CBSE Class 10 Mathematics
Important Questions
Chapter 4
Quadratic Equations

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1 Marks Questions

1. Which of the following is quadratic equation?

(a) $x^3 - 2x - \sqrt{5} - x = 0$

(b) $3x^2 - 5x + 9 = x^2 - 7x + 3$

(c) $\left(x + \frac{1}{x}\right)^2 = 3\left(x + \frac{1}{x}\right) + 4$

(d) $x^3 + x + 3 = 0$

Ans. (b) $3x^2 - 5x + 9 = x^2 - 7x + 3$

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2. Factor of $a^2x^2 - 3abx + 2b^2 = 0$ **is**

(a) $\frac{2b}{a}, \frac{b}{a}$

(b) $\frac{3b}{a}, \frac{a}{b}$

(c) $\frac{b}{a}, \frac{a}{b}$

(d) $\frac{a}{b}, \frac{a}{b}$

Ans. (a) $\frac{2b}{a}, \frac{b}{a}$

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3. Which of the following have real root

(a) $2x^2 + x - 1 = 0$

(b) $x^2 + x + 1 = 0$

(c) $x^2-6x+6=0$

(d) $2x^2+15x+30=0$

Ans. (c) $x^2-6x+6=0$

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4. Solve for x: $x=\cfrac{1}{2-\cfrac{1}{2-\cfrac{1}{2-x}}}$

(a) $x=2$

(b) $x=-1$

(c) $x=1$

(d) $x=3$

Ans. (b) $x=-1$

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5. Solve by factorization $\sqrt{3}x^2+10x+7\sqrt{3}=0$

(a) $x=-\sqrt{3},-\frac{7}{\sqrt{3}}$

(b) $x=-\sqrt{3},\frac{7}{\sqrt{3}}$

(c) $x=2,\frac{1}{2}$

(d) ± 3

Ans. (a) $x=-\sqrt{3},-\frac{7}{\sqrt{3}}$

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6. The quadratic equation whose roots are 3 and -3 is

(a) $x^2-9=0$

(b) $x^2-3x-3=0$

(c) $x^2-2x+2=0$

(d) $x^2+9=0$

Ans. (a) $x^2-9=0$

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7. Discriminant of $-x^2+\frac{1}{2}x+\frac{1}{2}=0$ **is**

(a) $-\frac{1}{2},1$

(b) $\frac{1}{2},1$

(c) $\frac{-1}{2},-1$

(d) $\frac{1}{2},\frac{-1}{2}$

Ans. (a) $-\frac{1}{2},1$

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8. For equal root, $kx(x-2)+6=0$ **, value of k is**

(a) $k=6$

(b) $k=3$

(c) $k=2$

(d) $k=8$

Ans. (a) $k=6$

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9. Quadratic equation whose roots are $2+\sqrt{5},\ 2-\sqrt{5}$ **is**

(a) $x^2-4x-1=0$

(b) $x^2+4x+1=0$

(c) $x^2+\left(x+\sqrt{5}\right)x-\left(2\sqrt{5}\right)=0$

(d) $x^2-4x+2=0$

Ans. (a) $x^2-4x-1=0$

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10. If α and β are roots of the equation $3x^2+5x-7=0$, then $\alpha\beta$ equal to

(a) $\frac{7}{3}$

(b) $\frac{-7}{3}$

(c) $\frac{-5}{3}$

(d) 21

Ans. (b) $\frac{-7}{3}$

2 Marks Questions

1. Solve the following problems given:

(i) $x^2-45x+324=0$

(ii) $x^2-55x+750=0$

Ans. (i) $x^2-45x+324=0$

$\Rightarrow x^2-36x-9x+324=0$

$\Rightarrow x(x-36)-9(x-36)=0$

$\Rightarrow (x-9)(x-36)=0$

$\Rightarrow x=9,36$

(ii) $x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-30)(x-25)=0$

$\Rightarrow x=30,25$

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2. Find two numbers whose sum is 27 and product is 182.

Ans. Let first number be x and let second number be $(27-x)$

According to given condition, the product of two numbers is 182.

Therefore,

$x(27-x)=182$

$\Rightarrow 27x-x^2=182$

$\Rightarrow x^2-27x+182=0$

$\Rightarrow x^2-14x-13x+182=0$

$\Rightarrow x(x-14) - 13(x-14)=0$

$\Rightarrow (x-14)(x-13)=0$

$\Rightarrow x=14,13$

Therefore, the first number is equal to 14 or 13

And, second number is = 27 – x=27 – 14=13 or Second number = 27 – 13=14

Therefore, two numbers are 13 and 14.

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3. Find two consecutive positive integers, sum of whose squares is 365.

Ans. Let first number be $x$and let second number be $(x+1)$

According to given condition,

$x^2+(x+1)^2=365$ $\{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow x^2+x^2+1+2x=365$

$\Rightarrow 2x^2+2x- 364=0$

Dividing equation by 2

$\Rightarrow x^2+x- 182=0$

$\Rightarrow x^2+14x-13x-182=0$

$\Rightarrow x(x+14)-13(x+14)=0$

$\Rightarrow (x+14)(x-13)=0$

$\Rightarrow x=13,-14$

Therefore, first number = 13{We discard -14 because it is negative number)

Second number = $x+1=13+1=14$

Therefore, two consecutive positive integers are 13 and 14 whose sum of squares is equal to 365.

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4. The altitude of right triangle is 7 cm less than its base. If, hypotenuse is 13 cm. Find the other two sides.

Ans. Let base of triangle be x cm and let altitude of triangle be $(x-7)$ cm

It is given that hypotenuse of triangle is 13 cm

According to Pythagoras Theorem,

$13^2=x^2+(x-7)^2$ $(a+b)^2=a^2+b^2+2ab$

$\Rightarrow 169=x^2+x^2+49-14x$

$\Rightarrow 169=2x^2-14x+49$

$\Rightarrow 2x^2-14x-120=0$

Dividing equation by 2

$\Rightarrow x^2-7x-60=0$

$\Rightarrow x^2-12x+5x-60=0$

$\Rightarrow x(x-12)+5(x-12)=0$

$\Rightarrow (x-12)(x+5)$

$\Rightarrow x=-5,12$

We discard $x=-5$ because length of side of triangle cannot be negative.

Therefore, base of triangle = 12 cm

Altitude of triangle = $(x-7)=12-7=5$ cm

5. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.

Ans. Let cost of production of each article be Rs x

We are given total cost of production on that particular day = Rs 90

Therefore, total number of articles produced that day = 90/x

According to the given conditions,

$$x = 2\left(\frac{90}{x}\right) + 3$$

$\Rightarrow$ $x = \frac{180}{x} + 3$

$\Rightarrow$ $x = \frac{180 + 3x}{x}$

$\Rightarrow x^2 = 180 + 3x$

$\Rightarrow x^2 - 3x - 180 = 0$

$\Rightarrow x^2 - 15x + 12x - 180 = 0$

$\Rightarrow x(x-15) + 12(x-15) = 0$

$\Rightarrow (x-15)(x+12) = 0$

$\Rightarrow x = 15, -12$

Cost cannot be in negative, therefore, we discard $x = -12$

Therefore, $x = Rs15$ which is the cost of production of each article.

Number of articles produced on that particular day = $\frac{90}{15}$ = 6

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6. In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Ans. Let Shefali's marks in Mathematics = x

Let Shefali's marks in English = $30-x$

If, she had got 2 marks more in Mathematics, her marks would be = $x+2$

If, she had got 3 marks less in English, her marks in English would be = $30 - x-3 = 27-x$

According to given condition:

$(x+2)(27-x)=210$

$\Rightarrow 27x-x^2+54-2x=210$

$\Rightarrow x^2-25x+156=0$

Comparing quadratic equation $x^2-25x+156=0$ with general form $ax^2+bx+c=0$,

We get $a=1, b=-25$ and $c=156$

Applying Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{25\pm\sqrt{(25)^2-4(1)(156)}}{2\times 1}$$

$\Rightarrow x=\frac{25\pm\sqrt{625-624}}{2}$

$\Rightarrow x=\frac{25\pm\sqrt{1}}{2}$

$\Rightarrow x=\frac{25+1}{2}, \frac{25-1}{2}$

$\Rightarrow x=13,12$

Therefore, Shefali's marks in Mathematics = 13 or 12

Shefali's marks in English = $30 - x=30 - 13=17$

Or Shefali's marks in English = $30 - x=30 - 12=18$

Therefore, her marks in Mathematics and English are (13,17) or (12,18).

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7. The diagonal of a rectangular field is 60 metres more than the shorter side. If, the longer side is 30 metres more than the shorter side, find the sides of the field.

Ans. Let shorter side of rectangle = x metres

Let diagonal of rectangle = (x+60) metres

Let longer side of rectangle = (x+30) metres

According to pythagoras theorem,

$(x+60)^2=(x+30)^2+x^2$

$\Rightarrow x^2+3600+120x=x^2+900+60x+x^2$

$\Rightarrow x^2-60x-2700=0$

Comparing equation $x^2-60x-2700=0$ with standard form $ax^2+bx+c=0$,

We get $a=1, b=-60$ and $c=-2700$

Applying quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{60\pm\sqrt{(60)^2-4(1)(-2700)}}{2\times1}$$

$$\Rightarrow x=\frac{60\pm\sqrt{3600+10800}}{2}$$

$$\Rightarrow x=\frac{60\pm\sqrt{14400}}{2}=\frac{60\pm120}{2}$$

$$\Rightarrow x=\frac{60+120}{2},\frac{60-120}{2}$$

$\Rightarrow x=90, -30$

We ignore −30. Since length cannot be in negative.

Therefore, x=90 which means length of shorter side =90 metres

And length of longer side = x+30 = 90+30=120 metres

Therefore, length of sides are 90 and 120 in metres.

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8. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Ans. Let smaller number = x and let larger number = y

According to condition:

$y^2 - x^2 = 180$ ... (1)

Also, we are given that square of smaller number is 8 times the larger number.

$\Rightarrow x^2 = 8y$ ... (2)

Putting equation (2) in (1), we get

$y^2 - 8y = 180$

$\Rightarrow y^2 - 8y - 180 = 0$

Comparing equation $y^2 - 8y - 180 = 0$ with general form $ay^2 + by + c = 0$,

We get $a=1, b=-8$ and $c=-180$

Using quadratic formula $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$y = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-180)}}{2 \times 1}$

$\Rightarrow y = \frac{8 \pm \sqrt{64 + 720}}{2}$

$\Rightarrow y = \frac{8 \pm \sqrt{784}}{2} = \frac{8 \pm 28}{2}$

$\Rightarrow y = \frac{8+28}{2}, \frac{8-28}{2}$

$\Rightarrow y = 18, -10$

Using equation (2) to find smaller number:

$x^2 = 8y$

$\Rightarrow x^2 = 8y = 8 \times 18 = 144$

$\Rightarrow x = \pm 12$

And, $x^2 = 8y = 8 \times -10 = -80$ {No real solution for x}

Therefore, two numbers are (12,18) or (−12,18)

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9. A train travels 360 km at a uniform speed. If, the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Ans. Let the speed of the train = x km/hr

If, speed had been 5km/hr more, train would have taken 1 hour less.

So, according to this condition

$$\frac{360}{x}=\frac{360}{x+5}+1$$

$$\Rightarrow 360\left(\frac{1}{x}-\frac{1}{x+5}\right)=1$$

$$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right)=1$$

$\Rightarrow 360\times5=x^2+5x$

$\Rightarrow x^2+5x-1800=0$

Comparing equation $x^2+5x-1800=0$ with general equation $ax^2+bx+c=0$,

We get $a=1, b=5$ and $c=-1800$

Applying quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-5\pm\sqrt{(5)^2-4(1)(-1800)}}{2\times1}$$

$$\Rightarrow x=\frac{-5\pm\sqrt{25+7200}}{2}$$

$$\Rightarrow x=\frac{-5\pm\sqrt{7225}}{2}=\frac{-5\pm85}{2}$$

$$\Rightarrow x=\frac{-5+85}{2},\frac{-5-85}{2}$$

$\Rightarrow x=40,-45$

Since speed of train cannot be in negative. Therefore, we discard $x=-45$

Therefore, speed of train = 40 km/hr

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10. Find the value of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2+kx+3=0$

(ii) $kx(x-2)+6=0$

Ans. (i) $2x^2+kx+3=0$

We know that quadratic equation has two equal roots only when the value of discriminant isequal to zero.

Comparing equation $2x^2+kx+3=0$ with general quadratic equation $ax^2+bx+c=0$, we get $a=2, b=k$ and $c=3$

Discriminant = $b^2-4ac=k^2-4(2)(3)=k^2-24$

Putting discriminant equal to zero

$k^2-24=0$

$\Rightarrow k^2=24$

$\Rightarrow k=\pm\sqrt{24}=\pm2\sqrt{6}$

$\Rightarrow k=2\sqrt{6},-2\sqrt{6}$

(ii) $kx(x-2)+6=0$

$\Rightarrow kx^2-2kx+6=0$

Comparing quadratic equation $kx^2-2kx+6=0$ with general form $ax^2+bx+c=0$, we get $a=k, b=-2k$ and $c=6$

Discriminant = $b^2-4ac=(-2k)^2-4(k)(6)=4k^2-24k$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.

Putting discriminant equal to zero

$4k^2-24k=0$

$\Rightarrow 4k(k-6)=0$

$\Rightarrow k=0,6$

The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2+bx+c=0$, where $a\neq0$.

Therefore, in equation $kx^2-2kx+6=0$, we cannot have k =0.

Therefore, we discard k=0.

Hence the answer is k=6.

11. Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2. If so, find its length and breadth.

Ans. Let breadth of rectangular mango grove = x metres

Let length of rectangular mango grove = $2x$ metres

Area of rectangle = length × breadth = $x \times 2x = 2x^2$ m^2

According to given condition:

$2x^2=800$

$\Rightarrow 2x^2 - 800=0$

$\Rightarrow x^2 - 400=0$

Comparing equation $x^2 - 400=0$ with general form of quadratic equation $ax^2+bx+c=0$, we get$a=1,b=0$ and $c=-400$

Discriminant = $b^2-4ac=(0)^2 - 4(1)(-400)=1600$

Discriminant is greater than 0 means that equation has two disctinct real roots.

Therefore, it is possible to design a rectangular grove.

Applying quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$$x=\frac{0\pm\sqrt{1600}}{2\times 1}=\frac{\pm 40}{2}=\pm 20$$

$\Rightarrow x=20,-20$

We discard negative value of x because breadth of rectangle cannot be in negative.

Therefore, x = breadth of rectangle = 20 metres

Length of rectangle = $2x=2\times 20=40$ metres

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12. Is the following situation possible? If so, determine their present ages.

The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Ans. Let age of first friend = x years and let age of second friend = $(20-x)$ years

Four years ago, age of first friend = (x−4) years

Four years ago, age of second friend = (20−x)−4 = (16−x) years

According to given condition,

$(x-4)(16-x)=48$

$\Rightarrow 16x-x^2-64+4x=48$

$\Rightarrow 20x-x^2-112=0$

$\Rightarrow x^2-20x+112=0$

Comparing equation, $x^2-20x+112=0$ with general quadratic equation $ax^2+bx+c=0$, we get$a=1, b=-20$ and $c=112$

Discriminant $=b^2-4ac=(-20)^2-4(1)(112)=400-448=-48<0$

Discriminant is less than zero which means we have no real roots for this equation.

Therefore, the give situation is not possible.

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13. Value of x for $x^2-8x+15=0$ is quadratic formula is

(a) 3, 2

(b) 5, 2

(c) 5, 3

(d) 2, 3

Ans. (c) 5, 3

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14. Discriminate of $\sqrt{3}x^2-2\sqrt{2}x-2\sqrt{3}=0$ is

(a) 30

(b) 31

(c) 32

(d) 35

Ans. (c) 32

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15. Solve $12abx^2-9a^2x+8b^2x-6ab=0$.

Ans. $12abx^2-9a^2x+8b^2x-6ab=0$

$$\Rightarrow 3ax(4bx-3x)+2b(4bx-3a)=0$$
$$\Rightarrow (4bx-3a)(3ax+2b)=0$$

$$\Rightarrow 4bx-3a=0 \ or \ 3ax+2b=0$$
$$\Rightarrow x=\frac{3a}{4b} \ or \ x=-\frac{2b}{3a}$$

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16. Solve for x **by quadratic formula** $p^2x^2+(p^2-q^2)x-q^2=0$

Ans. $p^2x^2+(p^2-q^2)x-q^2=0$

$$a=p^2, b=p^2-q^2, c=-q^2$$
$$D=b^2-4ac$$
$$=(p^2-q^2)-4\times p^2(-q^2)$$

$$=p^4+q^2-2p^2q^2+4p^2q^2$$
$$=(p^2+q^2)^2$$

$$x=\frac{-b\pm\sqrt{D}}{29}$$
$$=\frac{-(p^2q^2)\pm\sqrt{(p^2+q^2)^2}}{2\times p^2}$$

$$=\frac{-p^2+q^2+p^2+q^2}{2p^2}$$
$$or \ x=\frac{-p^2+q^2-p^2}{2p^2}$$

$$x=\frac{2q^2}{2p^2} \ or \ x=\frac{-2p^2}{2p^2}$$

$$x=\frac{q^2}{p^2} \ or \ x=-1$$

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17. Find the value of k for which the quadratic equation $kx^2+2x+1=0$ **has real and distinct root.**

Ans. $kx^2+2x+1=0$

$a=k,\ \ b=2,\ \ c=1$
$b=b^2-4ac$
$=(2)^2-4\times k\times 1=4-4k$

For real and distinct roots,

$D>0$

$4-4k>0$
$\Rightarrow -4k>-4$
$\Rightarrow k<1$

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18. If one root of the equations $2x^2+ax+3=0$ **is 1, find the value of a.**

(a) $=-4$

(b) $=-5$

(c) $=-3$

(d) $=-1$

Ans. (b) $=-5$

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19. Find k for which the quadratic equation $4x^2-3kx+1=0$ **has equal root.**

(a) $=\pm\frac{3}{4}$

(b) $=\frac{3}{4}$

(c) $=\pm\frac{4}{3}$

(d) $=\pm\frac{2}{3}$

Ans. (c) $=\pm\frac{4}{3}$

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20. Determine the nature of the roots of the quadratic equation $9a^2b^2x^2-24abcdx+16c^2d^2=0$

Ans. $D=b^2-4ac$

$=(-24abcd)^2-4\times 9a^2b^2\times 16c^2d^2$

$=576a^2b^2c^2-576a^2b^2c^2d^2=0$

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21. Find the discriminant of the equation $(x-1)(2x-1)=0.$

Ans. $(x-1)(2x-1)=0$

$\Rightarrow 2x^2-x-2x+1=0$
$\Rightarrow 2x^2-3x+1=0$
$Here,\ a=2,\ \ c=-3,\ \ c=1$
$D=b^2-4ac$

$=(-3)^2-4\times 2\times 1$
$=9-8=1$

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22. Find the value of k so that $(x-1)$ **is a factor of** $k^2x^2-2kx-3.$

Ans. Let $P(x)=k^2x^2-2x-3$

$P(1)=k^2(1)^2-2K(1)-3$
$\Rightarrow 0=K^2-2k-3$
$\Rightarrow k^2-3k+k-3$

$\Rightarrow k(k-3)+1(k-3)=0$

$\Rightarrow (k-3)(k+1)=0$
$\Rightarrow k=3\ or\ k=-1$

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23. The product of two consecutive positive integers is 306. Represent these in quadratic equation.

(a) $x^2+x-306=0$

(b) $x^2-x+306=0$

(c) $x^2+2x-106=0$

(d) $x^2-x-306=0$

Ans. (a) $x^2+x-306=0$

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24. Which is a quadratic equation?

(a) $x^2+x+2=0$

(b) $x^3+x^2+2=0$

(c) $x^4+x^2+2=0$

(d) $x+2=0$

Ans. (a) $x^2+x+2=0$

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25. The sum of two numbers is 16. The sum of their reciprocals is $\frac{1}{3}$. Find the numbers.

Ans. Let no. be x

According to question,

$$\frac{1}{x}+\frac{1}{16-x}=\frac{1}{3}$$

$$\Rightarrow \frac{16}{16x-x^2}=\frac{1}{3}$$

$$\Rightarrow x^2-16x+48=0$$
$$\Rightarrow x^2-12x-4x+48=0$$

$\Rightarrow x=12$ or $x=4$

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26. Solve for x : $\sqrt{217-x}=x-7$

Ans. $\sqrt{217-x}=(x-7)$

$\Rightarrow (217-x)=(x-7)^2$
$\Rightarrow 217-x=x2+49-14x$
$\Rightarrow x^2-14x+x+49-217=0$
$\Rightarrow x^2-13x-168=0$
$\Rightarrow x^2-21x+8x-168=0$

$\Rightarrow x=21$ or $x=-8$

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27. Solve for x by factorization: $x+\frac{1}{x}=11\frac{1}{11}$

Ans. $\frac{x^2+1}{x}=\frac{122}{11}$

$\Rightarrow 11x^2-12x+11=0$

$\Rightarrow 11x^2-121x-1x+11=0$
$\Rightarrow 11x(x-11)-1(x-11)=0$
$\Rightarrow (11x-1)(x-11)=0$
$\Rightarrow x=11\ or\ x=\frac{1}{11}$

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28. Find the ratio of the sum and product of the roots of $7x^2-12x+18=0.$

Ans. $7x^2-12x+18=0$

$\alpha+\beta=\frac{-b}{a}=\frac{12}{7}\ and\ \alpha\beta=\frac{c}{a}=\frac{18}{17}$

$\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{12}{7}}{\frac{18}{17}}=\frac{12}{7}\times\frac{17}{18}=\frac{34}{21}$

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29. If α **and** β **are the roots of the equation** $x^2+kx+12=0,$ **such that** $\alpha-\beta=1$**, then**

Ans. $\alpha+\beta=\frac{-k}{1},$

$\alpha-\beta=1.$
$\alpha\beta=\frac{12}{1}$

$(\alpha+\beta)^2=(\alpha-\beta)^2+4\alpha\beta$

$\Rightarrow(-k)^2=(1)^2+4\times 12$

$\Rightarrow k^2=49$

$\Rightarrow k=\pm 7$

3 Marks Questions

1. Check whether the following are Quadratic Equations.

(i) $(x+1)^2=2(x-3)$

(ii) $x^2-2x=(-2)(3-x)$

(iii) $(x-2)(x+1)=(x-1)(x+3)$

(iv) $(x-3)(2x+1)=x(x+5)$

(v) $(2x-1)(x-3)=(x+5)(x-1)$

(vi) $x^2+3x+1=(x-2)^2$

(vii) $(x+2)^3=2x(x^2-1)$

(viii) $x^3-4x^2-x+1=(x-2)^3$

Ans. (i) $(x+1)^2=2(x-3)$ $\{(a+b)^2=a^2+2ab+b^2\}$

$\Rightarrow x^2+1+2x=2x-6$

$\Rightarrow x^2+7=0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

(ii) $x^2-2x=(-2)(3-x)$

$\Rightarrow x^2-2x=-6+2x$

$\Rightarrow x^2-2x-2x+6=0$

$\Rightarrow x^2-4x+6=0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

(iii) $(x-2)(x+1)=(x-1)(x+3)$

$\Rightarrow x^2+x-2x-2=x^2+3x-x-3=0$

$\Rightarrow x^2+x-2x-2-x^2-3x+x+3=0$

$\Rightarrow x-2x-2-3x+x+3=0$

$\Rightarrow -3x+1=0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

(iv) $(x-3)(2x+1)=x(x+5)$

$\Rightarrow 2x^2+x-6x-3=x^2+5x$

$\Rightarrow 2x^2+x-6x-3-x^2-5x=0$

$\Rightarrow x^2-10x-3=0$

Here, degree of equation is 2.

Therefore, it is a quadratic equation.

(v) $(2x-1)(x-3)=(x+5)(x-1)$

$\Rightarrow 2x^2-6x-x+3=x^2-x+5x-5$

$\Rightarrow 2x^2-7x+3-x^2+x-5x+5=0$

$\Rightarrow x^2-11x+8=0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

(vi) $x^2+3x+1=(x-2)^2$ $\{(a-b)^2=a^2-2ab+b^2\}$

$\Rightarrow x^2+3x+1=x^2+4-4x$

$\Rightarrow x^2+3x+1-x^2+4x-4=0$

$\Rightarrow 7x-3=0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

(vii) $(x+2)^3=2x(x^2-1)$ $\{(a+b)^3=a^3+b^3+3ab(a+b)\}$

$\Rightarrow x^3+2^3+3(x)(2)(x+2)=2x(x^2-1)$

$\Rightarrow x^3+8+6x(x+2)=2x^3-2x$

$\Rightarrow 2x^3-2x-x^3-8-6x^2-12x=0$

$\Rightarrow x^3-6x^2-14x-8=0$

Here, degree of Equation is 3.

Therefore, it is not a quadratic Equation.

(viii) $x^3-4x^2-x+1=(x-2)^3$ $\{(a-b)^3=a^3-b^3-3ab(a-b)\}$

$\Rightarrow x^3-4x^2-x+1=x^3-2^3-3(x)(2)(x-2)$

$\Rightarrow -4x^2-x+1=-8-6x^2+12x$

$\Rightarrow 2x^2-13x+9=0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

---

2. Represent the following situations in the form of Quadratic Equations:

(i) The area of rectangular plot is 528 *m*2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive numbers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) after 3 years will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at uniform speed. If, the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find speed of the train.

Ans. (i) We are given that area of a rectangular plot is 528 m^2.

Let breadth of rectangular plot be x metres

Length is one more than twice its breadth.

Therefore, length of rectangular plot is $(2x+1)$ metres

Area of rectangle = length × breadth

$\Rightarrow 528=x(2x+1)$

$\Rightarrow 528=2x^2+x$

$\Rightarrow 2x^2+x-528=0$

This is a Quadratic Equation.

(ii) Let two consecutive numbers be x and (x+1).

It is given that $x(x+1)=306$

$\Rightarrow x^2+x=306$

$\Rightarrow x^2+x-306=0$

This is a Quadratic Equation.

(iii) Let present age of Rohan = x years

Let present age of Rohan's mother = (x +26) years

Age of Rohan after 3 years = (x+3) years

Age of Rohan's mother after 3 years = x+26+3 = (x+29) years

According to given condition:

$(x+3)(x+29)=360$

$\Rightarrow x^2+29x+3x+87=360$

$\Rightarrow x^2+32x-273=0$

This is a Quadratic Equation.

(iv) Let speed of train be x km/h

Time taken by train to cover 480 km = $480x$ hours

If, speed had been 8km/h less then time taken would be $(480x-8)$ hours

According to given condition, if speed had been 8km/h less then time taken is 3 hours less.

Therefore, $480x-8=480x+3$

$\Rightarrow 480(1x-8-1x)=3$

$\Rightarrow 480(x-x+8)\ (x)\ (x-8)=3$

$\Rightarrow 480\times8=3(x)(x-8)$

$\Rightarrow 3840=3x^2-24x$

$\Rightarrow 3x^2-24x-3840=0$

Dividing equation by 3, we get

$\Rightarrow x^2 - 8x - 1280 = 0$

This is a Quadratic Equation.

---

3. Find the roots of the following Quadratic Equations by factorization.

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Ans. (i) $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0$

$\Rightarrow x = 5, -2$

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (2x-3)(x+2) = 0$

$\Rightarrow x = \frac{3}{2}, -2$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\Rightarrow (\sqrt{2}x + 5)(x + \sqrt{2}) = 0$

$\Rightarrow x = \frac{-5}{\sqrt{2}}, -\sqrt{2}$

$\Rightarrow$ $x=\frac{-5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}},-\sqrt{2}$

$\Rightarrow$ $x=\frac{-5\sqrt{2}}{2},-\sqrt{2}$

(iv) $2x^2-x+\frac{1}{8}=0$

$\Rightarrow$ $\frac{16x^2-8x+1}{8}=0$

$\Rightarrow 16x^2-8x+1=0$

$\Rightarrow 16x^2-4x-4x+1=0$

$\Rightarrow 4x(4x-1)-1(4x-1)=0$

$\Rightarrow (4x-1)(4x-1)=0$

$\Rightarrow x=\frac{1}{4},\frac{1}{4}$

(v) $100x^2-20x+1=0$

$\Rightarrow 100x^2-10x-10x+1=0$

$\Rightarrow 10x(10x-1)-1(10x-1)=0$

$\Rightarrow (10x-1)(10x-1)=0$

$\Rightarrow x=\frac{1}{10},\frac{1}{10}$

---

4. Find the roots of the following equations:

(i) $\frac{x-1}{x}=3, x\neq 0$

(ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x\neq -4,7$

Ans. (i) $x-\frac{1}{x}=3\ where\ x\neq 0$

$\Rightarrow$ $\frac{x^2-1}{x}=3$

$\Rightarrow x^2 - 1 = 3x$

$\Rightarrow x^2 - 3x - 1 = 0$

Comparing equation $x^2-3x-1=0$ with general form $ax^2+bx+c=0$,

We get$a=1, b=-3$ and $c=-1$

Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$$x=\frac{3\pm\sqrt{(3)^2-4(1)(-1)}}{2\times 1}$$

$$\Rightarrow x=\frac{3\pm\sqrt{13}}{2}$$

$$\Rightarrow x=\frac{3+\sqrt{13}}{2},\frac{3-\sqrt{13}}{2}$$

(ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}\ where\ x\neq -4,7$

$$\Rightarrow \frac{(x-7)-(x+4)}{(x-4)(x-7)}=\frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x-4)(x-7)}=\frac{11}{30}$$

$\Rightarrow -30 = x^2-7x+4x-28$

$\Rightarrow x^2-3x+2=0$

Comparing equation $x^2-3x+2=0$ with general form $ax^2+bx+c=0$,

We get $a=1, b=-3$ and $c=2$

Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,

$$x=\frac{3\pm\sqrt{(3)^2-4(1)(2)}}{2\times 1}$$

$$\Rightarrow x=\frac{3\pm\sqrt{1}}{2}$$

$$\Rightarrow x=\frac{3+\sqrt{1}}{2},\frac{3-\sqrt{1}}{2}$$

$\Rightarrow x=2,1$

---

5. The sum of reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 13. Find his present age.

Ans. Let present age of Rehman= x years

Age of Rehman 3 years ago = $(x-3)$ years.

Age of Rehman after 5 years = $(x+5)$ years

According to the given condition:

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3}$$

$\Rightarrow 3(2x+2)=(x-3)(x+5)$

$\Rightarrow 6x+6=x^2-3x+5x-15$

$\Rightarrow x^2-4x-15-6=0$

$\Rightarrow x^2-4x-21=0$

Comparing quadratic equation $x^2-4x-21=0$ with general form $ax^2+bx+c=0$,

We get $a=1, b=-4$ and $c=-21$

Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{4\pm\sqrt{(4)^2-4(1)(-21)}}{2\times1}$$

$$\Rightarrow x=\frac{4\pm\sqrt{16+84}}{2}$$

$$\Rightarrow x=\frac{4\pm\sqrt{100}}{2}=\frac{4\pm10}{2}$$

$$\Rightarrow x=\frac{4+10}{2},\frac{4-10}{2}$$

$\Rightarrow x=7,-3$

We discard $x=-3$. Since age cannot be in negative.

Therefore, present age of Rehman is 7 years.

---

6. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Ans. Let time taken by tap of smaller diameter to fill the tank = x hours

Let time taken by tap of larger diameter to fill the tank = $(x-10)$ hours

It means that tap of smaller diameter fills $\frac{1}{x}^{th}$ part of tank in 1 hour. ... (1)

And, tap of larger diameter fills $\frac{1}{x-10}^{th}$ part of tank in 1 hour. ... (2)

When two taps are used together, they fill tank in 758 hours.

In 1 hour, they fill $\frac{8}{75}^{th}$ part of tank $\left(\frac{1}{\frac{75}{8}}=\frac{8}{75}\right)$ ... (3)

From (1), (2) and (3),

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$

$$\Rightarrow \frac{x-10+x}{x(x-10)}=\frac{8}{75}$$

$\Rightarrow 75(2x-10)=8(x^2-10x)$

$\Rightarrow 150x-750=8x^2-80x$

$\Rightarrow 8x^2-80x-150x+750=0$

$\Rightarrow 4x^2-115x+375=0$

Comparing equation $4x^2-115x+375=0$ with general equation $ax^2+bx+c=0$,

We get $a=4, b=-115$ and $c=375$

Applying quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{115\pm\sqrt{(-115)^2-4(4)(375)}}{2\times4}$$

$$\Rightarrow x=\frac{115\pm\sqrt{13225-6000}}{8}$$

$$\Rightarrow x=\frac{115\pm\sqrt{7225}}{8}$$

$$\Rightarrow x=\frac{115\pm85}{8}$$

$$\Rightarrow x=\frac{115+85}{8},\frac{115-85}{8}$$

$\Rightarrow x=25,3.75$

Time taken by larger tap = $x-10=3.75-10=-6.25$ hours

Time cannot be in negative. Therefore, we ignore this value.

Time taken by larger tap = $x-10=25-10=15$ hours

Therefore, time taken by larger tap is 15 hours and time taken by smaller tap is 25 hours.

---

7. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.

(i) $2x^2-3x+5=0$

(ii) $3x^2-4\sqrt{3}x+4=0$

(iii) $2x^2-6x+3=0$

Ans. (i) $2x^2-3x+5=0$

Comparing this equation with general equation $ax^2+bx+c=0$,

We get $a=2, b=-3$ and $c=5$

Discriminant = $b^2-4ac=(-3)^2-4(2)(5)=9-40=-31$

Discriminant is less than 0 which means equation has no real roots.

(ii) $3x^2-4\sqrt{3}x+4=0$

Comparing this equation with general equation $ax^2+bx+c=0$, We get a=3,b=$-4\sqrt{3}$ and c=4

Discriminant = $b^2-4ac=\left(-4\sqrt{3}\right)^2$ −4(3)(4)=48 − 48=0

Discriminant is equal to zero which means equations has equal real roots.

Applying quadratic $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find roots,

$$x=\frac{4\sqrt{3}\pm\sqrt{0}}{6}=\frac{2\sqrt{3}}{3}$$

Because, equation has two equal roots, it means $x=\frac{2\sqrt{3}}{3},\frac{2\sqrt{3}}{3}$

(iii) $2x^2 - 6x + 3 = 0$

Comparing equation with general equation $ax^2+bx+c=0$,

We get a=2,b=−6, and c=3

Discriminant = $b^2-4ac=(-6)^2 - 4(2)(3)=36 - 24=12$

Value of discriminant is greater than zero.

Therefore, equation has distinct and real roots.

Applying quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to find roots,

$$x=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm2\sqrt{3}}{4}$$

$\Rightarrow x=\frac{3\pm\sqrt{3}}{2}$

$\Rightarrow x=\frac{3+\sqrt{3}}{2},\frac{3-\sqrt{3}}{2}$

---

8. If -4 is a root of the quadratic equation x^2+px-4 and the quadratic equation $x^2+px+k=0$ has equal root, find the value of *k*.

Ans. -4 is root of $x^2+px-4=0$

$\therefore (-4)^2 + p(-4) - 4 = 0$
$\Rightarrow 16 - 4p - 4 = 0$
$\Rightarrow -4p = -12$
$\Rightarrow p = 3$

$x^2 + px + k = 0$ (Given)

$x^2 + 3x + k = 0$
$D = b^2 - 4ac$

$\Rightarrow 0 = (3)^2 - 4 \times 1 \times k$ [For equal roots D = 0]

$\Rightarrow 4k = 9$
$\Rightarrow k = \frac{9}{4}$

---

9. Solve for $x : 5^{x+1} + 5^{1-x} = 26$

Ans. $5^{x+1} + 5^{1-x} = 26$

$5^x.5^1 + 5^1.5^{-x} = 26$
$\Rightarrow 5^x.5 + \frac{5^1}{5^x} = 26$

Put $5^x = y$

$\frac{5y}{1} + \frac{5}{y} = 26$

$\Rightarrow 5y^2 - 26y + 5 = 0$

$\Rightarrow 5y^2 - 25y - y + 5 = 0$
$\Rightarrow 5y(y-5) - 1(y-5) = 0$
$\Rightarrow (y-5)(5y-1) = 0$
$\Rightarrow y = 5 \quad \text{or} \quad y = \frac{1}{5}$

But
$5^x = 5^1$ and $5^x = \frac{1}{5}$
$\Rightarrow x = 1$ and $5^x = 5^{-1} \Rightarrow x = -1$

---

10. $\frac{1}{p+q+x}=\frac{1}{p}+\frac{1}{q}+\frac{1}{x}$ **solve for** x **by factorization method.**

Ans. $\frac{1}{p+q+x}=\frac{1}{p}+\frac{1}{q}+\frac{1}{x}$

$$\Rightarrow \frac{1}{p+q+x}-\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$$

$$\Rightarrow \frac{x-p-q-x}{x^2+px+qx}=\frac{p+q}{pq}$$

$$\Rightarrow \frac{-(p+q)}{x^2+px+qx}=\frac{p+q}{pq}$$

$$\Rightarrow \frac{-1}{x^2+px+qx}=\frac{1}{pq}$$

$$\Rightarrow x^2+px+qx=-pq$$

$$\Rightarrow x^2+px+qx+pq=0$$

$$\Rightarrow x(x+p)+q(x+p)=0$$

$$\Rightarrow (x+p)(x+q)=0$$

$$\Rightarrow x=-p \ or \ x=-q$$

---

11. $5x^2-6x-2=0,$ **solve for** x **by the method of completing the square.**

Ans. $5x^2-6x-2=0$

$$\Rightarrow x^2-\frac{6}{5}x-\frac{2}{5}=0$$

$$\Rightarrow x^2-\frac{6}{5}x+\left(\frac{3}{5}\right)^2-\frac{2}{5}=0$$

$$\Rightarrow \left(x-\frac{3}{5}\right)^2=\frac{9}{25}+\frac{2}{5}$$

$$\Rightarrow \left(x-\frac{3}{5}\right)^2=\frac{9+10}{25}$$

$$\Rightarrow \left(x-\frac{3}{5}\right)^2=\frac{19}{25}$$

$$\Rightarrow x-\frac{3}{5}=\pm\frac{\sqrt{19}}{5}$$

$$\Rightarrow x=\frac{3}{5}\pm\frac{\sqrt{19}}{5}$$
$$\Rightarrow x=\frac{3+\sqrt{19}}{5} \text{ or } x=\frac{3-\sqrt{19}}{5}$$

---

12. Solve for x **:** $a^2b^2x^2+b^2x-a^2x-1=0$

Ans. $a^2b^2x^2+b^2+x-a^2x-1=0$

$$\Rightarrow b^2x\left(a^2x+1\right)-1\left(a^2x+1\right)=0$$
$$\Rightarrow \left(a^2x+1\right)\left(b^2x-1\right)=0$$
$$\Rightarrow x=\frac{-1}{a^2}\ or\ x=\frac{1}{b^2}$$

---

13. Using quadratic formula, solve for x **:** $9x^2-9(a+b)x+\left(2a^2+5ab+2b^2\right)=0$

Ans. $D=b^2-4ac$

$$=\left(-9(a+b)\right)^2-4\times 9\times\left(2a^2+5ab+2ab^2\right)$$

$$=81(a+b)^2-36\left(2a^2+5ab+2b^2\right)$$
$$=9\left[9\left(a^2+b^2+2ab-8a^2-20ab-8b^2\right)\right]$$
$$=9\left[a^2+b^2-2ab\right]$$
$$=9(a-b)^2$$

$$x=\frac{-b\pm\sqrt{D}}{2a}\ =\frac{9(a+b)\pm\sqrt{9(a-b)^2}}{2\times 9}$$

$$\Rightarrow x=3\frac{\left[3(a+b)\pm(a-b)\right]}{2\times 9}$$
$$\Rightarrow x=\frac{(3a+3b)\pm(a-b)}{6}$$
$$\Rightarrow x=\frac{3a+3b+a-b}{6}\ or\ x=\frac{3a+3b+a-b}{6}$$
$$\Rightarrow x=\frac{4a+2b}{6}\ or\ x=\frac{4a+2b}{6}$$
$$\Rightarrow x=\frac{2a+b}{3}\ or\ x=\frac{2a+b}{3}$$

---

14. In a cricket match, Kapil took one wicket less than twice the number of wickets taken by Ravi. If the product of the numbers of wickets taken by these two is 15, find the number of wickets taken by each.

Ans. Let no. of wicket taken by Ravi = x

No. of wicket taken by Kapil = $2x-1$

According to question,

$(2x-1).x=15$

$\Rightarrow 2x^2-x-15=0$

$\Rightarrow x=3 \ or \ x=\frac{-5}{2}$ (Neglects)

So, no. of wickets taken by Ravi is $x=3$

---

15. The sum of a number and its reciprocal is $\frac{17}{4}$. Find the number.

Ans. Let no. be x

According to question,

$\frac{x}{1}+\frac{1}{x}=\frac{17}{4}$

$\Rightarrow \frac{x^2+1}{x}=\frac{17}{4}$

$\Rightarrow 4x^2+4=17x$

$\Rightarrow 4x^2-17x+4=0$

$\Rightarrow 4x^2-16x-x+4=0$

$\Rightarrow 4x(x-4)-1(x-4)=0$

$\Rightarrow (x-4)(4x-1)=0$

$\Rightarrow x=4 \ or \ x=\frac{1}{4}$

4 Marks Questions

1. Find the roots of the following Quadratic Equations by applying quadratic formula.
(i) $2x^2-7x+3=0$
(ii) $2x^2+x-4=0$
(iii) $4x^2+4\sqrt{3}x+3=0$
(iv) $2x^2+x+4=0$

Ans . (i) $2x^2 - 7x + 3 = 0$
Comparing quadratic equation $2x^2 - 7x + 3 = 0$ with general form $ax^2+bx+c=0$, we get $a=2$, $b=-7$ and $c=3$

Putting these values in quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{7\pm\sqrt{(-7)^2-4(2)(3)}}{2\times 2}$$

$\Rightarrow x=\dfrac{7\pm\sqrt{49-24}}{4}$

$\Rightarrow x=\dfrac{7\pm 5}{4}$

$\Rightarrow x=\dfrac{7+5}{4},\dfrac{7-5}{4}$

$\Rightarrow x=3, \frac{1}{2}$

(ii) $2x^2 + x - 4 = 0$
Comparing quadratic equation $2x^2 + x - 4 = 0$ with the general form $ax^2+bx+c=0$, we get $a=2$, $b=1$ and $c=-4$

Putting these values in quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-1\pm\sqrt{(1)^2-4(2)(-4)}}{2\times 2}$$

$\Rightarrow x=\dfrac{-1\pm\sqrt{33}}{4}$

$\Rightarrow x=\dfrac{-1+\sqrt{33}}{4},\dfrac{-1-\sqrt{33}}{4}$

(iii) $4x^2+4\sqrt{3}x+3=0$

Comparing quadratic equation $4x^2+4\sqrt{3}x+3=0$ with the general form $ax^2+bx+c=0$, we get $a=4$, $b=4\sqrt{3}$ and $c=3$

Putting these values in quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-4\sqrt{3}\pm\sqrt{(4\sqrt{3})^2-4(4)(3)}}{2\times 4}$$

$\Rightarrow x=\dfrac{-4\sqrt{3}\pm\sqrt{0}}{8}$

$\Rightarrow x=\dfrac{-\sqrt{3}}{2}$

A quadratic equation has two roots. Here, both the roots are equal.

Therefore, $x=\dfrac{-\sqrt{3}}{2},\dfrac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$
Comparing quadratic equation $2x^2 + x + 4 = 0$ with the general form $ax^2+bx+c=0$, we get $a=2, b=1$ and $c= 4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(4)}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

But, square root of negative number is not defined.
Therefore, Quadratic Equation $2x^2 + x + 4 = 0$ has no solution.

---

2. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If, the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of two trains.

Ans. Let average speed of passenger train = x km/h
Let average speed of express train = $(x+11)$ km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours
According to the given condition,

$$\frac{132}{x} = \frac{132}{x+11} + 1$$

$$\Rightarrow 132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$$

$$\Rightarrow 132\left(\frac{x+11-x}{x(x+11)}\right) = 1$$

$\Rightarrow 132(11)=x(x+11)$
$\Rightarrow 1452=x^2+11x$
$\Rightarrow x^2+11x- 1452=0$
Comparing equation $x^2+11x- 1452=0$ with general quadratic equation $ax^2+bx+c=0$, we get $a=1, b=11$ and $c=-1452$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(-1452)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-11 \pm \sqrt{121 + 5808}}{2}$$

$$\Rightarrow x = \frac{-11 \pm \sqrt{5929}}{2}$$

$\Rightarrow x=\frac{-11\pm 77}{2}$

$\Rightarrow x=\frac{-11+77}{2},\frac{-11-77}{2}$

$\Rightarrow x=33,-44$

As speed cannot be in negative.Therefore, speed of passenger train = 33 km/h

And, speed of express train = $x+11=33+11=44$ km/h

---

3. Sum of areas of two squares is 468 m^2. If, the difference of their perimeters is 24 metres, find the sides of the two squares.

Ans. Let perimeter of first square = x metres

Let perimeter of second square = $(x+24)$ metres

Length of side of first square = $\frac{x}{4}$ metres {Perimeter of square = 4 × length of side}

Length of side of second square = $\left(\frac{x+24}{4}\right)$ metres

Area of first square = side × side = $\frac{x}{4}\times\frac{x}{4}=\frac{x^2}{16}m^2$

Area of second square = $\left(\frac{x+24}{4}\right)^2 m^2$

According to given condition:

$\frac{x^2}{16}+\left(\frac{x+24}{4}\right)^2=468$

$\Rightarrow \frac{x^2}{16}+\frac{x^2+576+48x}{16}=468$

$\Rightarrow \frac{x^2+x^2+576+48x}{16}=468$

$\Rightarrow 2x^2+576+48x=468\times 16$

$\Rightarrow 2x^2+48x+576=7488$

$\Rightarrow 2x^2+48x-6912=0$

$\Rightarrow x^2+24x-3456=0$

Comparing equation $x^2+24x-3456=0$ with standard form $ax^2+bx+c=0$,

We get $a=1, b=24$ and $c=-3456$

Applying Quadratic Formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-24\pm\sqrt{(24)^2-4(1)(-3456)}}{2\times 1}$

$\Rightarrow x=\frac{-24\pm\sqrt{576+13824}}{2}$

$\Rightarrow x=\frac{-24\pm\sqrt{14400}}{2}=\frac{-24\pm 120}{2}$

$\Rightarrow x=\frac{-24+120}{2},\frac{-24-120}{2}$
$\Rightarrow x=48,-72$
Perimeter of square cannot be in negative. Therefore, we discard $x=-72$.
Therefore, perimeter of first square = 48 metres
And, Perimeter of second square = $x+24=48+24=72$ metres
$\Rightarrow$ Side of First square = $\frac{Perimeter}{4}=\frac{48}{4}=12\,m$
And, Side of second Square = $\frac{Perimeter}{4}=\frac{72}{4}=18\,m$

---

4. Is it possible to design a rectangular park of perimeter 80 metres and area 400 m^2. If so, find its length and breadth.

Ans. Let length of park = x metres
We are given area of rectangular park = 400 m^2
Therefore, breadth of park = $\frac{400}{x}$ metres {Area of rectangle = length ×*breadth*}
Perimeter of rectangular park = 2(*length*+*breath*)= $2\left(x+\frac{400}{x}\right)$ metres
We are given perimeter of rectangle = 80 metres
According to condition:
$2\left(x+\frac{400}{x}\right)=80$
$\Rightarrow 2\left(\frac{x^2+400}{x}\right)=80$
$\Rightarrow 2x^2+800=80x$
$\Rightarrow 2x^2-80x+800=0$
$\Rightarrow x^2-40x+400=0$
Comparing equation, $x^2-40x+400=0$ with general quadratic equation $ax^2+bx+c=0$, we get$a=1,b=-40$ and $c=400$
Discriminant = $b^2-4ac=(-40)^2-4(1)(400)=1600-1600=0$
Discriminant is equal to 0.
Therefore, two roots of equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area 400 m^2.
Using quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve equation,
$x=\frac{40\pm\sqrt{0}}{2}=\frac{40}{2}=20$
Here, both the roots are equal to 20.
Therefore, length of rectangular park = 20 metres
Breadth of rectangular park = $\frac{400}{x}=\frac{400}{20}=20\,m$

---

5. If I had walked 1 km per hour faster, I would have taken 10 minutes less to walk 2 km. Find the rate of my walking.

Ans. Distance = 2 km
Let speed = x km/hr
New speed = (x+1) km/hr

Time taken by normal speed = $\frac{2}{x} hr$

Time taken by new speed = $\frac{2}{x+1} hr$
According to question,

$$\frac{2}{x} - \frac{2}{x+1} = \frac{10}{60}$$

$$\Rightarrow \frac{2x+2-2x}{x^2+x} = \frac{1}{6}$$

$$\Rightarrow x^2+x=12$$

$$\Rightarrow x^2+x-12=0$$

$$\Rightarrow x^2+4x-3x-12=0$$

$$\Rightarrow x(x+4)-3(x+4)=0$$

$$\Rightarrow (x+4)(x-3)=0$$

$$\Rightarrow x=-4 \; or \; x=3$$

So, speed is $x=3$ km/hr

---

6. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

Ans. Let B takes x days to finish the work, then A alone can finish it in $(x-6)$ days
According to question,

$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$

$$\Rightarrow \frac{x-6+x}{x^2-6x} = \frac{1}{4}$$

$$\Rightarrow \frac{2x-6}{x^2-6x} = \frac{1}{4}$$

$$\Rightarrow x^2-6x=8x-24$$

$$\Rightarrow x^2-14x+24=0$$

$$\Rightarrow x^2-12x-2x+24=0$$

$$\Rightarrow x(x-12)-2(x-12)=0$$

$$\Rightarrow (x-12)(x-2)=0$$

$$\Rightarrow x=12 \; or \; x=2$$

$x=2$ (Neglect)
So, B takes $x=12$ days.

---

7. A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.

Ans. Let usual speed $=x$ km/hr

New speed $=(x+250)$ km/hr

Total distance = 1500 km

Time taken by usual speed = $\frac{1500}{x}$ hr

Time taken by new speed = $\frac{1500}{x+250}$ hr

According to question,

$$\frac{1500}{x}-\frac{1500}{x+250}=\frac{1}{2}$$

$$\Rightarrow \frac{1500x+1500\times 250-1500x}{x^2+250x}=\frac{1}{2}$$

$$\Rightarrow x^2+250x=\frac{1500\times 250}{2}$$

$$\Rightarrow x^2+250x=750000$$

$$\Rightarrow x^2+250x-750000=0$$

$$\Rightarrow x^2+1000x-750x-750000=0$$

$$\Rightarrow x(x+1000)-750(x+1000)=0$$

$$\Rightarrow x=750 \text{ or } x=-1000$$

Therefore, usual speed is 750 km/hr, -1000 is neglected.

---

8. A motor boat, whose speed is 15 km/hr in still water, goes 30 km downstream and comes back in a total time of 4 hr 30 minutes, find the speed of the stream.

Ans. Speed of motor boat in still water = 15 km/hr

Speed of stream = x km/hr

Speed in downward direction = $15+x$

Speed in downward direction = $15-x$

According to question,

$$\frac{30}{15+x}+\frac{30}{15-x}=4\frac{1}{2}$$

$$\Rightarrow \frac{30(15-x)+30(15+x)}{(15+x)(15-x)}=\frac{9}{2}$$

$$\Rightarrow \frac{450-30x+450+30x}{225-x^2}=\frac{9}{2}$$

$$\Rightarrow 9(225-x^2)=1800$$

$$\Rightarrow 225-x^2=200$$

$$\Rightarrow x=5$$

Speed of stream = 5 km/hr

---

9. A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously fill the pool in the same time during which the pool is the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately.

Ans. Let x be the number of hours required by the second pipe alone to till the pool and first pipe $(x+5)$ hour while third pipe $(x-4)$ hour

$\frac{1}{x+5}+\frac{1}{x}=\frac{1}{x-4}$

$\Rightarrow \frac{x+x+5}{x^2+5x}=\frac{1}{x-4}$

$\Rightarrow x^2-8x-20=0$

$\Rightarrow x^2-10x+2x-20=0$

$\Rightarrow x(x-10)+2(x-10)=0$

$\Rightarrow (x-10)(x+2)=0$

$\Rightarrow x=10 \text{ or } x=-2 \ (Neglected)$

---

10. A two-digit number is such that the product of its digits is 18. When 63 is subtracted from the number the digit interchange their places. Find the number.

Ans. Let digit on unit's place = x

Digit on ten's place = y

$xy=18$ (given)

Number = 10.$y+x$

= $10\left(\frac{18}{x}\right)+x$

According to question,

$10\left(\frac{18}{x}\right)+x-63=10x+\frac{18}{x}$

$\Rightarrow \frac{180}{x}+\frac{x-63}{1}=\frac{10x^2+18}{x}$

$\Rightarrow \frac{180+x^2-63x}{x}=\frac{10x^2+18}{x}$

$\Rightarrow 9x^2+63x-162=0$

$\Rightarrow 9(x^2+9x-2x-18)=0$

$\Rightarrow x(x+9)-2(x+9)=0$

$\Rightarrow x=2 \text{ or } x=-9$

Number = $10\left(\frac{18}{2}\right)+2$ = 92

---

11. A factory kept increasing its output by the same percent ago every year. Find the percentage if it is known that the output is doubled in the last two years.

Ans. According to question,

2P $= p\left(1+\frac{r}{100}\right)^2$

$\Rightarrow \frac{\sqrt{2}}{1} = 1+\frac{r}{100}$

$\Rightarrow \sqrt{2}-1 = \frac{r}{100}$

$\Rightarrow r = \left(\sqrt{2}-1\right)100$

---

12. Two pipes running together can fill a cistern in $3\frac{1}{13}$ minutes. **if one pipe takes 3 minutes more than the other to fill it, find the time in which each pipe would fill the cistern.**

Ans. Let the faster pipe takes x minutes to fill the cistern and the slower pipe will take $(x+3)$ minutes.

According to question,

$\frac{1}{x}+\frac{1}{x+3} = \frac{1}{\frac{40}{13}}$

$\Rightarrow \frac{1}{x}+\frac{1}{x+3} = \frac{13}{40}$

$\Rightarrow \frac{x+3+x}{x^2+3x} = \frac{13}{40}$

$\Rightarrow 13x^2-41x-120 = 0$

$\Rightarrow 13x^2-65x+24x-120 = 0$

$\Rightarrow 13x(x-5)+24(x-5) = 0$

$\Rightarrow x = 5 \text{ or } x = \frac{-24}{13} (Neglected)$

---

13. If the roots of the equation $(a-b)x^2+(b-c)x+(c-a) = 0$ **are equal, prove that** $2a = b+c$.

Ans. $(a-b)x^2+(b-c)x+(c-a)=0$

$D=b^2-4ac$

$=(b-c)^2-4\times(a-b)\times(c-a)$

$=b^2+c^2-2bc-4(ac-a^2-bc+ab)$

$=b^2+c^2-2bc-4ac+4a^2+4bc-4ab$

$=(b)^2+(c)^2+(2a)^2+2bc-4ac-4ab$

$=(b+c-2a)^2$

For equal root s,
D = 0

$\Rightarrow(b+c-2a)^2=0$

$\Rightarrow(b+c-2a)=0$

$\Rightarrow b+c=2a$

---

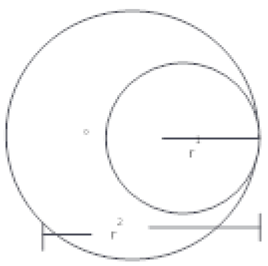
14. Two circles touch internally. The sum of their areas is 116 π cm[2] and the distance between their centres is 6 cm. Find the radii of the circles.

Ans. Let r_1 and r_2 be the radius of two circles
According to question,

$\pi r_1^2+\pi r_2^2=116\pi$

$\Rightarrow r_1^2+r_2^2=116........(i)$

$r_2-r_1=6$ (Given)



$\Rightarrow r_2=6+r_1$

Put the value of r_2 in eq. … (i)

$r_1^2+(6+r_1)^2=116$

$\Rightarrow r_1^2+36+r_1^2+12r_1=116$

$\Rightarrow 2r_1^2+12r_1-80=0$

$\Rightarrow r_1^2+6r_1-40=0$

$\Rightarrow r_1^2+10r_1-4r_1-40=0$

$\Rightarrow r_1(r_1+10)-4(r_1+10)=0$

$\Rightarrow(r_1+10)(r_1-4)=0$

$when\ r_1 = 4\ cm$

$r_2 = 6 + r_1$

$= 6 + 4$

$r_2 = 10\ cm$

$\Rightarrow r_1 = -10$ (Neglect) or r_1 = 4 cm

---

15. A piece of cloth costs Rs. 200. If the piece was 5 m longer and each metre of cloth costs Rs. 2 less the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

Ans. Let the length of piece = x m

Rate per metre = $\frac{200}{x}$

New length = (x+5)

New rate per metre = $\frac{200}{x+5}$

According to question,

$\frac{200}{x} - \frac{200}{x+5} = 2$

$\Rightarrow \frac{200(x+5) - 200x}{(x+5)} = \frac{2}{1}$

$\Rightarrow \frac{200x + 1000 - 200x}{x^2 + 5x} = \frac{2}{1}$

$\Rightarrow x^2 + 5x = 500$

$\Rightarrow x^2 + 5x - 500 = 0$

$\Rightarrow x^2 + 25x - 20x - 500 = 0$

$\Rightarrow x(x+25) - 20(x+25) = 0$

$\Rightarrow (x+25)(x-20) = 0$

$\Rightarrow x = -25\ (Neglect)\ or\ x = 20$

Rate per metre = 10

---

16. $ax^2+bx+x = 0$, $a \neq 0$ solve by quadratic formula.

Ans. $ax^2+bx+x = 0$

$$\Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\Rightarrow x^2+\frac{b}{a}x\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\Rightarrow x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$\Rightarrow x=\frac{-b}{2a}\pm\sqrt{\frac{b^2-4ac}{2a}}$$

$$\Rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

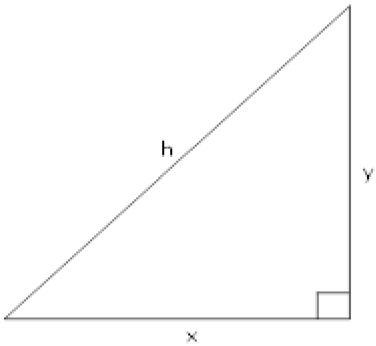
$$\Rightarrow x=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$or\ x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

---

17. The length of the hypotenuse of a right-angled triangleexceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

Ans. Let base = x
Altitude = y
Hypotenuse = h



According to question,

$h = x+2$
$h = 2y+1$
$\Rightarrow x+2 = 2y+1$
$\Rightarrow x+2-1 = 2y$
$\Rightarrow x-1 = 2y$
$\Rightarrow \frac{x-1}{2} = y$
$And\ x^2 + y^2 = h^2$
$\Rightarrow x^2 + \left(\frac{x-1}{2}\right)^2 = (x+2)^2$
$\Rightarrow x^2 - 15x + x - 15 = 0$

$\Rightarrow x^2 - 15x + x - 15 = 0$
$\Rightarrow (x-15)(x+1) = 0$
$\Rightarrow x = 15\ or\ x = -1$

Base = 15 cm

Altitude = $\frac{x+1}{2} = 8\ cm$

Hypotenuse = 17 cm

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18. Find the roots of the following quadratic equations if they exist by the method of completing square.

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Ans. (i) $2x^2 - 7x + 3 = 0$

First we divide equation by 2 to make coefficient of x^2 equal to 1,
$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

We divide middle term of the equation by $2x$, we get $\frac{7}{2}x \times \frac{1}{2x} = \frac{7}{4}$

We add and subtract square of $\frac{7}{4}$ from the equation $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$,

$$x^2-\frac{7}{2}x+\frac{3}{2}+\left(\frac{7}{4}\right)^2-\left(\frac{7}{4}\right)^2=0$$

$\Rightarrow$ $x^2+\left(\frac{7}{4}\right)^2-\frac{7}{2}x+\frac{3}{2}+-\left(\frac{7}{4}\right)^2=0$ $\{(a-b)^2=a^2+b^2-2ab\}$

$\Rightarrow$ $\left(x-\frac{7}{4}\right)^2+\frac{24-49}{16}=0$ $\Rightarrow$ $\left(x-\frac{7}{4}\right)^2=\frac{49-24}{16}$

Taking Square root on both sides,

$\Rightarrow$ $x-\frac{7}{4}=\pm\frac{5}{4}$

$\Rightarrow$ $x=\frac{5}{4}+\frac{7}{4}=\frac{12}{4}=3 \text{ and } x=-\frac{5}{4}+\frac{7}{4}=\frac{2}{4}=\frac{1}{2}$

Therefore, $x=\frac{1}{2},3$

(ii) $2x^2+x-4=0$

Dividing equation by 2,

$$x^2+\frac{x}{2}-2=0$$

Following procedure of completing square,

$$x^2+\frac{x}{2}-2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$$

$\Rightarrow$ $x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-2-\frac{1}{16}=0$ $\{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow$ $\left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$

$\Rightarrow$ $\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$

Taking square root on both sides,

$\Rightarrow$ $x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$

$\Rightarrow$ $x=\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{\sqrt{33}-1}{4} \text{ and } x=-\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{-\sqrt{33}-1}{4}$

Therefore, $x=\frac{\sqrt{33}-1}{4},\frac{-\sqrt{33}-1}{4}$

(iii) $4x^2+4\sqrt{3}x+3=0$

Dividing equation by 4,

$x^2+\sqrt{3}x+\frac{3}{4}=0$

Following the procedure of completing square,

$\Rightarrow x^2+\sqrt{3}x+\frac{3}{4}+\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2=0$

$\Rightarrow x^2+\left(\frac{\sqrt{3}}{2}\right)^2+\sqrt{3}x+\frac{3}{4}-\frac{3}{4}=0$ $\{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)^2=0$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right)=0$

Taking square root on both sides,

$\Rightarrow x+\frac{\sqrt{3}}{2}=0, x+\frac{\sqrt{3}}{2}=0$

$\Rightarrow x=-\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2}$

(iv) $2x^2+x+4=0$

Dividing equation by 2,

$x^2+\frac{x}{2}+2=0$

Following the procedure of completing square,

$\Rightarrow x^2+\frac{x}{2}+2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow x^2+\left(\frac{1}{4}\right)^2+\frac{x}{2}+2-\left(\frac{1}{4}\right)^2=0$ $\{(a+b)^2=a^2+b^2+2ab\}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2+2-\frac{1}{16}=0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{1}{16}-2=\frac{1-32}{16}$

Taking square root on both sides

Right hand side does not exist because square root of negative number does not exist.

Therefore, there is no solution for quadratic equation $2x^2+x+4=0$