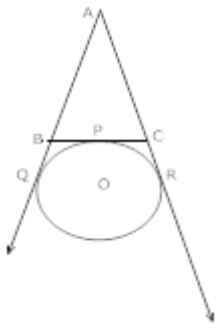
$\Delta TRP \sim \Delta TRQ$ [By AA similarity]

$$\therefore \frac{TP}{PO} = \frac{RP}{RO}$$

$$\Rightarrow \frac{TP}{5} = \frac{4}{3}$$

$$\Rightarrow TP = \frac{20}{3} \text{ cm}$$

7. A circle is touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that $AQ = \frac{1}{2}$ (perimeter of ΔABC).



Ans. We know that the two tangents drawn to a circle from an external point are equal.

$$\therefore AQ = AR, BP = BQ, CP = CR$$

$$\therefore \text{Perimeter of } \triangle ABC = AB + BC + AC$$

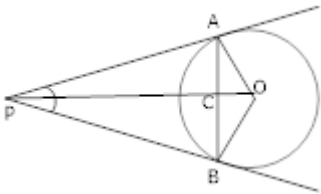
$$= AB + BP + PC + AC$$

$$= AB + BQ + CR + AC \quad [\because BP = BQ, PC = CQ]$$

$$= AQ + AR = 2AQ = 2AR \quad [\because AQ = AR]$$

$$= AQ = AR = \frac{1}{2} \text{ [perimeter of } \triangle ABC]$$

8. If PA and PB are two tangents drawn from a point P to a circle with centre O touching it at A and B. Prove that OP is the perpendicular bisector of AB.



Ans. Let OP intersect AB at a point C, we have to prove that $AC = CB$ and $\angle ACP = \angle BCP = 90^\circ$

$\because PA, PB$ are two tangents from a point P to the circle with centre O

$$\therefore \angle APO = \angle BPO \quad [\because O \text{ lies on the bisector of } \angle APB]$$

In two $\triangle S$, $\triangle ACP$ and $\triangle BCP$, we have

$$AP = BP \quad [\because \text{tangents from P to the circle are equal}]$$

$$PC = PC \quad [\text{Common}]$$

$$\angle APO = \angle BPO \quad [\text{Proved}]$$

$$\therefore \triangle ACP \cong \triangle BCP \quad [\text{By SAS rule}]$$

$$\therefore AC = CB \quad [\text{CPCT}]$$

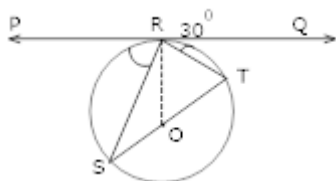
And $\angle ACP = \angle BCP$ [CPCT]

But $\angle ACP + \angle BCP = 180^\circ$

$$\Rightarrow \angle ACP = \angle BCP = 90^\circ$$

Hence, OP is perpendicular bisector of AB.

9. In the given figure, PQ is tangent at point R of the circle with centre O. If $\angle TRQ = 30^\circ$, find $m\angle PRS$.



Ans. Given PQ is tangent at point R and $\angle TRQ = 30^\circ$

$$\angle PRQ = 180^\circ$$

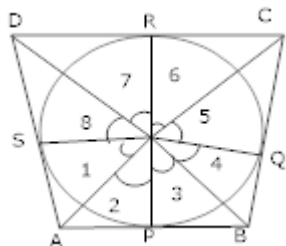
$$\angle QRT = 30^\circ$$

$$\angle TRS = 90^\circ \text{ [}\because \text{Tangent of a circle is perpendicular to Radius]}$$

$$\therefore \angle PRS = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore \angle PRS = 180^\circ - 120^\circ = 60^\circ$$

10. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.



Ans. Let the circle touch the sides AB, BC, CD and DA at the points P, Q, R, and S respectively.

Join OP, OQ, OR and OS.

Join OA, OB, OC and OD.

Since the two tangents drawn from an external point subtend equal angles at the centre.

$$\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$$

$$\text{But } \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^\circ$$

[\because Sum of all angles around a point = 360°]

$$\therefore 2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360^\circ$$

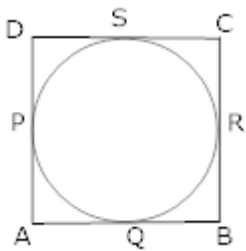
$$\text{And } 2(\angle 4 + \angle 5 + \angle 8 + \angle 1) = 360^\circ$$

$$\Rightarrow (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$$

$$\text{And } (\angle 4 + \angle 5) + (\angle 8 + \angle 1) = 180^\circ$$

$$\Rightarrow \angle AOB + \angle COD = 180^\circ \text{ and } \angle BOC + \angle AOD = 180^\circ$$

11. Prove that parallelogram circumscribing a circle is a rhombus.



Ans. Given ABCD is a parallelogram in which all the sides touch a given circle

To prove: ABCD is a rhombus

Proof:

\because ABCD is a parallelogram

$\therefore AB = DC$ and $AD = BC$

Again AP, AQ are tangents to the circle from the point A

$$\therefore AP = AQ$$

Similarly, $BR = BQ$

$$CR = CS$$

$$DP = DS$$

$$\therefore (AP + DP) + (BR + CR)$$

$$= AQ + DS + BQ + CS$$

$$= (AQ + BQ) + (CS + DS)$$

$$\Rightarrow AD + BC = AB + DC$$

$$\Rightarrow BC + BC = AB + AB$$

$$[\because AB = DC, AD = BC]$$

$$\Rightarrow 2BC = 2AB$$

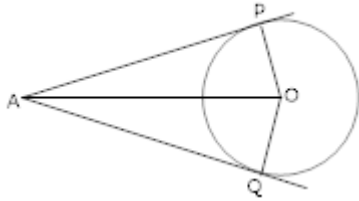
$$\Rightarrow BC = AB$$

Hence, parallelogram ABCD is a rhombus.

12. If two tangents are drawn to a circle from an external point then

(i) they subtend equal angles at the centre.

(ii) they are equally inclined to the segment joining the centre to that point.



Ans. Given on a circle C (O,r), two tangents AP and AQ are drawn from an external point A.

To prove:

(i) $\angle AOP = \angle AOQ$

(ii) $\angle OAP = \angle OAQ$

Construction: Join AO, PO and QO

Proof: In $\triangle APO$ and $\triangle AQO$,

AP = AQ [Length of the tangents drawn from an external point]

PO = QO [Radii of the same circle]

AO = AO [common]

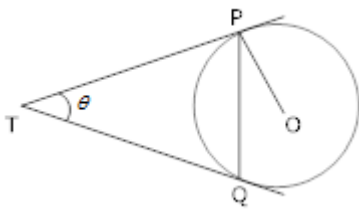
$\triangle APO \cong \triangle AQO$ [By SSS theorem of congruence]

(i) $\angle AOP = \angle AOQ$ [CPCT]

(ii) $\angle OAP = \angle OAQ$ [By CPCT.]

13. Two tangents TP and TQ are drawn to a circle with centre O from an external point T.

Prove that $\angle PTQ = 2\angle OPQ$.



Ans. Given A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove: $\angle PTQ = 2\angle OPQ$

Proof: Let $\angle PTQ = \theta$

Since TP, TQ are tangents drawn from point T to the circle.

TP = TQ

\therefore TPQ is an isosceles triangle

$$\therefore \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta)$$

$$= 90^\circ - \frac{\theta}{2}$$

Since, TP is a tangent to the circle at point of contact P

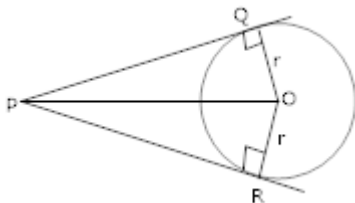
$$\therefore \angle OPT = 90^\circ$$

$$\therefore \angle OPQ = \angle OPT - \angle TPQ$$

$$= 90^\circ - \left(90^\circ - \frac{1}{2}\theta\right) = \frac{\theta}{2} = \frac{1}{2}\angle PTQ$$

Thus, $\angle PTQ = 2\angle OPQ$

14. Prove that the lengths of two tangents drawn from an external point to a circle are equal.



Ans. Given: P is an external point to the circle C(O,r).

PQ and PR are two tangents from P to the circle.

To Prove: PQ = PR

Construction: Join OP

Proof:

∵ A tangent to a circle is perpendicular to the radius through the point of contact

$$\therefore \angle OQP = 90^\circ = \angle ORP$$

Now in right triangles POQ and POR,

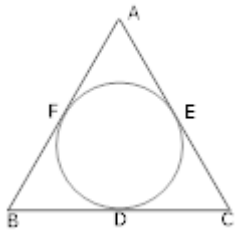
$$OQ = OR \text{ [Each radius } r\text{]}$$

Hypotenuse. OP = Hypotenuse. OP [common]

$$\therefore \triangle POQ \cong \triangle POR \text{ [By RHS rule]}$$

$$\therefore PQ = PR$$

15. The circle of $\triangle ABC$ touches the sides BC, CA and AB at D,E and F respectively. If $AB = AC$, prove that $BD = CD$.



Ans. ∵ Tangents drawn from an external point to a circle are equal in length

$$\therefore AF = AE \text{ [Tangents from A] ... (i)}$$

$$BF = BD \text{ [Tangents from B] ... (ii)}$$

$$CD = CE \text{ [Tangents from C] ... (iii)}$$

Adding (i), (ii) and (iii), we get

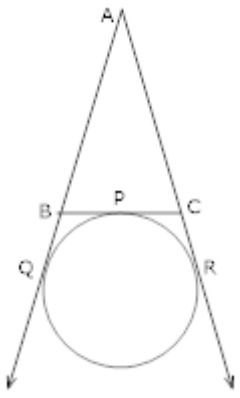
$$AF + BF + CD = AE + BD + CE$$

$$\Rightarrow AB + CD = AC + BD$$

But $AB = AC$ (given)

$$CD = BD$$

16. A circle touches the side BC of a $\triangle ABC$ at a point P and touches AB and AC when produced at Q and R respectively, show that $AQ = \frac{1}{2}$ [Perimeter of $\triangle ABC$].



Ans. Since the tangents from an external point to a circle are equal in length,

$$BP = BQ \dots(i) \text{ [from point B]}$$

$$CP = CR \dots(ii) \text{ [from point C]}$$

$$\text{And } AQ = AR \dots(iii) \text{ [From point A]}$$

From (iii), we have

$$AQ = AR$$

$$\Rightarrow AB + BQ = AC + CR$$

$$\Rightarrow AB + BP = AC + CP \dots(iv) \text{ [Using (i) and (ii)]}$$

Now perimeter of $\triangle ABC$

$$AB + BC + AC = AB + (BP + PC) + AC$$

$$= (AB + BP) + (AC + PC)$$

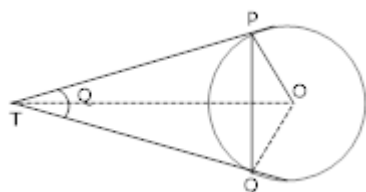
$$= 2 (AB + BP) \text{ [using (iv)]}$$

$$= 2 (AB + BQ) \text{ [using (i)]}$$

$$= 2 AQ$$

$$\therefore AQ = \frac{1}{2} (\text{perimeter of } \triangle ABC)$$

17. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Ans. Given: A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P and Q are the points of contact.

To prove: $\angle PTQ = 2\angle OPQ$

Proof: Let $\angle PTQ = \theta$

In $\triangle TPQ$, we have

$$TP = TQ$$

[Length of the tangents drawn from an external point to a circle are equal]

So, TPQ is an isosceles triangle.

$$\therefore \angle TPQ = \angle TQP \dots\dots\dots(i)$$

In $\triangle TPQ$, we have

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ \quad [\because \text{Sum of three angles of a } \Delta \text{ is } 180^\circ]$$

$$\Rightarrow 2\angle TPQ + \theta = 180^\circ \dots\dots\dots(i)$$

$$\Rightarrow 2\angle TPQ = 180^\circ - \theta$$

$$\Rightarrow \angle TPQ = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta \dots\dots\dots(ii)$$

But $\angle OPT = 90^\circ \dots\dots\dots(iii)$

[Angle between the tangent and radius of a circle is 90°]

$$\text{Now } \angle OPQ = \angle OPT - \angle TPQ$$

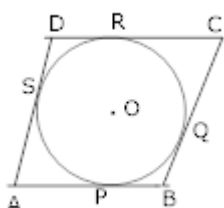
$$= 90^\circ - \left[90^\circ - \frac{1}{2}\theta \right]$$

$$= \frac{1}{2}\theta = \frac{1}{2}\angle PTQ$$

$$\Rightarrow \angle OPQ = \frac{1}{2}\angle PTQ$$

$$\Rightarrow \angle PTQ = 2\angle OPQ$$

18. Prove that the parallelogram circumscribing a circle is a rhombus



Ans. Given: ABCD be the parallelogram circumscribing a circle with centre O such that the sides AB, BC, CD and DA touch a circle at P, Q, R and S respectively.

To prove: ||gm ABCD is a rhombus.

Proof: $AP = AS \dots(i)$

$BP = BQ \dots(ii)$

$CR = CQ \dots(iii)$

$DR = DS \dots(iv)$

[Tangents drawn from an external point to a circle are equal]

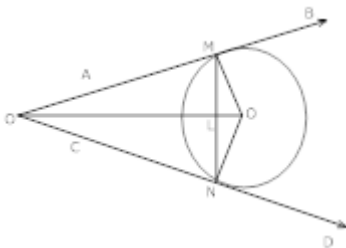
Adding (i), (ii), (iii) and (iv), we get

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

$$\Rightarrow AB + CD = AD + BC$$

19. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with chord.



Ans. Let NM be chord of circle with centre C.

Let tangents at M.N meet at the point O.

Since OM is a tangent

$$\therefore OM \perp CM \quad \text{i.e. } \angle OMC = 90^\circ$$

$\because ON$ is a tangent

$$\therefore ON \perp CN \quad \text{i.e. } \angle ONC = 90^\circ$$

Again in $\triangle CMN$, $CM = CN = r$

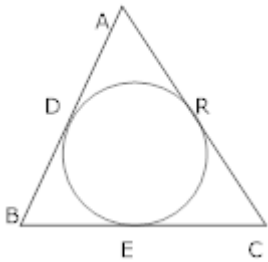
$$\therefore \angle CMN = \angle CNM$$

$$\therefore \angle OMC - \angle CMN = \angle ONC - \angle CNM$$

$$\Rightarrow \angle OML = \angle ONL$$

Thus, tangents make equal angle with the chord.

20. In the given figure, if $AB = AC$, prove that $BE = EC$.



Ans. Since tangents from an exterior point A to a circle are equal in length

$$\therefore AD = AR \dots\dots\dots(i)$$

Similarly, tangents from an exterior point B to a circle are equal in length

$$\therefore BD = BE \dots\dots\dots(2)$$

Similarly, for C

$$CE = CR \dots\dots\dots(3)$$

Now $AB = AC$

$$\therefore AB - AD = AC - AR$$

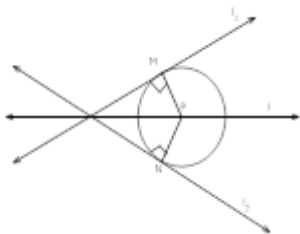
$$\Rightarrow AB - AD = AC - AR \dots\dots\dots[By (i)]$$

$$\Rightarrow BD = CR$$

$$\Rightarrow BE = CR \dots\dots\dots[By (ii)]$$

$$\Rightarrow BE = CE [\because BD = BE, CE = CR] [By (iii)]$$

21. Find the locus of centre of circle with two intersecting lines.



Ans. Let l_1, l_2 be two intersection lines.

Let a circle with centre P touch the two lines l_1 and l_2 at M and N respectively.

$$PM = PN \text{ [Radii of same circle]}$$

\therefore P is equidistance from the lines l_1 and l_2

Similarly, centre of any other circle which touch the two intersecting lines l_1, l_2 will be equidistant from l_1 and l_2

\therefore P lies on l a bisector of the angle between l_1 and l_2

[\because The locus of points equidistant from two intersecting lines is the pair of bisectors of the angle between the lines]

Hence, locus of centre of circles which touch two intersecting lines is the pair of bisectors of the angles between the two lines.

22. In the given figure, a circle is inscribed in a quadrilateral ABCD in which $\angle B = 90^\circ$. If AD = 23 cm, AB = 29 cm and DS = 5 cm, find the radius of the circle.

Ans. In the given figure, $OP \perp BC$ and $OQ \perp BA$

Also, $OP = OQ = r$

$\therefore OPBQ$ is a square

$\therefore BP = BQ = r$

But $DR = DS = 5$ cm ... (i)

$$\begin{aligned}\therefore AR &= AD - DR \\ &= 23 - 5 = 18 \text{ cm}\end{aligned}$$

$$AQ = AR = 18 \text{ cm}$$

$$\begin{aligned}BQ &= AB - AQ \\ &= 29 - 18 = 11 \text{ cm}\end{aligned}$$

$$r = 11 \text{ cm}$$