CBSE Class 9 Mathematics
Important Questions
Chapter 1
Number Systems

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1 Marks Questions

1. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Ans. We know that square root of every positive integer will not yield an integer.

We know that $\sqrt{4}$ is 2, which is an integer. But, $\sqrt{7}$ or $\sqrt{10}$ will give an irrational number.

Therefore, we conclude that square root of every positive integer is not an irrational number.

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2. Write three numbers whose decimal expansions are non-terminating non-recurring.

Ans. The three numbers that have their expansions as non-terminating on recurring decimal are given below.

0.04004000400004….

0.07007000700007….

0.013001300013000013….

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3. Find three different irrational numbers between the rational numbers $\frac{5}{11}$ and $\frac{9}{11}$.

Ans. Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, to get

$\frac{5}{7} = 0.714285....$ and $\frac{9}{11} = 0.818181....$.

Three irrational numbers that lie between $0.714285....$ and $0.818181....$ are:

0.73073007300073….

0.74074007400074….

0.76076007600076….

4. Which of the following rational numbers have terminating decimal representation

$(i)\ \frac{3}{5}$ $(ii)\ \frac{2}{13}$

$(iii)\ \frac{40}{27}$ $(iv)\ \frac{23}{7}$

Ans. $(i)\ \frac{3}{5}$

5. How many rational numbers can be found between two distinct rational numbers?

(i)Two

(ii) Ten

(iii) Zero

(iv) Infinite

Ans. (iv) Infinite

6. The value of $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$ **in**

(i)1

(ii)-1

(iii) 2

(iv) none of these

Ans. (i) 1

7. $(27)^{-2/3}$ **is equal to**

(i) 9

(ii) 1/9

(iii) 3

(iv) none of these

Ans. (ii) 1/9

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8. Every natural number is

(i) not an integer

(ii) always a whole number

(iii) an irrational number

(iv) not a fraction

Ans. (ii) always a whole number

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9. Select the correct statement from the following

(i) $\frac{7}{9} > \frac{4}{5}$ (ii) $\frac{2}{6} < \frac{3}{9}$

(iii) $\frac{-2}{3} > \frac{-4}{5}$ (iv) $\frac{-5}{7} < \frac{-3}{4}$

Ans. (iii) $\frac{-2}{3} > \frac{-4}{5}$

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10. $7.\overline{2}$ **is equal to**

(i) $\frac{68}{9}$ (ii) $\frac{64}{9}$

(iii) $\frac{65}{9}$ (iv) $\frac{63}{9}$

Ans. (iii) $\frac{65}{9}$

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11. 0.83458456……………is

(i) an irrational number

(ii) rational number

(iii) a natural number

(iv) a whole number.

Ans. (i) an irrational number

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12. A terminating decimal is

(i) a natural number

(ii) a rational number

(iii) a whole number

(iv) an integer.

Ans. (ii) a rational number

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13. The $\frac{p}{q}$ form of the number 0.8 is

$(i)\ \frac{8}{10}$ $\quad (ii)\ \frac{8}{100}$

$(iii)\ \frac{1}{8}$ $\quad (iv)\ 1$

Ans. $(i)\ \frac{8}{10}$

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14. The value of $\sqrt[3]{1000}$ is

(i) 1

(ii) 10

(iii) 3

(iv) 0

Ans. (ii) 10

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15. The sum of rational and an irrational number

(i) may be natural

(ii) may be irrational

(iii) is always irrational

(iv) is always rational

Ans. (iii) is always irrational

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16. The rational number not lying between $\frac{3}{5}$ and $\frac{2}{3}$ is

$(A)\frac{49}{75}$ $(B)\frac{50}{75}$

$(C)\frac{47}{75}$ $(D)\frac{46}{75}$

Ans. $(B)\frac{50}{75}$

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17. $0.12\overline{3}$ is equal to

(a) $\frac{122}{990}$

(b) $\frac{122}{100}$

(c) $\frac{122}{99}$

(d) None of these

Ans. (a) $\frac{122}{990}$

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18. The number $\left(1+\sqrt{3}\right)^2$ is

(a) natural number

(b) irrational number

(c) rational number

(d) integer

Ans. (b) irrational number

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19. The simplest form of $\sqrt{600}$ **is**

$(A)10\sqrt{60}$ $(B)100\sqrt{6}$

$(C)20\sqrt{3}$ $(D)10\sqrt{6}$

Ans. $(D)10\sqrt{6}$

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20. The value of $0.\overline{23}$ **+** $0.\overline{22}$ **is**

$(A)0.4\bar{5}$ $(B)0.4\bar{4}$

$(C)0.\overline{45}$ $(D)0.\overline{44}$

Ans. (A) $0.\overline{23}=0.232323...$

$0.\overline{22}=0.222222...$

$0.\overline{23}+0.\overline{22}=0.454545...$

$=0.\overline{45}$

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21. The value of $2^{\frac{1}{3}}\times 2^{-\frac{4}{3}}$ **is**

$(A)2$ $(B)\frac{1}{2}$

$(C)3$ (D) None of these

Ans. (B) $2^{\frac{1}{3}}\times 2^{\frac{-4}{3}}=2^{\frac{1}{3}-\frac{4}{3}}=2^{\frac{1-4}{3}}=2^{\frac{-3}{3}}$

= $2^{-1}=\frac{1}{2}$

22. $16\sqrt{13} \div 9\sqrt{52}$ **is equal to**

$(A)\frac{3}{9}$ $(B)\frac{9}{8}$

$(C)\frac{8}{9}$ (D) None of these

Ans. $16\sqrt{13} \div 9\sqrt{52}$

$\frac{16\sqrt{13}}{9\sqrt{52}} = \frac{16}{9} \times \sqrt{\frac{13}{52_4}} = \frac{16_8}{9} \times \frac{1}{2} = \frac{8}{9}$

23. $\sqrt{8}$ **is an**

(A) natural number

(B) rational number

(C) integer

(D) irrational number

Ans. (D) $\sqrt{8}$ **is an irrational number**

$\because \sqrt{4 \times 2} = 2\sqrt{2}$

2 Marks Questions

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$?

Ans. Consider the definition of a rational number.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

Zero can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{4}, \frac{0}{5}.....$.

So, we arrive at the conclusion that 0 can be written in the form of $\frac{p}{q}$, where *q* is any integer.

Therefore, zero is a rational number.

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2. Find six rational numbers between 3 and 4.

Ans. We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where *p* and *q* are integer sand $q \neq 0$.

We know that the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 all lie between 3 and 4.

We need to rewrite the numbers $3.1, 3.2, 3.3, 3.4, 3.5$ and 3.6 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{31}{10}, \frac{32}{10}, \frac{33}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$.

We can further convert the rational numbers $\frac{32}{10}, \frac{34}{10}, \frac{35}{10}$ and $\frac{36}{10}$ into lowest fractions.

On converting the fractions into lowest fractions, we get $\frac{16}{5}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

Therefore, six rational numbers between 3 and 4 are $\frac{31}{10}, \frac{16}{5}, \frac{33}{10}, \frac{17}{5}, \frac{7}{2}$ and $\frac{18}{5}$.

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3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans. We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where *p* and *q* are

Integers and $q \neq 0$.

We know that the numbers $\frac{3}{5}$ and $\frac{4}{5}$ can also be written as 0.6 and 0.8.

We can conclude that the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 all lie between 0.6 and 0.8

We need to rewrite the numbers $0.61, 0.62, 0.63, 0.64$ and 0.65 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{61}{100}, \frac{62}{100}, \frac{63}{100}, \frac{64}{100}$ and $\frac{65}{100}$.

We can further convert the rational numbers $\frac{62}{100}, \frac{64}{100}$ and $\frac{65}{100}$ into lowest fractions.

On converting the fractions, we get $\frac{31}{50}, \frac{16}{25}$ and $\frac{13}{20}$.

Therefore, six rational numbers between 3 and 4 are $\frac{61}{100}, \frac{31}{50}, \frac{63}{100}, \frac{16}{25}$ and $\frac{13}{50}$.

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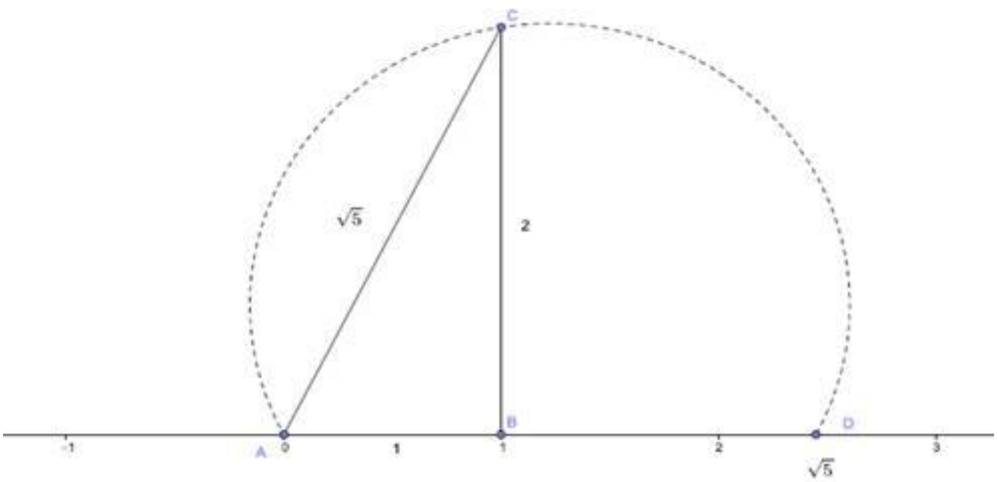
4. Show how $\sqrt{5}$ can be represented on the number line.

Ans. According to the Pythagoras theorem, we can conclude that

$$\left(\sqrt{5}\right)^2 = (2)^2 + (1)^2.$$

We need to draw a line segment *AB* of 1 unit on the number line. Then draw a straight line segment *BC* of 2 units. Then join the points *C* and *A*, to form a line segment *BC*.

Then draw the arc *ACD*, to get the number $\sqrt{5}$ on the number line.



5. You know that $\frac{1}{7}=0.142857.....$. Can you predict what the decimal expansions of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Ans. We are given that $\frac{1}{7}=0.\overline{142857}$ or $\frac{1}{7}=0.142857.....$.

We need to find the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

We know that, $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2\times\frac{1}{7},3\times\frac{1}{7},4\times\frac{1}{7},5\times\frac{1}{7}$ and $6\times\frac{1}{7}$.

On substituting value of $\frac{1}{7}$ as $0.142857.....$, we get

$$2\times\frac{1}{7}=2\times0.142857.....=0.285714.........$$

$$3\times\frac{1}{7}=3\times0.142857.....=0.428571$$

$$4\times\frac{1}{7}=4\times0.142857...=0.571428$$

$$5\times\frac{1}{7}=5\times0.142857.....=0.714285$$

$$6\times\frac{1}{7}=6\times0.142857.....=0.857142$$

Therefore, we conclude that, we can predict the values of $\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, to get

$$\frac{2}{7}=0.\overline{285714},\frac{3}{7}=0.\overline{428571},\frac{4}{7}=0.\overline{571428},\frac{5}{7}=0.\overline{714285},\text{ and }\frac{6}{7}=0.\overline{857142}$$

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6. Express $0.99999....$ in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

Ans. $\text{Let } x=0.99999....\qquad ....(a)$

We need to multiply both sides by 10 to get

$10x = 9.9999....$ $....(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{l} 10x = 9.99999..... \\ \underline{-\ x = 0.99999....} \\ \ 9x = 9 \end{array}$$

We can also write $9x = 9$ as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting $0.99999....$ in the $\frac{p}{q}$ form, we get the answer as 1.

Yes, at a glance we are surprised at our answer.

But the answer makes sense when we observe that 0.9999……… goes on forever. SO there is not gap between 1 and 0.9999……. and hence they are equal.

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7. Visualize 3.765 on the number line using successive magnification.

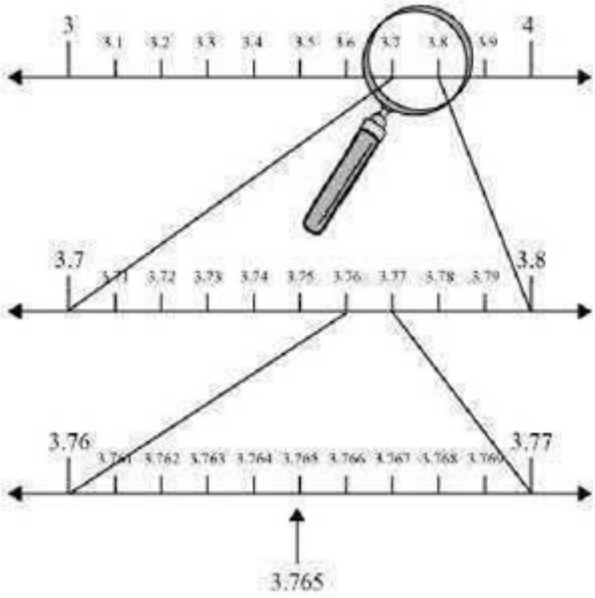
Ans. We know that the number 3.765 will lie between 3.764 and 3.766.

We know that the numbers 3.764 and 3.766 will lie between 3.76 and 3.77.

We know that the numbers 3.76 and 3.77 will lie between 3.7 and 3.8.

We know that the numbers 3.7 and 3.8 will lie between 3 and 4.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 3 and 4 on the number line



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8. Visualize $4.\overline{26}$ on the number line, up to 4 decimal places.

Ans. We know that the number $4.\overline{26}$ can also be written as $4.262....$.

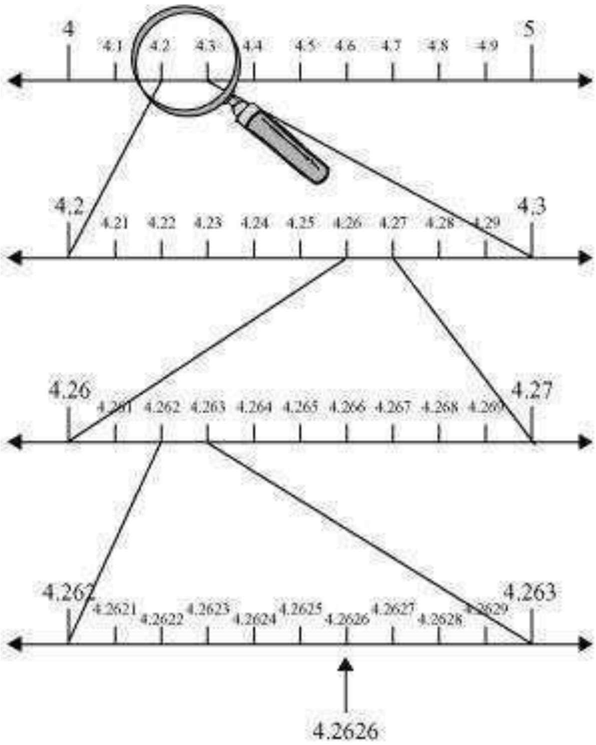
We know that the number $4.262....$ will lie between 4.261 and 4.263.

We know that the numbers 4.261 and 4.263 will lie between 4.26 and 4.27.

We know that the numbers 4.26 and 4.27 will lie between 4.2 and 4.3.

We know that the numbers 4.2 and 4.3 will lie between 4 and 5.

Therefore, we can conclude that we need to use the successive magnification, after locating numbers 4 and 5 on the number line.



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9. Recall, π is defined as the ratio of the circumference (say *c*) of a circle to its diameter (say *d*). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Ans. We know that when we measure the length of a line or a figure by using a scale or any device, we do not get an exact measurement. In fact, we get an approximate rational value. So, we are not able to realize that either circumference (*c*) or diameter(*d*) of a circle is irrational.

Therefore, we can conclude that as such there is not any contradiction regarding the value of π and we realize that the value of π is irrational.

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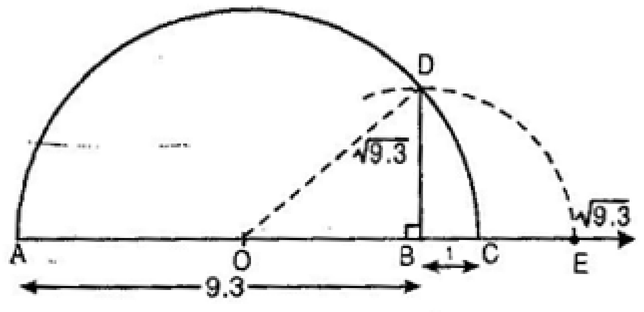
10. Represent 9.3 on the number line.

Ans. Mark the distance 9.3 units from a fixed point A on a given line to obtain a point B such that AB = 9.3 units. From B mark a distance of 1 unit and call the new point as C. Find the mid-point of AC and call that point as O. Draw a semi-circle with centre O and radius OC = 5.15 units. Draw a line perpendicular to AC passing through B cutting the semi-circle at D. Then BD = $\sqrt{9.3}$.



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11. Find: (i) $64^{\frac{1}{5}}$ **(ii)** $32^{\frac{1}{5}}$ **(iii)** $125^{\frac{1}{3}}$

Ans. (i) $64^{\frac{1}{2}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $64^{\frac{1}{2}}$ can also be written as $\sqrt[2]{64} = \sqrt[2]{8 \times 8}$

$\sqrt[2]{64} = \sqrt[2]{8 \times 8} = 8.$

Therefore, the value of $64^{\frac{1}{2}}$ will be 8.

(ii) $32^{\frac{1}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $32^{\frac{1}{5}}$ can also be written as $\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2}$

$\sqrt[5]{32} = \sqrt[5]{2 \times 2 \times 2 \times 2 \times 2} = 2$

Therefore, the value of $32^{\frac{1}{5}}$ will be 2.

(iii) $125^{\frac{1}{3}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We conclude that $125^{\frac{1}{3}}$ can also be written as $\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5}$

$\sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

Therefore, the value of $125^{\frac{1}{3}}$ will be 5.

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12. Simplify: $\sqrt[3]{2}\times\sqrt[4]{3}$

Ans. $\sqrt[3]{2}\times\sqrt[4]{3}$

$2^{\frac{1}{3}}, 3^{\frac{1}{4}}$

The LCM of 3 and 4 is 12

$\therefore 2^{\frac{1}{3}} = 2^{\frac{4}{12}} = (2^4)^{\frac{1}{12}} = 16^{\frac{1}{12}}$

$3^{\frac{1}{4}} = 3^{\frac{3}{12}} = (3^3)^{\frac{1}{12}} = 27^{\frac{1}{12}}$

$2^{\frac{1}{3}}\times 3^{\frac{1}{4}} = 16^{\frac{1}{12}}\times 27^{\frac{1}{12}} = (16\times 27)^{\frac{1}{12}}$

$= (432)^{\frac{1}{12}}$

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13. Find the two rational numbers between $\frac{1}{2}$ and $\frac{1}{3}$.

Ans. First rational number between $\frac{1}{2}$ and $\frac{1}{3}$

$= \frac{1}{2}\left[\frac{1}{2}+\frac{1}{3}\right] \Rightarrow \frac{1}{2}\left[\frac{3+2}{6}\right] \Rightarrow \frac{5}{12}$

$= \frac{1}{2}, \frac{5}{12}$ and $\frac{1}{3}$

Second rational number between $\frac{1}{2}$ *and* $\frac{1}{3}$

$= \frac{1}{2}\left[\frac{1}{2}+\frac{5}{12}\right] \Rightarrow \frac{1}{2}\left[\frac{6+5}{12}\right] \Rightarrow \frac{11}{24}$

$= \frac{5}{12}$ *and* $\frac{11}{24}$ are two rational numbers between $\frac{1}{2}$ *and* $\frac{1}{3}$

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14. Find two irrational numbers between 2 and 3.

$$\sqrt{2\times3}=\sqrt{6}$$

Ans. Irrational number between 2 and 3 is

Irrational number between 2 and $\sqrt{6}$ is

$$\sqrt{2\times\sqrt{6}}=2^{\frac{1}{2}}\times6^{\frac{1}{4}}=2^{2\times\frac{1}{4}}\times6^{\frac{1}{4}}$$
$$=\left(2^2\right)^{\frac{1}{4}}\times6^{\frac{1}{4}}=4^{\frac{1}{4}}\times6^{\frac{1}{4}}=(24)^{\frac{1}{4}}=\sqrt[4]{24}$$

$\sqrt{6}$ and $\sqrt{24}$ are two irrational numbers between 2 and 3.

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15. Multiply $\left(3-\sqrt{5}\right)$ $by\left(6+\sqrt{2}\right)$

Ans. $\left(3-\sqrt{5}\right)\left(6+\sqrt{2}\right)$

$$=3\left(6-\sqrt{2}\right)-\sqrt{5}\left(6+\sqrt{2}\right)$$
$$=18+3\sqrt{2}-6\sqrt{5}-\sqrt{5}\times\sqrt{2}$$
$$=18+3\sqrt{2}-6\sqrt{5}-\sqrt{10}$$

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16. Evaluate $(i)\ \sqrt[3]{125}\ (ii)\ \sqrt[4]{1250}$

Ans. (i) $\sqrt[3]{125}=(5\times5\times5)^{\frac{1}{3}}=\left(5^{3}\right)^{\frac{1}{3}}=5$

(ii) $\sqrt[4]{1250}=(2\times5\times5\times5\times5)^{\frac{1}{4}}=\left(2\times5^4\right)^{\frac{1}{4}}$

$$=2^{\frac{1}{4}}\times5^{4\times\frac{1}{4}}=5\times\sqrt[4]{2}$$

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17. Find rationalizing factor of $\sqrt{300}$

Ans. $\sqrt{300}=\sqrt{2\times2\times3\times5\times5}$

$$=\sqrt{2^2\times3\times5^2}$$

$=2\times5\sqrt{3}=10\sqrt{3}$ Rationalizing factor is $\sqrt{3}$

18. Rationalize the denominator $\frac{1}{\sqrt{5}+\sqrt{2}}$ and subtract it from $\sqrt{5}-\sqrt{2}$

Ans. $\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2} = \frac{\sqrt{5}-\sqrt{2}}{5-2} = \frac{\sqrt{5}-\sqrt{2}}{3}$

Difference between $\left(\sqrt{5}-\sqrt{2}\right)$ and $\left(\sqrt{5}-\frac{\sqrt{2}}{3}\right)$

$= \sqrt{5}-\sqrt{2}-\left(\frac{\sqrt{5}-\sqrt{2}}{3}\right)$

$= \sqrt{5}-\sqrt{2}-\frac{\sqrt{5}}{3}+\frac{\sqrt{2}}{3}$

$= \left(\sqrt{5}-\frac{\sqrt{5}}{3}\right)-\left(\sqrt{2}-\frac{\sqrt{2}}{3}\right)$

$= \frac{2\sqrt{5}}{3}-\frac{2\sqrt{2}}{3} = \frac{2}{3}\left(\sqrt{5}-\sqrt{2}\right)$

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19. Shove that $\sqrt{7}-3$ is irrational

Ans. suppose $\sqrt{7}-3$ is rational

let $\sqrt{7}-3=x$ (x is a rational number)

$\sqrt{7}=x+3$

X is a rational number 3 is also rational number

$\therefore$ x+3 is rational number

but is $\sqrt{7}$ irrational number which is contradiction

$\therefore$ $\sqrt{7}-3$ is irrational number

20. Find two rational numbers between 7 and 5.

Ans. First rational number = $\frac{1}{2}[7+5]=\frac{12}{2}=6$

Second rational number = $\frac{1}{2}[7+6]=\frac{1}{2}\times 13=\frac{13}{2}$

Two rational numbers between 7 and 5 are 6 and $\frac{13}{2}$

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21. Show that $5+\sqrt{2}$ is not a rational number.

Ans. let $5+\sqrt{2}$ is rational number.

Say $5+\sqrt{2}=x$ è $\sqrt{2}=x-5$

x is a rational number 5 is also rational number

$\therefore x-5$ is also rational number

But $\sqrt{2}$ is irrational number which is a contradiction

$\therefore 5+\sqrt{2}$ is irrational number

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22. Simplify $(\sqrt{5}+\sqrt{2})^2$

Ans. $\left(\sqrt{5+}\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{5}\times\sqrt{2}$ $=5+2+2\sqrt{10}=7+2\sqrt{2}$

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23. Evaluate $\frac{11^{\frac{5}{2}}}{11^{\frac{3}{2}}}$

Ans. $\frac{11^{\frac{5}{2}}}{11^{\frac{3}{2}}}=11^{\frac{5}{2}-\frac{3}{2}}\left[\because \frac{a^m}{a^n}=a^m-n\right]$

$= 11^{\frac{5-3}{2}} = 11^{\frac{2}{2}}$

$= 11$

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24. Find four rational numbers between $\frac{3}{7}$ *and* $\frac{4}{7}$

Ans.

$\frac{3}{7}\ \frac{31}{70}, \frac{32}{70}, \frac{33}{70}, \frac{34}{70}, \frac{35}{70}$ $\qquad\qquad \frac{4}{7}$

$\frac{3}{7} \times \frac{10}{10} = \frac{30}{70}$ and $\frac{4}{7} \times \frac{10}{10} = \frac{40}{70}$

Take any four rational numbers between $\frac{30}{70}$ and $\frac{40}{70}$ i.e. rational numbers between $\frac{3}{7}$ and $\frac{4}{7}$ are $\frac{31}{70}, \frac{32}{70}, \frac{33}{70}, \frac{34}{70}, \frac{35}{70}$

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25. Writ the following in decimal form $(i)\frac{36}{100} \quad (ii)\frac{2}{11}$

Ans.(i) $\frac{36}{100} = 0.36$

(ii) $\frac{2}{11} = 0.\overline{18}$

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26. Express $2.417\overline{8}$ in the form $\frac{a}{b}$

Ans. $x = 2.4\overline{178}$

$10x = 24.\overline{178}$ _____(1) [Multiplying both sides by 10]

$10x = 24.178178178...$

$1000 \times 10x = 1000 \times 24.178178178....$[Multiplying both sides by 1000]

$10,000x = 24178.178178....$

$10000x = 24178.\overline{178}$ _______(2)

Subtracting (1) from (2)

$10,000x - x = 24178.\overline{178} - 24.\overline{178}$

$9990x = 24154$

$x = \frac{24154}{9990}$

$2.4\overline{178} = \frac{24154}{9990} = \frac{12077}{4995}$

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27. Multiply $\sqrt{3}$ by $\sqrt[3]{5}$

Ans. $\sqrt{3}$ and $\sqrt[3]{5}$

Or $3^{\frac{1}{2}}$ and $5^{\frac{1}{3}}$

LCM of 2 and 3 is 6

$3^{\frac{1}{2}} = 3^{\frac{1}{2}\times\frac{3}{3}} = \left(3^3\right)^{\frac{1}{6}} = \left(27\right)^{\frac{1}{6}}$

$5^{\frac{1}{3}} = 5^{\frac{1}{3}\times\frac{2}{2}} = \left(5^2\right)^{\frac{1}{6}} = \left(25\right)^{\frac{1}{6}}$

$\sqrt{3}\times\sqrt[3]{5} = \left(27\right)^{\frac{1}{6}}\times\left(25\right)^{\frac{1}{6}} = \left(27\times 25\right)^{\frac{1}{6}}$

$= 675^{\frac{1}{6}} = \sqrt[6]{675}$

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28. Find the value of $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}}$ if $\sqrt{5} = 2.236$ and $\sqrt{10} = 3.162$

Ans. $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}+5}{5}$ = $\frac{3.162+5}{5} = \frac{8.162}{5} = 1.6324$

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29. Convert $0.\overline{25}$ into rational number

Ans. Let $x = 0.\overline{25}$........(i)

$x = 0.252525...$

Multiplying both sides by 100

100x =25.252525…

100x = $25.\overline{25}$..........(ii)

Subtracting (i) from (ii)

$100x - x = 25.\overline{25} - 0.\overline{25}$

$99x = 25$

$x = \frac{25}{99}$

---

30. Simplify $\left(3\sqrt{3}+2\sqrt{2}\right)\left(2\sqrt{3}+3\sqrt{2}\right)$

Ans. $\left(3\sqrt{3}+2\sqrt{2}\right)\left(2\sqrt{3}+3\sqrt{2}\right)$

$3\sqrt{3}\left(2\sqrt{3}+3\sqrt{2}\right)+2\sqrt{2}\left(2\sqrt{3}+3\sqrt{2}\right)$

= $6\times3+9\sqrt{3}.\sqrt{2}+4\sqrt{2}.\sqrt{3}+6\times2$

= $18+9\sqrt{6}+4\sqrt{6}+12$

= $30+(9+4)\sqrt{6}=30+13\sqrt{6}$

---

31. Simplify $\frac{9^{\frac{3}{2}}.9^{-\frac{4}{2}}}{9^{\frac{1}{2}}}$

Ans. $\frac{9^{\frac{3}{2}}.9^{\frac{-4}{2}}}{9^{\frac{1}{2}}} = \frac{9^{\frac{3}{2}-\frac{4}{2}}}{9^{\frac{1}{2}}}$ $\quad a^m.a^n = a^{m+n}$

$$= \frac{9^{\frac{-1}{2}}}{9^{\frac{1}{2}}} = \frac{1}{9^{\frac{1}{2}+\frac{1}{2}}} \quad a^{-m} = a^{\frac{1}{m}}$$

$$= \frac{1}{9^{\frac{2}{2}}} = \frac{1}{9}$$

1. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Ans. (i) Consider the whole numbers and natural numbers separately.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.

We know that natural number series is $1, 2, 3, 4, 5 \ldots\ldots$.

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where $q = 1$.

Now, considering the series of integers, we have $\ldots\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots\ldots$.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.

We can conclude that all the numbers of whole number series lie in the series of integers. But every number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots\ldots$.

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1} \ldots$.

We conclude that every number of the whole number series is a rational number. But, every rational number does not appear in the whole number series.

Therefore, we conclude that every rational number is not a whole number.

---

2. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.

(iii) Every real number is an irrational number.

Ans.

(i) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

Therefore, we conclude that, yes every irrational number is a real number.

(ii) Consider a number line. We know that on a number line, we can represent negative as well as positive numbers.

We know that we cannot get a negative number after taking square root of any number.

Therefore, we conclude that not every number point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.

(iii) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

So, we can conclude that every irrational number is a real number. But every real number is not an irrational number.

Therefore, we conclude that, every real number is not a rational number.

---

3. Express the following in the form $\frac{p}{q}$, where *p* and *q* are integers and *q* $\neq$ 0.

(i) $0.\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.\overline{001}$

Ans. (i) $\text{Let } x = 0.\overline{6}$

$\Rightarrow x = 0.6666.....\ (a)$

We need to multiply both sides by 10 to get

$10x = 6.6666.....\ ....(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{l} 10x = 6.6666..... \\ -\ x = 0.6666..... \\ \hline 9x = 6 \end{array}$$

We can also write $9x = 6$ as $x = \frac{6}{9}$ or $x = \frac{2}{3}$.

Therefore, on converting $0.\overline{6}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{2}{3}$.

(ii) $\text{Let } x = 0.4\overline{7} \Rightarrow x = 0.47777.....(a)$

We need to multiply both sides by 10 to get

$10x = 4.7777.........(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{l} 10x = 4.7777..... \\ -\ x = 0.4777.... \\ \hline 9x = 4.3 \end{array}$$

We can also write $9x = 4.3$ as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$.

Therefore, on converting $0.4\overline{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) $\text{Let } x = 0.\overline{001}$

$\Rightarrow x = 0.001001.....(a)$

We need to multiply both sides by 1000 to get

$1000x = 1.001001..... \quad ....(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{r} 1000x = 1.001001..... \\ -x = 0.001001.... \\ \hline 999x = 1 \end{array}$$

We can also write $999x = 1$ as $x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

---

4. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Ans. We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

$$
\begin{array}{r}
0.0588235294117647.... \\
17 \overline{)\ 1} \\
-0 \\
\hline 10 \\
-0 \\
\hline 100 \\
-85 \\
\hline 150 \\
-136 \\
\hline 140 \\
-136 \\
\hline 40 \\
-34 \\
\hline 60 \\
-51 \\
\hline 90 \\
-85 \\
\hline 50 \\
-34 \\
\hline 160 \\
-153 \\
\hline 70 \\
-68 \\
\hline 20 \\
-17 \\
\hline 30 \\
-17 \\
\hline 130 \\
-119 \\
\hline 110 \\
-102 \\
\hline 80 \\
-68 \\
\hline 120 \\
-119 \\
\hline 1
\end{array}
$$

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that $\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$, which is a non-terminating decimal and recurring decimal.

---

5. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where *p* and *q* are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property *q* must satisfy?

Ans. Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which *q* must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that *q* must have powers of 2, 5 or both.

---

6. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Ans. (i) $2-\sqrt{5}$

We know that $\sqrt{5} = 2.236....,$ which is an irrational number.

$2-\sqrt{5} = 2-2.236....$

$= -0.236....,$ which is also an irrational number.

Therefore, we conclude that $2-\sqrt{5}$ is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

$\left(3+\sqrt{23}\right)-\sqrt{23}=3+\sqrt{23}-\sqrt{23}$ = 3.

Therefore, we conclude that $\left(3+\sqrt{23}\right)-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

We can cancel $\sqrt{7}$ in the numerator and denominator, as $\sqrt{7}$ is the common number in numerator as well as denominator, to get

$\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

Therefore, we conclude that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

We know that $\sqrt{2}=1.414....,$ which is an irrational number.

We can conclude that, when 1 is divided by $\sqrt{2}$, we will get an irrational number.

Therefore, we conclude that $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

We know that $\pi=3.1415....,$ which is an irrational number.

We can conclude that 2π will also be an irrational number.

Therefore, we conclude that 2π is an irrational number.

---

7. Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

(iv) $\left(5-\sqrt{2}\right)\left(5+\sqrt{2}\right)$

Ans. (i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$.

$\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$, we get $6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$.

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$.

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=3\left(3-\sqrt{3}\right)+\sqrt{3}\left(3-\sqrt{3}\right)$

$=9-3\sqrt{3}+3\sqrt{3}-3$

= 6

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$, we get 6.

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

We need to apply the formula $(a+b)^2=a^2+2ab+b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\times\sqrt{5}\times\sqrt{2}+\left(\sqrt{2}\right)^2$

$=5+2\sqrt{10}+2$

$=7+2\sqrt{10}.$

Therefore, on simplifying $\left(\sqrt{5}+\sqrt{2}\right)^2$, we get $7+2\sqrt{10}$.

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$$\begin{aligned}\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)&=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2\\&=5-2\\&=3.\end{aligned}$$

Therefore, on simplifying $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$, we get 3.

---

8. Find:

(i) $9^{\frac{3}{2}}$

(ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Ans.

(i) $9^{\frac{3}{2}}$

We know that $a^{\frac{1}{n}}=\sqrt[n]{a},\ \text{where } a>0.$

We conclude that $9^{\frac{3}{2}}$ can also be written as $\sqrt[2]{(9)^3}=\sqrt[2]{9\times9\times9}=\sqrt[2]{3\times3\times3\times3\times3\times3}$

$\sqrt[2]{(9)^3}=\sqrt[2]{3\times3\times3\times3\times3\times3}$

$=3\times3\times3$

= 27

Therefore, the value of $9^{\frac{3}{2}}$ will be 27.

(ii) $32^{\frac{2}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}, \text{ where } a > 0.$

We conclude that $32^{\frac{2}{5}}$ can also be written as

$\sqrt[5]{(32)^2} = \sqrt[5]{(2\times2\times2\times2\times2)(2\times2\times2\times2\times2)}$

$= 2\times2$

= 4

Therefore, the value of $32^{\frac{2}{5}}$ will be 4.

(iii) $16^{\frac{3}{4}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}, \text{ where } a > 0.$

We conclude that $16^{\frac{3}{4}}$ can also be written as

$\sqrt[4]{(16)^3} = \sqrt[4]{(2\times2\times2\times2)(2\times2\times2\times2)(2\times2\times2\times2)}$

$= 2\times2\times2$

= 8

Therefore, the value of $16^{\frac{3}{4}}$ will be 8.

(iv) $125^{\frac{-1}{3}}$

We know that $a^{-n} = \frac{1}{a^n}$

We conclude that $125^{\frac{-1}{3}}$ can also be written as $\frac{1}{125^{\frac{1}{3}}}$, or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$.

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We know that $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ can also be written as

$$\sqrt[3]{\left(\frac{1}{125}\right)} = \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)}$$

$$= \frac{1}{5}.$$

Therefore, the value of $125^{\frac{-1}{3}}$ will be $\frac{1}{5}$.

---

9. Simplify:

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

(ii) $\left(3^{\frac{1}{3}}\right)^{7}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Ans.

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

We know that $a^{m} \cdot a^{n} = a^{(m+n)}$.

We can conclude that $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$.

$$2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{10+3}{15}} \quad = (2)^{\frac{13}{15}}$$

Therefore, the value of $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ will be $(2)^{\frac{13}{15}}$.

(ii) $\left(3^{\frac{1}{3}}\right)^7$

We know that $a^m \times a^n = a^{m+n}$

We conclude that $\left(3^{\frac{1}{3}}\right)^7$ can also be written as $\left(3^{\frac{7}{3}}\right)$.

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

We know that $\frac{a^m}{a^n} = a^{m-n}$

We conclude that $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$.

$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}}$

$= 11^{\frac{1}{4}}$

Therefore, the value of $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ will be $11^{\frac{1}{4}}$.

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

We know that $a^m \cdot b^m = (a \times b)^m$.

We can conclude that $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$.

$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = (56)^{\frac{1}{2}}$

Therefore, the value of $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$ will be $(56)^{\frac{1}{2}}$

---

10. Express 0.8888… in the form p/q.

Ans. $\text{Let } x = 0.8888$

$x = 0.\bar{8} ---(1)$

$10x = 10 \times 0.8888$ (multiplying both sides by 10)

$10x = 8.8888$

$10x = 8.\bar{8} ----(2)$

$10x - x = 8.\bar{8} - 0.\bar{8}$ [subtracting (1) from (2)]

$9x = 8$

$x = \frac{8}{9}$

---

11. Simply by rationalizing denominator $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$

Ans. $\frac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \frac{7+3\sqrt{5}}{7+3\sqrt{5}}$ **(Rationalizing by denominator)**

$\frac{\left(7+3\sqrt{5}\right)^2}{7^2-\left(3\sqrt{5}\right)^2} = \frac{7^2+\left(3\sqrt{5}\right)^2+2\times7\times3\sqrt{5}}{49-3^2\times5}$

$= \frac{49+9\times5+42\sqrt{5}}{49-45} = \frac{49+45+42\sqrt{5}}{4}$

$= \frac{94+42\sqrt{5}}{4} = \frac{94}{4} + \frac{42}{4}\sqrt{5}$

$= \frac{47}{2} + \frac{21}{2}\sqrt{5}$

---

12. Simplify $\left\{\left[625^{\frac{-1}{2}}\right]^{-\frac{1}{4}}\right\}^2$

Ans. $\left\{\left(625^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right\}^2$

$=\left\{\left(\frac{1}{625^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2=\left\{\left(\frac{1}{(25^2)^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2 \left(a^{-m}=\frac{1}{a^m}\right)$

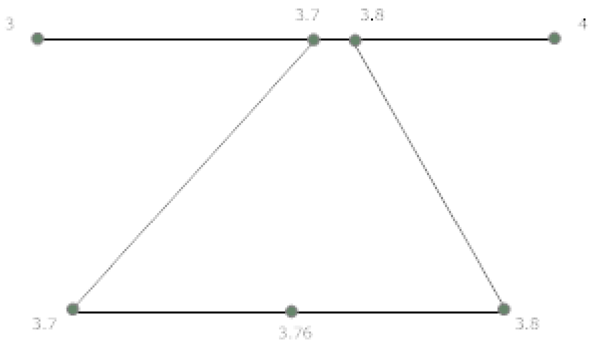
$=\left\{\left(\frac{1}{25}\right)^{-\frac{1}{4}\times 2}\right\}$

$=\left(\frac{1}{25^{-\frac{1}{2}}}\right)=\frac{1}{(5^2)^{-\frac{1}{2}}}=\frac{1}{5^{-1}}=5$

---

13. Visualize 3.76 on the number line using successive magnification.

Ans.



---

14. Prove that $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}=1$

Ans. $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$

$\frac{1}{1+x^b.x^{-a}+x^c.x^{-a}}+\frac{1}{1+x^a.x^{-b}+x^c.x^{-b}}+\frac{1}{1+x^a.x^{-c}+x^b.x^{-c}}$

$\frac{1}{x^{-a}.x^a+x^b.x^{-a}+x^c.x^{-a}}+\frac{1}{x^b.x^{-b}+x^a.x^{-b}+x^c.x^{-b}}+\frac{1}{x^c.x^{-c}+x^a.x^{-c}+x^b.x^{-c}}$

$$\frac{1}{x^{-a}\left(x^a+x^b+x^c\right)}+\frac{1}{x^{-b}\left(x^a+x^b+x^c\right)}+\frac{1}{x^{-c}\left(x^a+x^b+x^c\right)}$$

$$\frac{x^a}{\left(x^a+x^b+x^c\right)}+\frac{x^b}{\left(x^a+x^b+x^c\right)}+\frac{x^c}{\left(x^a+x^b+x^c\right)}$$

$$=\frac{x^a+x^b+x^c}{x^a+x^b+x^c}=1$$

---

15. Represent $\sqrt{3}$ on number line

Ans. *Take* OA=AB=1 unit in same line

. and $\angle A=90^0$

In Δ OAB, $OB^2=1^2+1^2$

$OB^2=2$

$OB=\sqrt{2}$

$\therefore OB=$ OA$=\sqrt{2}=1.41\sqrt{2}$, BD=1 and $\angle OBD=90^0$

$OD^2=OB^2+BD^2$

$OD^2=\left(\sqrt{2}\right)^2+(1)^2$

$OD^2=2+1=3$

$OD=\sqrt{3}$

---

16. Simplify $\left(3\sqrt{2}+2\sqrt{3}\right)^2\left(3\sqrt{2}-2\sqrt{3}\right)^2$

Ans. $\left(3\sqrt{2}+2\sqrt{3}\right)^2\left(3\sqrt{2}-2\sqrt{3}\right)^2$

$\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$=\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$$=\left[\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2\right]\left[\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2\right]$$

$=[9\times2-4\times3][9\times2-4\times3]$

$=[18-12][18-12]$

$=6\times6=36$

---

17. Express $2.4\overline{178}$ in the from $\frac{p}{q}$

Ans. Let $\frac{p}{q}=2.4\overline{178}$

$\frac{p}{q}=2.4178178178$

Multiplying by 10

$10\frac{p}{q}=24.178178$

Multiplying by 1000

$10,000\frac{p}{q}=1000\times24.178178$

$10,000\frac{p}{q}=24178.178178$

$10000\frac{p}{q}-\frac{p}{q}=24178.178178-24.178178$

$9999\frac{p}{q}=24154$

$\frac{p}{q}=\frac{24154}{9999}$

---

18. Simplify $(27)^{-\frac{2}{3}}\div9^{\frac{1}{2}}.3^{-\frac{3}{2}}$

Ans. $(27)^{-\frac{2}{3}} \div 9^{\frac{1}{2}} 3^{-\frac{3}{2}}$

$$\frac{(3\times3\times3)^{-\frac{2}{3}}\times3^{\frac{3}{2}}}{(3\times3)^{\frac{1}{2}}} \quad \left[a^{-m}=\frac{1}{a^{m}}\right]$$

$$=\frac{(3^{3})^{-\frac{2}{3}}\times3^{\frac{3}{2}}}{(3^{2})^{\frac{1}{2}}}$$

$$=\frac{3^{\frac{3}{2}-2}}{3}=\frac{3^{-\frac{1}{3}}}{3}=\frac{1}{3^{1+\frac{1}{3}}}$$

$$=\frac{1}{3^{\frac{4}{3}}}=\frac{1}{\sqrt[3]{81}}$$

---

19. Find three rational numbers between $2.\overline{2}$ and $2.\overline{3}$

Ans. Three rational numbers between $2.\overline{2}$ and $2.\overline{3}$ are $2.212341365....$ and $2.2321453269....$

---

20. Give an example of two irrational numbers whose

(i) sum is a rational number

(ii) product is a rational number

(iii) Quotient is a rational number.

Ans. (i) $2+\sqrt{2}.$ and $2-\sqrt{2}$

Sum $2+\sqrt{2}+2-\sqrt{2}=4$ which is a rational number

(ii) $3\sqrt{2}$ and $6\sqrt{2}$

Product = $3\sqrt{2}\times6\sqrt{2}=18\times2=36$ rational

(iii) $2\sqrt{125}$ and $3\sqrt{5}$

Quotient = $\frac{2\sqrt{125}}{3\sqrt{5}} = \frac{2}{3}\sqrt{\frac{125^{25}}{5}} = \frac{2}{3}\sqrt{25}$ $= \frac{2}{3}\times 5 = \frac{10}{3}$

---

21. If $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$, find the value of $\frac{5}{\sqrt{2}+\sqrt{3}}$

Ans. $\frac{5}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ (Rationalizing denominator)

$= \frac{5\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2} = \frac{5\left(\sqrt{2}-\sqrt{3}\right)}{2-3}$

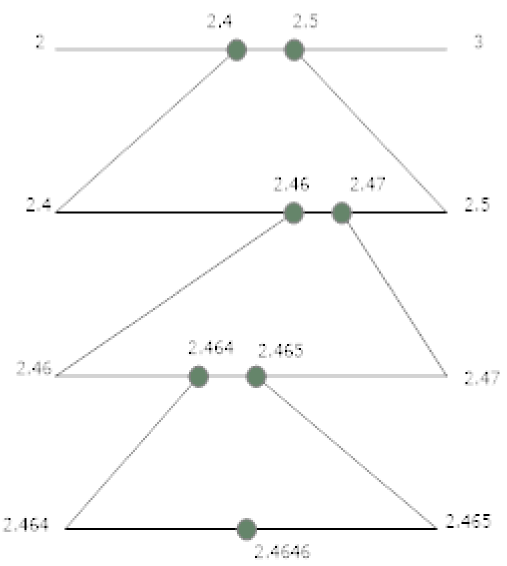
$= -5\left[1.414-1.732\right]$

$= -5\times -0.318 = 1.59$

---

22. Visualize 2.4646 on the number line using successive magnification.

Ans.



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23. Rationalize the denominator of $\frac{1}{4+2\sqrt{3}}$

Ans. $\frac{1}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}=\frac{4-2\sqrt{3}}{(4)^2-(2\sqrt{3})^2}$

$=\frac{4-2\sqrt{3}}{16-(2\sqrt{3})^2}=\frac{4-2\sqrt{3}}{16-4\times3}=\frac{4-2\sqrt{3}}{16-12}$

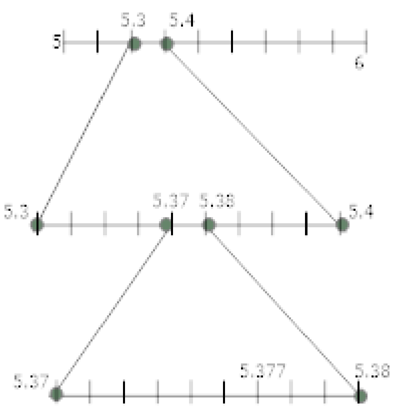
$=\frac{4-2\sqrt{3}}{4}=\frac{2(2-\sqrt{3})}{4_2}$

$=\frac{2-\sqrt{3}}{2}$

---

24. Visualize the representation of $5.3\overline{7}$ on the number line up to 3 decimal places.

Ans.



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25. Shone that $5\sqrt{2}$ is not rational number

Ans. let $5\sqrt{2}$ is rational number

$x=5\sqrt{2}$ (x is rational)

$\frac{x}{5}=\sqrt{2}$

x is rational number and 5 is also rational number.

$\therefore \frac{x}{5}$ is rational number

But $\frac{x}{5}=\sqrt{2}$ and $\sqrt{2}$ is irrational number in same line which is a contradiction

$\therefore 5\sqrt{2}$ is irrational number

---

26. Simplify $3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}$

Ans. $3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}$

$=3\sqrt[3]{5\times5\times5\times2}+7\sqrt[3]{2\times2\times2\times2}-4\sqrt[3]{3\times3\times3\times2}$

$=3\times5\sqrt[3]{2}+7\times2\sqrt[3]{2}-4\times3\sqrt[3]{2}$

$=15\sqrt[3]{2}+14\sqrt[3]{2}-12\sqrt[3]{2}$

$=[15+14-12]\sqrt[3]{2}=17\sqrt[3]{2}$

---

27. Simplify $3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}$

Ans. $3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}$

$=3\sqrt{16\times3}-\frac{5}{2}\sqrt{\frac{3}{3\times3}}+4\sqrt{3}$

$=3\times4\sqrt{3}-\frac{5}{2}\cdot\frac{1}{3}\sqrt{3}+4\sqrt{3}$

$=12\sqrt{3}-\frac{5}{6}\sqrt{3}+4\sqrt{3}$

$=\left(12-\frac{5}{6}+4\right)\sqrt{3}=\left(16-\frac{5}{6}\right)\sqrt{3}$

$=\frac{91}{6}\sqrt{3}$

---

28. If $\frac{1}{7}=0.\overline{142857}.$ find the value of $\frac{2}{7},\frac{3}{7},\frac{4}{7}$

Ans. $\frac{1}{7}=0.\overline{142857}$

$\frac{2}{7}=2\times 0.\overline{142857}$

$=0.\overline{285714}$

$\frac{3}{7}=3\times 0.\overline{142857}$ $=0.\overline{428571}$

$\frac{4}{7}=4\times 0.\overline{142857}$ $=0.\overline{571428}$

---

29. Find 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$

Ans. $\frac{6}{5},\frac{7}{5}$ is divided into 10 equal parts than the value of each part

$\frac{6.1}{5},\frac{6.2}{5},\frac{6.3}{5},\frac{6.4}{5},\frac{6.5}{5},\frac{6.6}{5},\frac{6.7}{5},\frac{6.8}{5},\frac{6.9}{5},\frac{7}{5}$

$\therefore$ 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$

$\frac{61}{50},\frac{62}{50},\frac{63}{50},\frac{64}{50},\frac{65}{50},\frac{66}{50}$

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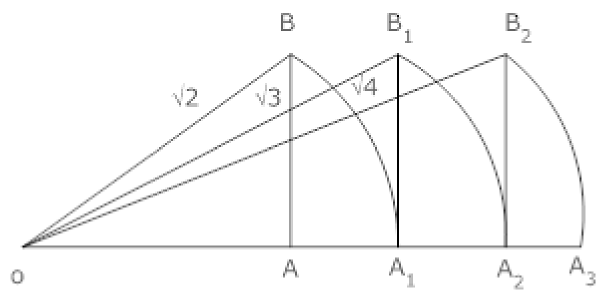
30. Show how $\sqrt{4}$ can be represented on the number line

Ans. Take $OA=AB=1$ unit



And $\angle A = 90^{\circ}$

In ΔOAB

$OB^2 = 1^2 + 1^2 = OB^2 = 2$

$OB = \sqrt{2}$

$OB = OA_1 = \sqrt{2}$

Take $A_1B_1 = 1$

And $\angle A_1 = 90^{\circ}$

In $\Delta OB_1A_1, OB_1^2 = \left(\sqrt{2}\right)^2 + 1^2$

$OB_1 = \sqrt{3}$

$OB_1 = OA_2 = \sqrt{3}$

Take $a_2b_2 = 1$ unit

$\angle A_2 = 90^{\circ}$

$OB_2^2 = \left(\sqrt{3}\right)^2 + 1^2 = OB_2^2 = 3 + 1$

$OB_2 = \sqrt{4} = 2$

3 Marks Questions

1. State whether the following statements are true or false. Give reasons for your answers.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Ans. (i) Consider the whole numbers and natural numbers separately.

We know that whole number series is $0, 1, 2, 3, 4, 5.....$.

We know that natural number series is $1, 2, 3, 4, 5.....$.

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

(ii) Consider the integers and whole numbers separately.

We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where $q = 1$.

Now, considering the series of integers, we have $\ldots -4, -3, -2, -1, 0, 1, 2, 3, 4 \ldots$.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots$.

We can conclude that all the numbers of whole number series lie in the series of integers. But every number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$, where $q \neq 0$.

We know that whole number series is $0, 1, 2, 3, 4, 5 \ldots$.

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as $\frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1} \ldots$.

We conclude that every number of the whole number series is a rational number. But, every rational number does not appear in the whole number series.

Therefore, we conclude that every rational number is not a whole number.

---

2. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.

(iii) Every real number is an irrational number.

Ans.

(i) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

Therefore, we conclude that, yes every irrational number is a real number.

(ii) Consider a number line. We know that on a number line, we can represent negative as well as positive numbers.

We know that we cannot get a negative number after taking square root of any number.

Therefore, we conclude that not every number point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.

(iii) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

So, we can conclude that every irrational number is a real number. But every real number is not an irrational number.

Therefore, we conclude that, every real number is not a rational number.

---

3. Express the following in the form $\frac{p}{q}$, where *p* and *q* are integers and *q* ≠ 0.

(i) $0.\overline{6}$

(ii) $0.4\overline{7}$

(iii) $0.\overline{001}$

Ans. (i) $\text{Let } x = 0.\overline{6}$

$\Rightarrow x = 0.6666..... (a)$

We need to multiply both sides by 10 to get

$10x = 6.6666.....\ ....(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{l} 10x = 6.6666..... \\ \underline{-\ x = 0.6666.....} \\ 9x = 6 \end{array}$$

We can also write $9x = 6$ as $x = \frac{6}{9}$ or $x = \frac{2}{3}$.

Therefore, on converting $0.\bar{6}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{2}{3}$.

(ii) $\text{Let } x = 0.4\bar{7} \Rightarrow x = 0.47777.....(a)$

We need to multiply both sides by 10 to get

$10x = 4.7777.........(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{l} 10x = 4.7777..... \\ \underline{-\ x = 0.4777....} \\ 9x = 4.3 \end{array}$$

We can also write $9x = 4.3$ as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$.

Therefore, on converting $0.4\bar{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) $\text{Let } x = 0.\overline{001}$

$\Rightarrow x = 0.001001.....(a)$

We need to multiply both sides by 1000 to get

$1000x = 1.001001.....\ \ ....(b)$

We need to subtract (*a*)from (*b*), to get

$$\begin{array}{l} 1000x = 1.001001..... \\ \underline{\quad -\ x = 0.001001....} \\ 999x = 1 \end{array}$$

We can also write $999x = 1$ as $x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

---

4. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Ans. We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

$$17\overline{)\,1\,}\quad 0.0588235294117647....$$

$$\begin{array}{r} 1 \\ \underline{-0} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-85} \\ 150 \\ \underline{-136} \\ 140 \\ \underline{-136} \\ 40 \\ \underline{-34} \\ 60 \\ \underline{-51} \\ 90 \\ \underline{-85} \\ 50 \\ \underline{-34} \\ 160 \\ \underline{-153} \\ 70 \\ \underline{-68} \\ 20 \\ \underline{-17} \\ 30 \\ \underline{-17} \\ 130 \\ \underline{-119} \\ 110 \\ \underline{-102} \\ 80 \\ \underline{-68} \\ 120 \\ \underline{-119} \\ 1 \end{array}$$

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that $\frac{1}{17} = 0.0588235294117647.....$ or $\frac{1}{17} = 0.\overline{0588235294117647}$, which is a non-terminating decimal and recurring decimal.

---

5. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where *p* and *q* are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property *q* must satisfy?

Ans. Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$\frac{5}{2} = 2.5$

$\frac{5}{4} = 1.25$

$\frac{2}{5} = 0.4$

$\frac{2}{10} = 0.2$

$\frac{5}{16} = 0.3125$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which *q* must satisfy in $\frac{p}{q}$, so that the rational number $\frac{p}{q}$ is a terminating decimal is that *q* must have powers of 2, 5 or both.

---

6. Classify the following numbers as rational or irrational:

(i) $2-\sqrt{5}$

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

(iv) $\frac{1}{\sqrt{2}}$

(v) 2π

Ans. (i) $2-\sqrt{5}$

We know that $\sqrt{5} = 2.236....,$ which is an irrational number.

$2-\sqrt{5} = 2-2.236....$

$= -0.236....,$ which is also an irrational number.

Therefore, we conclude that $2-\sqrt{5}$ is an irrational number.

(ii) $\left(3+\sqrt{23}\right)-\sqrt{23}$

$\left(3+\sqrt{23}\right)-\sqrt{23}=3+\sqrt{23}-\sqrt{23}$ = 3.

Therefore, we conclude that $\left(3+\sqrt{23}\right)-\sqrt{23}$ is a rational number.

(iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

We can cancel $\sqrt{7}$ in the numerator and denominator, as $\sqrt{7}$ is the common number in numerator as well as denominator, to get

$\frac{2\sqrt{7}}{7\sqrt{7}}=\frac{2}{7}$

Therefore, we conclude that $\frac{2\sqrt{7}}{7\sqrt{7}}$ is a rational number.

(iv) $\frac{1}{\sqrt{2}}$

We know that $\sqrt{2}=1.414\ldots$, which is an irrational number.

We can conclude that, when 1 is divided by $\sqrt{2}$, we will get an irrational number.

Therefore, we conclude that $\frac{1}{\sqrt{2}}$ is an irrational number.

(v) 2π

We know that $\pi=3.1415\ldots$, which is an irrational number.

We can conclude that 2π will also be an irrational number.

Therefore, we conclude that 2π is an irrational number.

---

7. Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

(iv) $\left(5-\sqrt{2}\right)\left(5+\sqrt{2}\right)$

Ans. (i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$.

$\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)=3\left(2+\sqrt{2}\right)+\sqrt{3}\left(2+\sqrt{2}\right)$

$=6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$, we get $6+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$.

(ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

We need to apply distributive law to find value of $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$.

$\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=3\left(3-\sqrt{3}\right)+\sqrt{3}\left(3-\sqrt{3}\right)$

$=9-3\sqrt{3}+3\sqrt{3}-3$

= 6

Therefore, on simplifying $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$, we get 6.

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$

We need to apply the formula $(a+b)^2=a^2+2ab+b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$\left(\sqrt{5}+\sqrt{2}\right)^2=\left(\sqrt{5}\right)^2+2\times\sqrt{5}\times\sqrt{2}+\left(\sqrt{2}\right)^2$

$=5+2\sqrt{10}+2$

$=7+2\sqrt{10}.$

Therefore, on simplifying $\left(\sqrt{5}+\sqrt{2}\right)^2$, we get $7+2\sqrt{10}$.

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ to find value of $\left(\sqrt{5}+\sqrt{2}\right)^2$.

$$\begin{aligned}\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right) &= \left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2 \\ &= 5-2 \\ &= 3.\end{aligned}$$

Therefore, on simplifying $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$, we get 3.

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8. Find:

(i) $9^{\frac{3}{2}}$

(ii) $32^{\frac{2}{5}}$

(iii) $16^{\frac{3}{4}}$

(iv) $125^{\frac{-1}{3}}$

Ans.

(i) $9^{\frac{3}{2}}$

We know that $a^{\frac{1}{n}}=\sqrt[n]{a}, \text{ where } a>0.$

We conclude that $9^{\frac{3}{2}}$ can also be written as $\sqrt[2]{(9)^3}=\sqrt[2]{9\times9\times9}=\sqrt[2]{3\times3\times3\times3\times3\times3}$

$\sqrt[2]{(9)^3}=\sqrt[2]{3\times3\times3\times3\times3\times3}$

$=3\times3\times3$

= 27

Therefore, the value of $9^{\frac{3}{2}}$ will be 27.

(ii) $32^{\frac{2}{5}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a},\ \text{where } a > 0.$

We conclude that $32^{\frac{2}{5}}$ can also be written as

$\sqrt[5]{(32)^2} = \sqrt[5]{(2\times2\times2\times2\times2)(2\times2\times2\times2\times2)}$

$= 2\times2$

= 4

Therefore, the value of $32^{\frac{2}{5}}$ will be 4.

(iii) $16^{\frac{3}{4}}$

We know that $a^{\frac{1}{n}} = \sqrt[n]{a},\ \text{where } a > 0.$

We conclude that $16^{\frac{3}{4}}$ can also be written as

$\sqrt[4]{(16)^3} = \sqrt[4]{(2\times2\times2\times2)(2\times2\times2\times2)(2\times2\times2\times2)}$

$= 2\times2\times2$

= 8

Therefore, the value of $16^{\frac{3}{4}}$ will be 8.

(iv) $125^{\frac{-1}{3}}$

We know that $a^{-n} = \frac{1}{a^n}$

We conclude that $125^{\frac{-1}{3}}$ can also be written as $\frac{1}{125^{\frac{1}{3}}}$, or $\left(\frac{1}{125}\right)^{\frac{1}{3}}$.

We know that $a^{\frac{1}{n}} = \sqrt[n]{a}$, where $a > 0$.

We know that $\left(\frac{1}{125}\right)^{\frac{1}{3}}$ can also be written as

$$\sqrt[3]{\left(\frac{1}{125}\right)} = \sqrt[3]{\left(\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}\right)}$$

$$= \frac{1}{5}.$$

Therefore, the value of $125^{\frac{-1}{3}}$ will be $\frac{1}{5}$.

---

9. Simplify:

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

(ii) $\left(3^{\frac{1}{3}}\right)^{7}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Ans.

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

We know that $a^{m} \cdot a^{n} = a^{(m+n)}$.

We can conclude that $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{2}{3}+\frac{1}{5}}$.

$$2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = (2)^{\frac{10+3}{15}} \quad = (2)^{\frac{13}{15}}$$

Therefore, the value of $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ will be $(2)^{\frac{13}{15}}$.

(ii) $\left(3^{\frac{1}{3}}\right)^7$

We know that $a^m \times a^n = a^{m+n}$

We conclude that $\left(3^{\frac{1}{3}}\right)^7$ can also be written as $\left(3^{\frac{7}{3}}\right)$.

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

We know that $\frac{a^m}{a^n} = a^{m-n}$

We conclude that $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}}$.

$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\frac{1}{2}-\frac{1}{4}} = 11^{\frac{2-1}{4}}$

$= 11^{\frac{1}{4}}$

Therefore, the value of $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ will be $11^{\frac{1}{4}}$.

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

We know that $a^m \cdot b^m = (a \times b)^m$.

We can conclude that $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$.

$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}} = (56)^{\frac{1}{2}}$

Therefore, the value of $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$ will be $(56)^{\frac{1}{2}}$

---

10. Express 0.8888… in the form p/q.

Ans. Let $x = 0.8888$

$x=0.\bar{8} ---(1)$

$10x = 10\times 0.8888$ (multiplying both sides by 10)

$10x = 8.8888$

$10x = 8.\bar{8} ----(2)$

$10x - x = 8.\bar{8} - 0.\bar{8}$ [subtracting (1) from (2)]

$9x = 8$

$x = \frac{8}{9}$

---

11. Simply by rationalizing denominator $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}$

Ans. $\frac{7+3\sqrt{5}}{7-3\sqrt{5}}\times\frac{7+3\sqrt{5}}{7+3\sqrt{5}}$ **(Rationalizing by denominator)**

$$\frac{\left(7+3\sqrt{5}\right)^2}{7^2-\left(3\sqrt{5}\right)^2} = \frac{7^2+\left(3\sqrt{5}\right)^2+2\times 7\times 3\sqrt{5}}{49-3^2\times 5}$$

$$= \frac{49+9\times 5+42\sqrt{5}}{49-45} = \frac{49+45+42\sqrt{5}}{4}$$

$$= \frac{94+42\sqrt{5}}{4} = \frac{94}{4}+\frac{42}{4}\sqrt{5}$$

$$= \frac{47}{2}+\frac{21}{2}\sqrt{5}$$

---

12. Simplify $\left\{\left[625^{\frac{-1}{2}}\right]^{-\frac{1}{4}}\right\}^2$

Ans. $\left\{\left(625^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right\}^2$

$=\left\{\left(\frac{1}{625^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2=\left\{\left(\frac{1}{\left(25^2\right)^{\frac{1}{2}}}\right)^{-\frac{1}{4}}\right\}^2\left(a^{-m}=\frac{1}{a^m}\right)$

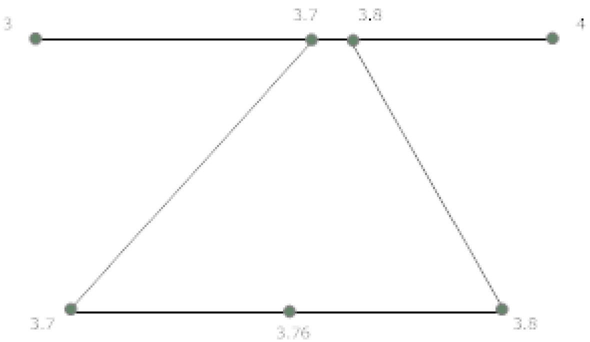
$=\left\{\left(\frac{1}{25}\right)^{-\frac{1}{4}\times 2}\right\}$

$=\left(\frac{1}{25^{-\frac{1}{2}}}\right)=\frac{1}{\left(5^2\right)^{-\frac{1}{2}}}=\frac{1}{5^{-1}}=5$

---

13. Visualize 3.76 on the number line using successive magnification.

Ans.



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14. Prove that $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}=1$

Ans. $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$

$\frac{1}{1+x^b.x^{-a}+x^c.x^{-a}}+\frac{1}{1+x^a.x^{-b}+x^c.x^{-b}}+\frac{1}{1+x^a.x^{-c}+x^b.x^{-c}}$

$\frac{1}{x^{-a}.x^a+x^b.x^{-a}+x^c.x^{-a}}+\frac{1}{x^b.x^{-b}+x^a.x^{-b}+x^c.x^{-b}}+\frac{1}{x^c.x^{-c}+x^a.x^{-c}+x^b.x^{-c}}$

$$\frac{1}{x^{-a}\left(x^a+x^b+x^c\right)}+\frac{1}{x^{-b}\left(x^a+x^b+x^c\right)}+\frac{1}{x^{-c}\left(x^a+x^b+x^c\right)}$$

$$\frac{x^a}{\left(x^a+x^b+x^c\right)}+\frac{x^b}{\left(x^a+x^b+x^c\right)}+\frac{x^c}{\left(x^a+x^b+x^c\right)}$$

$$=\frac{x^a+x^b+x^c}{x^a+x^b+x^c}=1$$

---

15. Represent $\sqrt{3}$ on number line

Ans. $Take$ OA=AB=1 unit in same line

. and $\angle A=90^0$

In Δ OAB, $OB^2=1^2+1^2$

$OB^2=2$

$OB=\sqrt{2}$

$\therefore OB=$ OA$=\sqrt{2}=1.41\sqrt{2}$, BD=1 and $\angle OBD=90^0$

$OD^2=OB^2+BD^2$

$OD^2=\left(\sqrt{2}\right)^2+(1)^2$

$OD^2=2+1=3$

$OD=\sqrt{3}$

---

16. Simplify $\left(3\sqrt{2}+2\sqrt{3}\right)^2\left(3\sqrt{2}-2\sqrt{3}\right)^2$

Ans. $\left(3\sqrt{2}+2\sqrt{3}\right)^2\left(3\sqrt{2}-2\sqrt{3}\right)^2$

$\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$=\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)\left(3\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}-2\sqrt{3}\right)$

$=\left[\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2\right]\left[\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2\right]$

$=[9\times2-4\times3][9\times2-4\times3]$

$=[18-12][18-12]$

$=6\times6=36$

---

17. Express $2.4\overline{178}$ in the from $\frac{p}{q}$

Ans. Let $\frac{p}{q}=2.4\overline{178}$

$\frac{p}{q}=2.4178178178$

Multiplying by 10

$10\frac{p}{q}=24.178178$

Multiplying by 1000

$10,000\frac{p}{q}=1000\times24.178178$

$10,000\frac{p}{q}=24178.178178$

$10000\frac{p}{q}-\frac{p}{q}=24178.178178-24.178178$

$9999\frac{p}{q}=24154$

$\frac{p}{q}=\frac{24154}{9999}$

---

18. Simplify $(27)^{-\frac{2}{3}}\div9^{\frac{1}{2}}.3^{-\frac{3}{2}}$

Ans. $(27)^{-\frac{2}{3}} \div 9^{\frac{1}{2}} 3^{-\frac{3}{2}}$

$\frac{(3\times3\times3)^{-\frac{2}{3}}\times3^{\frac{3}{2}}}{(3\times3)^{\frac{1}{2}}}$ $\left[a^{-m}=\frac{1}{a^m}\right]$

$= \frac{(3^{3})^{-\frac{2}{3}}\times3^{\frac{3}{2}}}{(3^{2})^{\frac{1}{2}}}$

$=\frac{3^{\frac{3}{2}-2}}{3}=\frac{3^{-\frac{1}{3}}}{3}=\frac{1}{3^{1+\frac{1}{3}}}$

$=\frac{1}{3^{\frac{4}{3}}}=\frac{1}{\sqrt[3]{81}}$

---

19. Find three rational numbers between $2.\overline{2}$ and $2.\overline{3}$

Ans. Three rational numbers between $2.\overline{2}$ and $2.\overline{3}$ are $2.212341365....$ and $2.2321453269....$

---

20. Give an example of two irrational numbers whose

(i) sum is a rational number

(ii) product is a rational number

(iii) Quotient is a rational number.

Ans. (i) $2+\sqrt{2}$. and $2-\sqrt{2}$

Sum $2+\sqrt{2}+2-\sqrt{2}=4$ which is a rational number

(ii) $3\sqrt{2}$ and $6\sqrt{2}$

Product = $3\sqrt{2}\times6\sqrt{2}=18\times2=36$ rational

(iii) $2\sqrt{125}$ and $3\sqrt{5}$

Quotient = $\frac{2\sqrt{125}}{3\sqrt{5}} = \frac{2}{3}\sqrt{\frac{\cancel{125}^{25}}{5}} = \frac{2}{3}\sqrt{25}$ = $\frac{2}{3}\times 5 = \frac{10}{3}$

---

21. If $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$, find the value of $\frac{5}{\sqrt{2}+\sqrt{3}}$

Ans. $\frac{5}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ (Rationalizing denominator)

= $\frac{5\left(\sqrt{2}-\sqrt{3}\right)}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2} = \frac{5\left(\sqrt{2}-\sqrt{3}\right)}{2-3}$

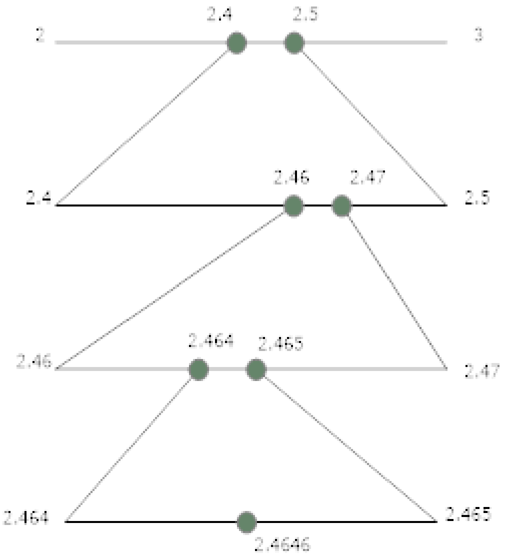
= $-5\left[1.414-1.732\right]$

= $-5\times -0.318 = 1.59$

---

22. Visualize 2.4646 on the number line using successive magnification.

Ans.



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23. Rationalize the denominator of $\frac{1}{4+2\sqrt{3}}$

Ans. $\frac{1}{4+2\sqrt{3}}\times\frac{4-2\sqrt{3}}{4-2\sqrt{3}}=\frac{4-2\sqrt{3}}{(4)^2-\left(2\sqrt{3}\right)^2}$

$=\frac{4-2\sqrt{3}}{16-\left(2\sqrt{3}\right)^2}=\frac{4-2\sqrt{3}}{16-4\times3}=\frac{4-2\sqrt{3}}{16-12}$

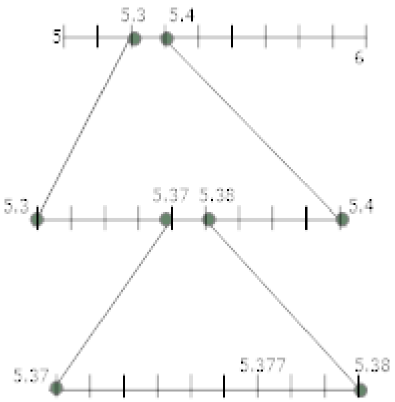
$=\frac{4-2\sqrt{3}}{4}=\frac{\cancel{2}\left(2-\sqrt{3}\right)}{\cancel{4}_2}$

$=\frac{2-\sqrt{3}}{2}$

---

24. Visualize the representation of $5.3\overline{7}$ on the number line up to 3 decimal places.

Ans.



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25. Shone that $5\sqrt{2}$ is not rational number

Ans. let $5\sqrt{2}$ is rational number

$x=5\sqrt{2}$ (x is rational)

$\frac{x}{5}=\sqrt{2}$

x is rational number and 5 is also rational number.

$\therefore \frac{x}{5}$ is rational number

But $\frac{x}{5}=\sqrt{2}$ and $\sqrt{2}$ is irrational number in same line which is a contradiction

$\therefore 5\sqrt{2}$ is irrational number

---

26. Simplify $3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}$

Ans. $3\sqrt[3]{250}+7\sqrt[3]{16}-4\sqrt[3]{54}$

$= 3\sqrt[3]{5\times5\times5\times2}+7\sqrt[3]{2\times2\times2\times2}-4\sqrt[3]{3\times3\times3\times2}$

$= 3\times5\sqrt[3]{2}+7\times2\sqrt[3]{2}-4\times3\sqrt[3]{2}$

$= 15\sqrt[3]{2}+14\sqrt[3]{2}-12\sqrt[3]{2}$

$= [15+14-12]\sqrt[3]{2}=17\sqrt[3]{2}$

---

27. Simplify $3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}$

Ans. $3\sqrt{48}-\frac{5}{2}\sqrt{\frac{1}{3}}+4\sqrt{3}$

$= 3\sqrt{16\times3}-\frac{5}{2}\sqrt{\frac{3}{3\times3}}+4\sqrt{3}$

$=3\times4\sqrt{3}-\frac{5}{2}.\frac{1}{3}\sqrt{3}+4\sqrt{3}$

$= 12\sqrt{3}-\frac{5}{6}\sqrt{3}+4\sqrt{3}$

$= \left(12-\frac{5}{6}+4\right)\sqrt{3}=\left(16-\frac{5}{6}\right)\sqrt{3}$

$= \frac{91}{6}\sqrt{3}$

---

28. If $\frac{1}{7}=0.\overline{142857}.$ find the value of $\frac{2}{7},\frac{3}{7},\frac{4}{7}$

Ans. $\frac{1}{7}=0.\overline{142857}$

$\frac{2}{7}=2\times0.\overline{142857}$

$=0.\overline{285714}$

$\frac{3}{7}=3\times0.\overline{142857}$ $=0.\overline{428571}$

$\frac{4}{7}=4\times0.\overline{142857}$ $=0.\overline{571428}$

---

29. Find 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$

Ans. $\frac{6}{5},\frac{7}{5}$ is divided into 10 equal parts than the value of each part

$\frac{6.1}{5},\frac{6.2}{5},\frac{6.3}{5},\frac{6.4}{5},\frac{6.5}{5},\frac{6.6}{5},\frac{6.7}{5},\frac{6.8}{5},\frac{6.9}{5},\frac{7}{5}$

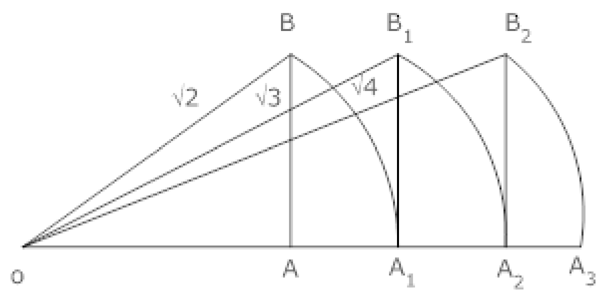
$\therefore$ 6 rational numbers between $\frac{6}{5}$ and $\frac{7}{5}$

$\frac{61}{50},\frac{62}{50},\frac{63}{50},\frac{64}{50},\frac{65}{50},\frac{66}{50}$

---

30. Show how $\sqrt{4}$ can be represented on the number line

Ans. Take $OA=AB=1$ unit



And $\angle A = 90^\circ$

In ΔOAB

$OB^2 = 1^2 + 1^2 = OB^2 = 2$

$OB = \sqrt{2}$

$OB = OA_1 = \sqrt{2}$

Take $A_1B_1 = 1$

And $\angle A_1 = 90^\circ$

In $\Delta OB_1A_1, OB_1^{\,2} = \left(\sqrt{2}\right)^2 + 1^2$

$OB_1 = \sqrt{3}$

$OB_1 = OA_2 = \sqrt{3}$

Take $a_2b_2 = 1$ unit

$\angle A_2 = 90^\circ$

$OB_2^{\,2} = \left(\sqrt{3}\right)^2 + 1^2 = OB_2^{\,2} = 3 + 1$

$OB_2 = \sqrt{4} = 2$

4 Marks Questions

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$

(v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

Ans.

(i) $\frac{36}{100}$

On dividing 36 by 100, we get

$$\begin{array}{r|l} & 0.36 \\ 100 & 36 \\ & -0 \\ & 360 \\ & -300 \\ & 600 \\ & -600 \\ & 0 \end{array}$$

Therefore, we conclude that $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11}$

On dividing 1 by 11, we get

$$\begin{array}{r|l} & 0.0909.... \\ 11 & 1 \\ & -0 \\ & 10 \\ & -0 \\ & 100 \\ & -99 \\ & 10 \\ & -0 \\ & 100 \\ & -99 \\ & 1 \end{array}$$

We can observe that while dividing 1 by 11, we got the remainder as 1, which will continue to be 1.

Therefore, we conclude that $\frac{1}{11} = 0.0909.....$ or $\frac{1}{11} = 0.\overline{09}$, which is a non-terminating decimal and recurring decimal.

(iii) $4\frac{1}{8} = \frac{33}{8}$

On dividing 33 by 8, we get

$$\begin{array}{r} 4.125 \\ 8\overline{)\ 33} \\ \underline{-32} \\ 10 \\ \underline{-8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ \underline{0} \end{array}$$

We can observe that while dividing 33 by 8, we got the remainder as 0.

Therefore, we conclude that $4\frac{1}{8} = \frac{33}{8} = 4.125$, which is a terminating decimal.

(iv) $\frac{3}{13}$

On dividing 3 by 13, we get

$$\begin{array}{r} 0.230769.... \\ 13 \overline{)\ \ \ \ \ \ \ \ \ 3} \\ \underline{-0} \\ 30 \\ \underline{-26} \\ 40 \\ \underline{-39} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-91} \\ 90 \\ \underline{-78} \\ 120 \\ \underline{-117} \\ \underline{3} \end{array}$$

We can observe that while dividing 3 by 13 we got the remainder as 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, we conclude that $\frac{3}{13} = 0.230769.....$ or $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating decimal and recurring decimal.

(v) $\frac{2}{11}$

On dividing 2 by 11, we get

$$\begin{array}{r} 0.1818.... \\ 11 \overline{)\ \ \ \ \ \ \ 2} \\ \underline{-0} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ \underline{2} \end{array}$$

We can observe that while dividing 2 by 11, first we got the remainder as 2 and then 9, which will continue to be 2 and 9 alternately.

Therefore, we conclude that $\frac{2}{11} = 0.1818.....$ or $\frac{2}{11} = 0\overline{18}$, which is a non-terminating decimal and recurring decimal.

(vi) $\frac{329}{400}$

On dividing 329 by 400, we get

$$\begin{array}{r|l} & 0.8225 \\ 400 & 329 \\ & -0 \\ \hline & 3290 \\ & -3200 \\ \hline & 900 \\ & -800 \\ \hline & 1000 \\ & -800 \\ \hline & 2000 \\ & -2000 \\ \hline & 0 \end{array}$$

We can observe that while dividing 329 by 400, we got the remainder as 0.

Therefore, we conclude that $\frac{329}{400} = 0.8225$, which is a terminating decimal.

---

2. Classify the following numbers as rational or irrational:

(i) 23

(ii) 225

(iii) 0.3796

(iv) 7.478478...

(v) 1.101001000100001...

Ans.

(i) $\sqrt{23}$

We know that on finding the square root of 23, we will not get an integer.

Therefore, we conclude that $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

We know that on finding the square root of 225, we get 15, which is an integer.

Therefore, we conclude that $\sqrt{225}$ is a rational number.

(iii)0.3796

We know that 0.3796 can be converted into $\frac{p}{q}$.

While, converting 0.3796 into $\frac{p}{q}$ form, we get

$$0.3796 = \frac{3796}{10000}.$$

The rational number $\frac{3796}{10000}$ can be converted into lowest fractions, to get $\frac{949}{2500}$.

We can observe that 0.3796 can be converted into a rational number.

Therefore, we conclude that 0.3796 is a rational number.

(iv)7.478478….

We know that 7.478478…. is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478…. into $\frac{p}{q}$ form, we get

$$x = 7.478478 \quad ....(a)$$

$$1000x = 7478.478478 \quad ......(b)$$

While, subtracting (a) from (b), we get

$$\begin{array}{r} 1000x = 7478.478478.... \\ -x = \quad 7.478478.... \\ \hline 999x = 7471 \end{array}$$

We know that $999x = 7471$ can also be written as $x = \frac{7471}{999}$.

Therefore, we conclude that 7.478478.... is a rational number.

(v)1.101001000100001....

We can observe that the number 1.101001000100001.... is a non-terminating on recurring decimal.

We know that non terminating and non-recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001.... is an irrational number.

---

3. Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Ans.

(i) $\frac{1}{\sqrt{7}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$, to get

$$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}}$, we get $\frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$, to get

$$\frac{1}{\sqrt{7}-\sqrt{6}}\times\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-\sqrt{6}}=\frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$=\frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$=\sqrt{7}+\sqrt{6}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$, we get $\sqrt{7}+\sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$, to get

$$\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$=\frac{\sqrt{7}+2}{7-4}$$

$$=\frac{\sqrt{7}+2}{3}$$

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$, we get $\frac{\sqrt{5}-\sqrt{2}}{3}$.

(iv) $\frac{1}{\sqrt{7}-2}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$, to get

$$\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$, we get $\frac{\sqrt{7}+2}{3}$.

5 Marks Questions

5 Marks Questions

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

(ii) $\frac{1}{11}$

(iii) $4\frac{1}{8}$

(iv) $\frac{3}{13}$

(v) $\frac{2}{11}$

(vi) $\frac{329}{400}$

Ans.

(i) $\frac{36}{100}$

On dividing 36 by 100, we get

$$
\begin{array}{r}
0.36 \\
100\overline{)\ 36} \\
\underline{-0} \\
360 \\
\underline{-300} \\
600 \\
\underline{-600} \\
\underline{0}
\end{array}
$$

Therefore, we conclude that $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii) $\frac{1}{11}$

On dividing 1 by 11, we get

$$
\begin{array}{r}
0.0909.... \\
11\overline{)\quad 1} \\
\underline{-0} \\
10 \\
\underline{-0} \\
100 \\
\underline{-99} \\
10 \\
\underline{-0} \\
100 \\
\underline{-99} \\
\underline{1}
\end{array}
$$

We can observe that while dividing 1 by 11, we got the remainder as 1, which will continue to be 1.

Therefore, we conclude that $\frac{1}{11}=0.0909.....$ or $\frac{1}{11}=0.\overline{09}$, which is a non-terminating decimal and recurring decimal.

(iii) $4\frac{1}{8}=\frac{33}{8}$

On dividing 33 by 8, we get

$$\begin{array}{r} 4.125 \\ 8\overline{)\;33} \\ -32 \\ \hline 10 \\ -8 \\ \hline 20 \\ -16 \\ \hline 40 \\ -40 \\ \hline 0 \\ \hline \end{array}$$

We can observe that while dividing 33 by 8, we got the remainder as 0.

Therefore, we conclude that $4\frac{1}{8}=\frac{33}{8}=4.125$, which is a terminating decimal.

(iv) $\frac{3}{13}$

On dividing 3 by 13, we get

$$\begin{array}{r} 0.230769.... \\ 13\overline{)\quad 3} \\ \underline{-0} \\ 30 \\ \underline{-26} \\ 40 \\ \underline{-39} \\ 10 \\ \underline{-0} \\ 100 \\ \underline{-91} \\ 90 \\ \underline{-78} \\ 120 \\ \underline{-117} \\ \underline{3} \end{array}$$

We can observe that while dividing 3 by 13 we got the remainder as 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, we conclude that $\frac{3}{13} = 0.230769.....$ or $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating decimal and recurring decimal.

(v) $\frac{2}{11}$

On dividing 2 by 11, we get

$$\begin{array}{r} 0.1818.... \\ 11\overline{)\quad 2} \\ \underline{-0} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ 20 \\ \underline{-11} \\ 90 \\ \underline{-88} \\ \underline{2} \end{array}$$

We can observe that while dividing 2 by 11, first we got the remainder as 2 and then 9, which will continue to be 2 and 9 alternately.

Therefore, we conclude that $\frac{2}{11} = 0.1818.....$ or $\frac{2}{11} = 0\overline{18}$, which is a non-terminating decimal and recurring decimal.

(vi) $\frac{329}{400}$

On dividing 329 by 400, we get

$$\begin{array}{r} 0.8225 \\ 400 \overline{) \quad 329} \\ \underline{-0} \\ 3290 \\ \underline{-3200} \\ 900 \\ \underline{-800} \\ 1000 \\ \underline{-800} \\ 2000 \\ \underline{-2000} \\ \underline{0} \end{array}$$

We can observe that while dividing 329 by 400, we got the remainder as 0.

Therefore, we conclude that $\frac{329}{400} = 0.8225$, which is a terminating decimal.

---

2. Classify the following numbers as rational or irrational:

(i) 23

(ii) 225

(iii) 0.3796

(iv) 7.478478...

(v) 1.101001000100001...

Ans.

(i) $\sqrt{23}$

We know that on finding the square root of 23, we will not get an integer.

Therefore, we conclude that $\sqrt{23}$ is an irrational number.

(ii) $\sqrt{225}$

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We know that 0.3796 can be converted into $\frac{p}{q}$.

While, converting 0.3796 into $\frac{p}{q}$ form, we get

$$0.3796 = \frac{3796}{10000}.$$

The rational number $\frac{3796}{10000}$ can be converted into lowest fractions, to get $\frac{949}{2500}$.

We can observe that 0.3796 can be converted into a rational number.

Therefore, we conclude that 0.3796 is a rational number.

(iv)7.478478….

We know that 7.478478…. is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478…. into $\frac{p}{q}$ form, we get

$$x = 7.478478.... \quad ....(a)$$
$$1000x = 7478.478478.... \quad ...(b)$$
While, subtracting (a) from (b), we get
$$1000x = 7478.478478....$$
$$-x = 7.478478....$$
$$999x = 7471$$

While, converting 7.478478 ….. into p/q form, we get

We know that $999x = 7471$ can also be written as $x = \frac{7471}{999}$.

Therefore, we conclude that 7.478478.... is a rational number.

(v) 1.101001000100001....

We can observe that the number 1.101001000100001.... is a non-terminating on recurring decimal.

We know that non terminating and non-recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001.... is an irrational number.

---

3. Rationalize the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

(iv) $\frac{1}{\sqrt{7}-2}$

Ans.

(i) $\frac{1}{\sqrt{7}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}}$ by $\sqrt{7}$, to get

$$\frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}}$, we get $\frac{\sqrt{7}}{7}$.

(ii) $\frac{1}{\sqrt{7}-\sqrt{6}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$ by $\sqrt{7}+\sqrt{6}$, to get

$$\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}-\sqrt{6}\right)\left(\sqrt{7}+\sqrt{6}\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{\left(\sqrt{7}\right)^2-\left(\sqrt{6}\right)^2}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$= \sqrt{7}+\sqrt{6}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$, we get $\sqrt{7}+\sqrt{6}$.

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$ by $\sqrt{5}-\sqrt{2}$, to get

$$\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

$$\frac{1}{\sqrt{7}-2} = \frac{\sqrt{7}+2}{\left(\sqrt{7}\right)^2-(2)^2}$$

$$= \frac{\sqrt{7}+2}{7-4}$$

$$= \frac{\sqrt{7}+2}{3}$$

$$\frac{1}{\sqrt{5}+\sqrt{2}}=\frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}.$$

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{5}+\sqrt{2}}$, we get $\frac{\sqrt{5}-\sqrt{2}}{3}$.

(iv) $\frac{1}{\sqrt{7}-2}$

We need to multiply the numerator and denominator of $\frac{1}{\sqrt{7}-2}$ by $\sqrt{7}+2$, to get

$$\frac{1}{\sqrt{7}-2}\times\frac{\sqrt{7}+2}{\sqrt{7}+2}=\frac{\sqrt{7}+2}{\left(\sqrt{7}-2\right)\left(\sqrt{7}+2\right)}.$$

We need to apply the formula $(a-b)(a+b)=a^2-b^2$ in the denominator to get

Therefore, we conclude that on rationalizing the denominator of $\frac{1}{\sqrt{7}-2}$, we get $\frac{\sqrt{7}+2}{3}$.

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4. It $\sqrt{5}=2.236$ and $\sqrt{3}=1.732$ Find the value of $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{7}{\sqrt{5}-\sqrt{3}}$

Ans. $\frac{2}{\sqrt{5}+\sqrt{3}}+\frac{7}{\sqrt{5}-\sqrt{3}}$

$$\frac{2\left(\sqrt{5}-\sqrt{3}\right)+7\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}=\frac{2\sqrt{5}-2\sqrt{3}+7\sqrt{5}+7\sqrt{3}}{5-3}$$

$$=\frac{9\sqrt{5}+5\sqrt{3}}{2}=\frac{2\times2.236+5\times1.732}{2}$$

$$=\frac{4.472+8.66}{2}=\frac{13.132}{2}=6.566$$

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5. Find the value of $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{7}{\sqrt{5}-\sqrt{2}}$, *If* $\sqrt{5}=2.236$ and $\sqrt{2}=1.414$

Ans. $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{7}{\sqrt{5}-\sqrt{2}}$

$= \frac{3\left(\sqrt{5}-\sqrt{2}\right)+7\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)}$

$= \frac{3\sqrt{5}-3\sqrt{2}+7\sqrt{5}+7\sqrt{2}}{5-2}$

$= \frac{10\sqrt{5}+4\sqrt{2}}{3}$

$= \frac{10\times 2.236+4\times 1.414}{3}$

$= 22.36+5.656$

$= 28.016$

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6. Simplify $\frac{2+\sqrt{5}}{2-\sqrt{5}}+\frac{2-\sqrt{5}}{2+\sqrt{5}}$

Ans. $\frac{2+\sqrt{5}}{2-\sqrt{5}}=\frac{2-\sqrt{5}}{2+\sqrt{5}}$

$= \frac{\left(2+\sqrt{5}\right)^2+\left(2-\sqrt{5}\right)^2}{\left(2-\sqrt{5}\right)\left(2+\sqrt{5}\right)}$

$= \frac{4+5+4\sqrt{5}+4+5-4\sqrt{5}}{(2)^2-\left(\sqrt{5}\right)^2}$

$= \frac{18}{4-5}$

$= \frac{18}{-1}$

= -18

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7. Find a and b if $\frac{3-\sqrt{6}}{3+2\sqrt{6}}=a\sqrt{6}-b$

Ans. $\frac{3-\sqrt{6}}{3+2\sqrt{6}} = a\sqrt{6} - b$

Rationalizing denominator of L.H.S.

$$\frac{3-\sqrt{6}}{3+2\sqrt{6}} \times \frac{3-2\sqrt{6}}{3-2\sqrt{6}} = a\sqrt{6} - b$$

$$\frac{9-6\sqrt{6}-3\sqrt{6}+2\times 6}{3^2-\left(2\sqrt{6}\right)^2} = a\sqrt{6} - b$$

$$\frac{9-9\sqrt{6}+12}{9-24} = a\sqrt{6} - b$$

$$\frac{-9\sqrt{6}+21}{-15} = a\sqrt{6} - b$$

$$\frac{\cancel{-9}_{3}\sqrt{6}}{\cancel{-15}_{5}} + \frac{\cancel{21}^{7}}{-\cancel{15}_{5}} = a\sqrt{6} - b$$

$$\frac{3}{5}\sqrt{6} - \frac{7}{5} = a\sqrt{6} - b$$

Comparing coefficient on both sides, we get

$a = \frac{3}{5}, +b = +\frac{7}{5}$ and b = $\frac{7}{5}$