CBSE Class 9 Mathematics
Important Questions
Chapter 8
Quadrilaterals

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1 Marks Questions

1. A quadrilateral ABCD is a parallelogram if

(a) AB = CD

(b) AB $\parallel$ BC

(c) $\angle A = 60^0, \angle C = 60^0, \angle B = 120^0$

(d) AB = AD

Ans. (c) $\angle A = 60^0, \angle C = 60^0, \angle B = 120^0$

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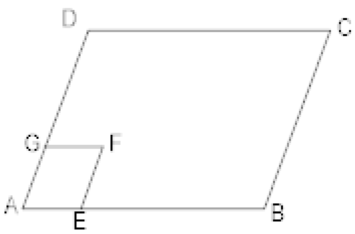
2. In figure, ABCD and AEFG are both parallelogram if $\angle C = 80^0, then \angle DGF$ *is*



(a) 100^0

(b) 60^0

(c) 80^0

(d) 120^0

Ans. (c) 80^0

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3. In a square ABCD, the diagonals AC and BD bisects at O. Then ΔAOB **is**

(a) acute angled

(b) obtuse angled

(c) equilateral

(d) right angled

Ans. (d) right angled

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4. ABCD is a rhombus. If $\angle ACB = 30^0$, *then* $\angle ADB$ *is*

(a) 30^0

(b) 120^0

(c) 60^0

(d) 45^0

Ans. (c) 60^0

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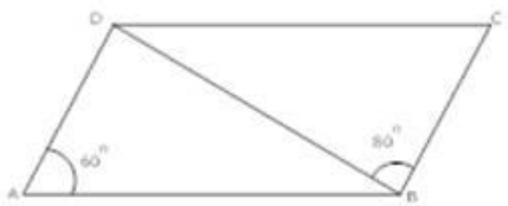
5. In fig ABCD is a parallelogram. If $\angle DAB = 60°$ **and** $\angle DBC = 80°$ **then** $\angle CDB$ **is**



$(A)\,80°$
$(B)\,60°$
$(C)\,20°$
$(D)\,40°$

Ans. $(D)\,40°$

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6. If the diagonals of a quadrilateral bisect each other, then the quadrilateral must be.

(a) Square

(b) Parallelogram

(c) Rhombus

(d) Rectangle

Ans. (b) Parallelogram

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7. The diagonal AC and BD of quadrilateral ABCD are equal and are perpendicular bisector of each other then quadrilateral ABCD is a

(a) Kite

(b) Square

(c) Trapezium

(d) Rectangle

Ans. (b) Square

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8. The quadrilateral formed by joining the mid points of the sides of a quadrilateral ABCD taken in order, is a rectangle if

(a) ABCD is a parallelogram

(b) ABCD is a rut angle

(c) Diagonals AC and BD are perpendicular

(d) AC=BD

Ans. (a) ABCD is a parallelogram

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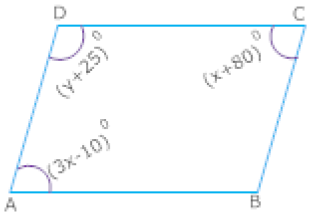
9. In the fig ABCD is a Parallelogram. The values of x and y are



(a) 30, 35

(b) 45, 30

(c) 45, 45

(d) 55, 35

Ans. (b) 45, 30

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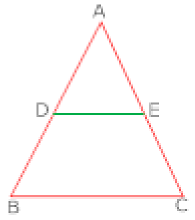
10. In fig if DE=8 cm and D is the mid-Point of AB, then the true statement is



(a) AB=AC

(b) DE||BC

(c) E is not mid-Point of AC

(d) DE $\neq$ BC

Ans. (c) E is not mid-Point of AC

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11. The sides of a quadrilateral extended in order to form exterior angler. The sum of these exterior angle is

(a) $180°$

(b) $270°$

(c) $90°$

(d) $360°$

Ans. (d) $360°$

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12. ABCD is rhombus with $\angle ABC = 40°$. **The measure of** $\angle ACD$ **is**

(a) $90°$

(b) $20°$

(c) $40°$

(d) $70°$

Ans. b) $20°$

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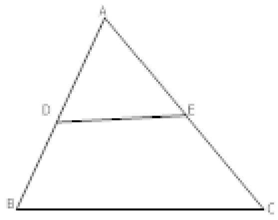
13. In fig D is mid-point of AB and DE $\parallel$ **BC then AE is equal to**



(a) AD

(b) EC

(c) DB

(d) BC

Ans. (b) EC

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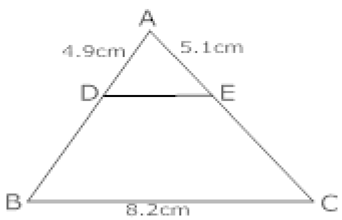
14. In fig D and E are mid-points of AB and AC respectively. The length of DE is



(a) 8.2 cm

(b) 5.1 cm

(c) 4.9 cm

(d) 4.1 cm

Ans. (d) 4.1 cm

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15. A diagonal of a parallelogram divides it into

(a) two congruent triangles

(b) two similes triangles

(c) two equilateral triangles

(d) none of these

Ans. (a) two congruent triangles

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16. A quadrilateral is a ________, if its opposite sides are equal:

(a) Kite

(b) trapezium

(c) cyclic quadrilateral

(d) parallelogram

Ans. (d) parallelogram

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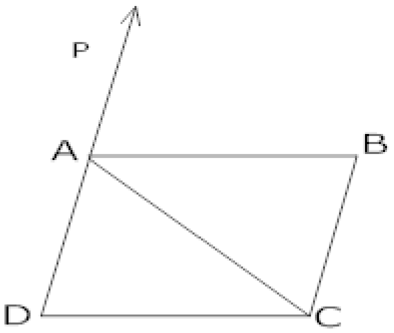
17. In the adjoining Fig. AB = AC. CD||BA and AD is the bisector of $\angle PAC$ prove that

(a) $\angle DAC = \angle BCA$ **and**



Ans. In ΔABC $AB = AC$

$\Rightarrow \angle BCA = \angle BAC$ [Opposite angle of equal sides are equal]

$\angle CAD = \angle BCA + \angle ABC$ [Exterior angle]

$\Rightarrow \angle PAC = \angle BCA$

Now $\angle PAC = \angle BCA$

$\Rightarrow AP \parallel BC$

Also CD||BA Given)

$\therefore ABCD$ is a parallelogram

(ii) ABCD is a parallelogram

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18. Which of the following is not a parallelogram?

(a) Rhombus

(b) Square

(c) Trapezium

(d) Rectangle

Ans. (c) Trapezium

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19. The sum of all the four angles of a quadrilateral is

(a) 180^0

(b) 360^0

(c) 270^0

(d) 90^0

Ans. (b) 360^0

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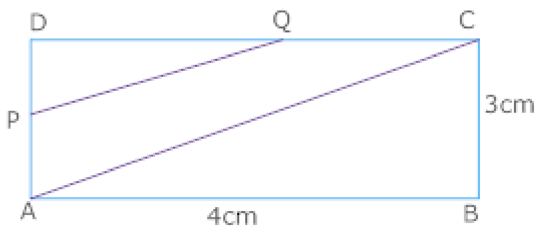
20. In Fig ABCD is a rectangle P and Q are mid-points of AD and DC respectively. Then length of PQ is



(a)5 cm

(b) 4 cm

(c) 2.5 cm

(d) 2 cm

Ans. (c) 2.5 cm

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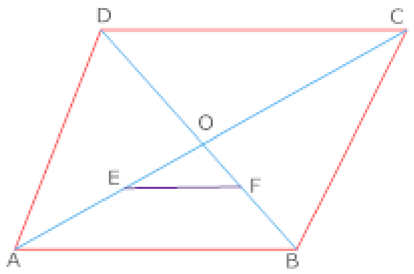
21. In Fig ABCD is a rhombus. Diagonals AC and BD intersect at O. E and F are mid points of AO and BO respectively. If AC = 16 cm and BD = 12 cm then EF is



(a)10 cm

(b) 5 cm

(c) 8 cm

(d) 6 cm

Ans. (b) 5 cm

2 Marks Questions

1. The angles of a quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all angles of the quadrilateral

Ans. Let in quadrilateral ABCD, ∠A = $3x$, ∠B = $5x$, ∠C = $9x$ and ∠D = $13x$.

Since, sum of all the angles of a quadrilateral = 360°

∴ ∠A + ∠B + ∠C + ∠D = 360° $\Rightarrow$ $3x+5x+9x+13x=360^\circ$

$\Rightarrow 30x=360^\circ \Rightarrow x=12^\circ$

Now ∠A = $3x=3\times12=36^\circ$

∠B = $5x=5\times12=60^\circ$

∠C = $9x=9\times12=108^\circ$

And ∠D = $13x=13\times12=156^\circ$

Hence angles of given quadrilateral are $36^\circ, 60^\circ, 108^\circ$ and 156°.

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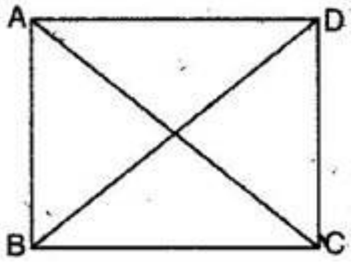
2. If the diagonals of a parallelogram are equal, show that it is a rectangle.

Ans. Given: ABCD is a parallelogram with diagonal AC = diagonal BD

To prove: ABCD is a rectangle.



Proof: In triangles ABC and ABD,

AB = AB [Common]

AC = BD [Given]

AD = BC [opp. Sides of a $\parallel$ gm]

$\therefore$ ΔABC $\cong$ ΔBAD [By SSS congruency]

$\Rightarrow$ $\angle$DAB = $\angle$CBA [By C.P.C.T.] ....(i)

But $\angle$DAB + $\angle$CBA = 180° ....(ii)

[$\because$ AD $\parallel$ BC and AB cuts them, the sum of the interior angles of the same side of transversal is 180°]

From eq. (i) and (ii),

$\angle$DAB = $\angle$CBA = 90°

Hence ABCD is a rectangle.

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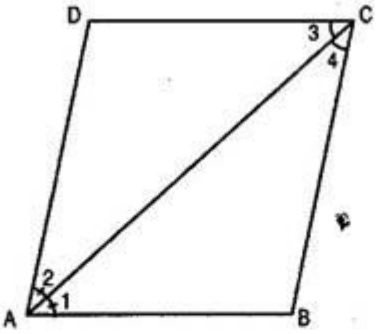
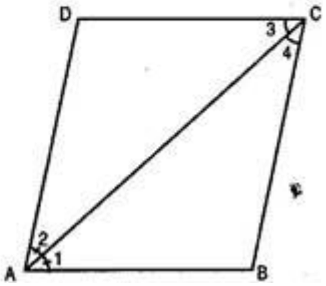
3. Diagonal AC of a parallelogram ABCD bisects $\angle$A (See figure). Show that:

(i) It bisects $\angle$C also.

(ii) ABCD is a rhombus.



Ans. Diagonal AC bisects $\angle$A of the parallelogram ABCD.



(i) Since AB $\parallel$ DC and AC intersects them.
$\therefore$ $\angle 1 = \angle 3$ [Alternate angles] ……….(i)
Similarly $\angle 2 = \angle 4$ ……….(ii)
But $\angle 1 = \angle 2$ [Given] ……….(iii)
$\therefore$ $\angle 3 = \angle 4$ [Using eq. (i), (ii) and (iii)]
Thus AC bisects $\angle$C.

(ii) $\angle 2 = \angle 3 = \angle 4 = \angle 1$
$\Rightarrow$ AD = CD [Sides opposite to equal angles]
$\therefore$ AB = CD = AD = BC
Hence ABCD is a rhombus.

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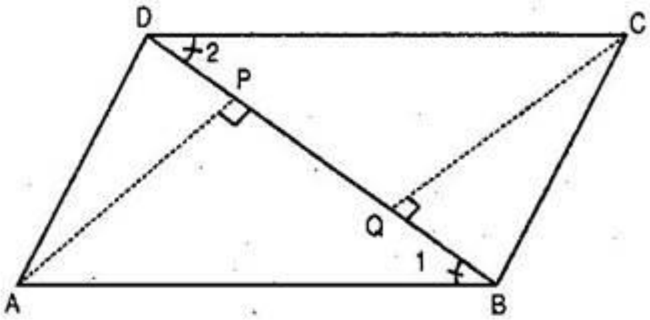
4. ABCD is a parallelogram and AP and CQ are the perpendiculars from vertices A and C on its diagonal BD (See figure). Show that:

(i) ΔAPB $\cong$ ΔCQD

(ii) AP = CQ



Ans. Given: ABCD is a parallelogram. AP $\perp$ BD and CQ $\perp$ BD

To prove: (i) ΔAPB $\cong$ ΔCQD (ii) AP = CQ

Proof: (i) In ΔAPB and ΔCQD,

$\angle 1 = \angle 2$ [Alternate interior angles]

AB = CD [Opposite sides of a parallelogram are equal]

$\angle APB = \angle CQD = 90^\circ$

$\therefore$ ΔAPB $\cong$ ΔCQD [By ASA Congruency]

(ii) Since ΔAPB $\cong$ ΔCQD

AP = CQ [By C.P.C.T.]

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5. ABCD is a quadrilateral in which P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively (See figure). AC is a diagonal. Show that:

(i) SR $\parallel$ AC and SR = $\frac{1}{2}$ AC

(ii) PQ = SR

(iii) PQRS is a parallelogram.

Ans. In Δ ABC, P is the mid-point of AB and Q is the mid-point of BC.

Then PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC

(i) In ΔACD, R is the mid-point of CD and S is the mid-point of AD.

Then SR $\parallel$ AC and SR = $\frac{1}{2}$ AC

(ii) Since PQ = $\frac{1}{2}$ AC and SR = $\frac{1}{2}$ AC

Therefore, PQ = SR

(iii) Since PQ $\parallel$ AC and SR $\parallel$ AC

Therefore, PQ $\parallel$ SR [two lines parallel to given line are parallel to each other]

Now PQ = SR and PQ $\parallel$ SR

Therefore, PQRS is a parallelogram.

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6. The angles of a quadrilateral are in the ratio 3:5:9:13. Find all the angles of the quadrilateral.

Ans. Suppose angles of quadrilateral ABCD are 3x, 5x, 9x, and 13x

$\angle A + \angle B + \angle C + \angle D = 360^0$ [sum of angles of a quadrilateral is 360^0]

$30x = 360^0$

$x = 12^0$

$\therefore \angle A = 3x = 3 \times 12 = 36^0$

$\angle B = 5x = 5 \times 12 = 60^0$

$\angle C = 9x = 9 \times 12 = 108^0$

$\angle D = 13x = 13 \times 12 = 156^0$

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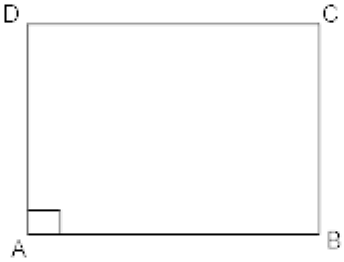
7. Show that each angle of a rectangle is a right angle.

Ans. We know that rectangle is a parallelogram whose one angle is right angle.



Let ABCD be a rectangle.

$\angle A = 90^0$

To prove $\angle B = \angle C = \angle D = 90^0$

Proof: $\because AD \parallel BC$ and AB is transversal

$\therefore \angle A + \angle B = 180^0$

$90^0 + \angle B = 180^0$

$\angle B = 180^0 - 90^0 = 90^0$

$\angle C = \angle A$

$\therefore \angle C = 90^0$

$\angle D = \angle B$

$\therefore \angle D = 90^0$

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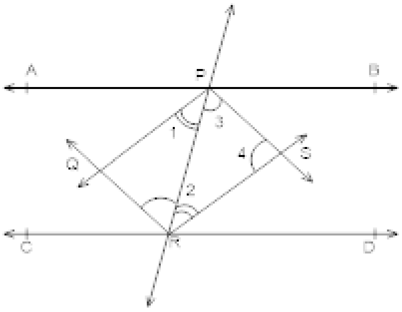
8. A transversal cuts two parallel lines prove that the bisectors of the interior angles enclose a rectangle.



Ans. $\because AB \parallel CD$ and EF cuts them at P and R.

$\therefore \angle APR = \angle PRD$ [alternate interior angles]

$\therefore \frac{1}{2}\angle APR = \frac{1}{2}\angle PRD$

$i.e.\ \angle 1 = \angle 2$

$\therefore PQ \parallel RS$ [alternate]

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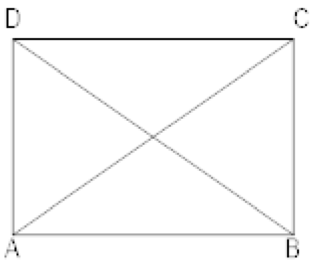
9. Prove that diagonals of a rectangle are equal in length.

Ans. ABCD is a rectangle and AC and BD are diagonals.



To prove AC = BD

Proof: $In\ \Delta\ DAB\ and\ CBA$

AD = BC [In a rectangle opposite sides are equal]

$\angle A = \angle B \quad [90^0 each]$

AB = AB common [common]

$\therefore \Delta DAB \cong \Delta CAB\ \ [By\ SAS]$

$\therefore AC = BD\ \ [By\ CPCT]$

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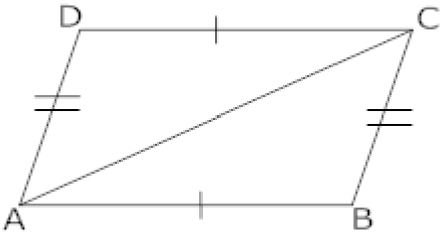
10. If each pair of opposite sides of a quadrilateral is equal, then prove that it is a parallelogram.

Ans. Given A quadrilateral ABCD in which AB = DC and AD = BC



To prove: ABCD is a parallelogram

Construction: Join AC

Proof: In ΔABC and ΔADC

AD=BC (Given)

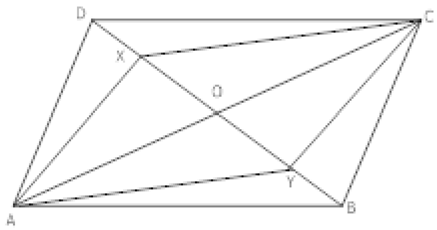
AB=DC

AC=AC [common]

$\therefore \Delta ABC \cong \Delta ADC$ [by SSS]

$\therefore \angle BAC = \angle DAC$ [By CPCT]

$\therefore ABCD$ is a parallelogram.



11.

Ans. ABCD is a parallelogram. The diagonals of a parallelogram bisect bisect each other

$\therefore OD = OB$

But $DX = BY$ [given]

$\therefore OD - DX = OB - BY$

Or OX=OY

Now in quadrilateral AYCX, the diagonals AC and XY bisect each other

$\therefore AYCX$ is a parallelogram.

In fig ABCD is a parallelogram and x, y are the points on the diagonal BD such that Dx<By show that AYCX is a parallelogram.

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12. Show that the line segments joining the mid points of opposite sides of a quadrilateral bisect each other.

Ans. Given ABCD is quadrilateral E, F, G, H are mid points of the side AB, BC, CD and DA respectively

To prove: EG and HF bisect each other.

In ΔABC, E is mid-point of AB and F is mid-point of BC

$\therefore EF \parallel AC$ And $EF = \frac{1}{2}AC ....(i)$

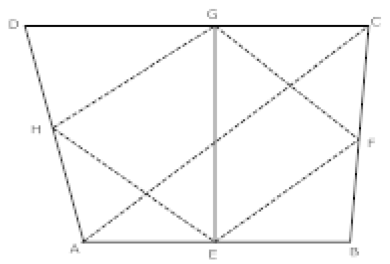
Similarly, $HG \parallel AC$ and $HG = \frac{1}{2}AC ......(ii)$

From (i) and (ii), $EF \parallel HG$ and $EF = GH$

$\therefore EFGH$ is a parallelogram and EG and HF are its diagonals

Diagonals of a parallelogram bisect each other

Thus, EG and HF bisect each other.



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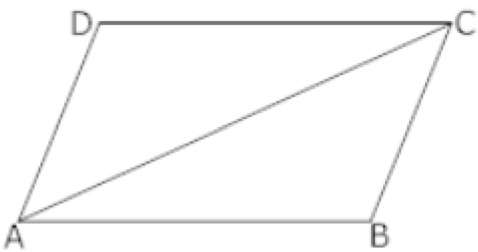
13. ABCD is a rhombus show that diagonal AC bisects $\angle A$ as well as $\angle C$ and diagonal BD bisects $\angle B$ as well as $\angle D$

Ans. ABCD is a rhombus



In ΔABC and ΔADC

$AB = AD$ [Sides of a rhombus]

$BC = DC$ [Sides of a rhombus]

$AC = AC$ [Common]

$\therefore \Delta ABC \cong \Delta ADC$ [By SSS Congruency]

$\therefore \angle CAB = \angle CAD$ And $\angle ACB = \angle ACD$

Hence AC bisects $\angle A$ as well as $\angle C$

Similarly, by joining B to D, we can prove that $\Delta ABD \cong \Delta CBD$

Hence BD bisects $\angle B$ as well as $\angle D$

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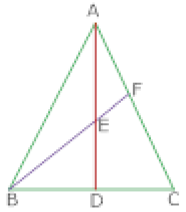
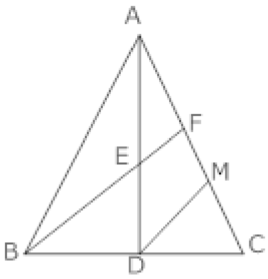
14. In fig AD is a median of $\Delta ABC, E$ is mid-Point of AD.BE produced meet AC at F. Show that $AF = \frac{1}{3}AC$



Ans. Let M is mid-Point of CF Join DM

$\therefore DM \parallel BF$.



In $\Delta ADM, E$ is mid- Point of AD and

$DM \parallel EF \Rightarrow F$ is mid-point of AM

$\therefore AF = FM$

FM=MC

$\therefore AF = FM = MC$

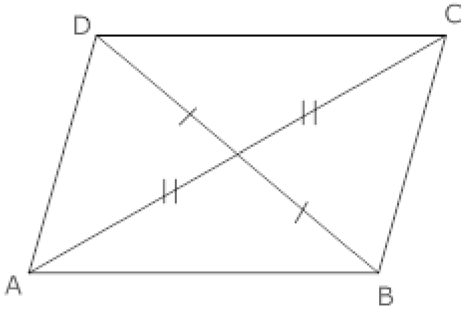
$\therefore AC = AF + FM + MC$

$= AF + AF + AF$

$AF = 3AF$

$\Rightarrow AF = \frac{1}{3}AC$

Hence Proved.



is a parallelogram if the diagonals bisect each other.

Ans. ABCD is a quadrilateral in which diagonals AC and BD intersect each other at O

In ΔAOB and ΔDOC

$OA = OC$ [Given]

$OB = OD$ [Given]

And $\angle AOB = \angle COD$ [Vertically apposite angle

$\therefore \Delta AOB \cong \Delta COD$ [By SAS]

$\therefore \angle OAB = \angle OCD$ [By C.P.C.T]

But this is Pair of alternate interior angles

$\therefore AB || CD$

$\therefore AB || CD$

Similarly AD||BC

$\therefore$Quadrilateral ABCD is a Parallelogram.

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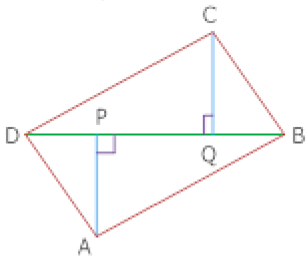
16. In fig ABCD is a Parallelogram. AP and CQ are Perpendiculars from the Vertices A and C on diagonal BD.



Show that

(i) $\Delta APB \cong \Delta CQD$

(ii) $AP = CQ$

Ans. (I) in ΔAPB and ΔCQD

AB=DC [opposite sides of a Parallelogram]

$\angle P = \angle Q$ [each 90°]

And $\angle ABP = \angle CDQ$

$\therefore \Delta APB \cong \Delta CQD$ [ASA]

(II) $\therefore AP = CQ$ (By C.P.C.T)

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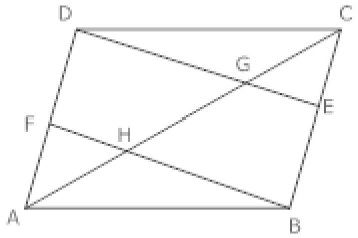
17. ABCD is a Parallelogram E and F are the mid-Points of BC and AD respectively. Show that the segments BF and DE trisect the diagonal AC.



Ans. FD||BE and FD=BE

$\therefore$BEDF Is a Parallelogram

EG||BH and E is the mid-Point of BC

$\therefore$G is the mid-point of HC

Or HG=GC…………..(i)

Similarly AH=HG………….(ii)

From (i) and (ii) we get

AH=HG=GC

Thus the segments BF and DE bisects the diagonal AC.

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18. Prove that if each pair of apposite angles of a quadrilateral is equal, then it is a parallelogram.

Ans. Given: ABCD is a quadrilateral in which $\angle A = \angle C$ and $\angle B = \angle D$

To Prove: ABCD is a parallelogram

Proof: $\angle A = \angle C$ [Given]

$\angle B = \angle D$ [Given]

$\angle A + \angle B = \angle C + \angle D ......(i)$

In quadrilateral. ABCD

$\angle A + \angle B + \angle C + \angle D = 360°$

$(\angle A + \angle B) + (\angle A + \angle B) = 360°$ [By....(i)]

$\angle A + \angle B = 180°$

$\angle A + \angle B = \angle C + \angle D = 180°$

These are sum of interior angles on the same side of transversal

$\therefore AD \parallel BC$ and $AB \parallel DC$

$\therefore$ABCD is a parallelogram**.**

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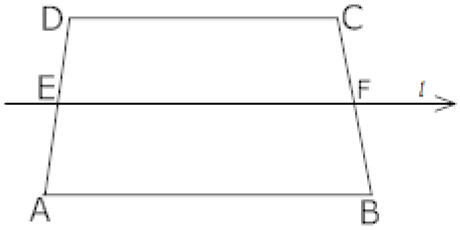
19. In Fig. ABCD is a trapezium in which AB||DC E is the mid-point of AD. A line through E is parallel to AB show that l bisects the side BC



Ans. Join AC

In ΔADC

E is mid-point of AD and EO||DC

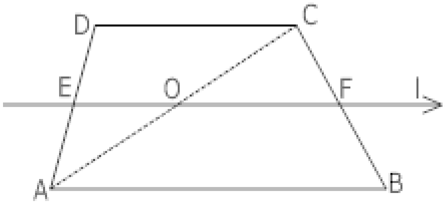
$\therefore$O is mid point of AC [A line segment joining the midpoint of one side of a Δparallel to second side and bisect the third side]

In ΔACB

O is mid point of AC

OF||AB ∴F is mid point of BC

∴l Bisect BC



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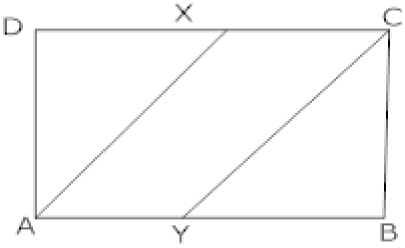
20. In Fig. ABCD is a parallelogram in which X and Y are the mid-points of the sides DC and AB respectively. Prove that AXCY is a parallelogram



Ans. In the given fig

ABCD is a parallelogram

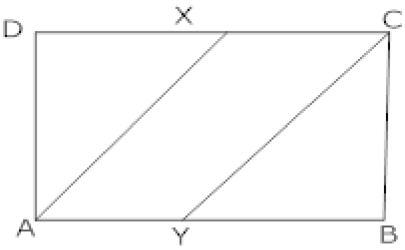
∴AB||CD and AB = CD

$\Rightarrow \frac{1}{2}AB \parallel \frac{1}{2}CD$ And $\frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow XC \parallel AY$ And $XC = AY$

[X and Y are mid-point of DC and AB respectively]

$\Rightarrow AXCY$ is a parallelogram



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21. The angles of quadrilateral are in the ratio 3:5:10:12 Find all the angles of the quadrilateral.

Ans. Suppose angles of quadrilaterals are

$3x,\ 5x,\ 10x,$ and $12x$

$\therefore \angle A = 3x,\ \angle B = 5x,\ \angle C = 10x,\ \angle D = 12x$

In a quadrilateral

$\angle A+\angle B+\angle C+\angle D=360^\circ$

$3x+5x+10x+12x=360^\circ$

30x=360

$x=\frac{360}{30}=12$

$\angle A=3\times 12=36^\circ,\ \angle B=5\times 12=60^\circ$

$\angle C=10\times 12=120^\circ,\ \angle D=12\times 12=144^\circ$

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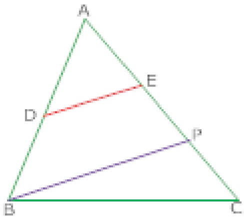
22. In fig D is mid-points of AB. P is on AC such that $PC=\frac{1}{2}AP$ and DE||BP show that $AE=\frac{1}{3}AC$



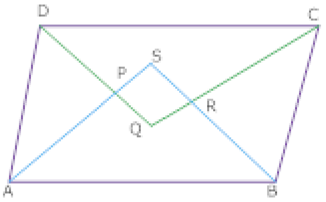
Ans. In ΔABP
D is mid points of AB and DE||BP
∴E is midpoint of AP

∴AE = EP also PC = $\frac{1}{2}$ AP

2PC = AP
2PC = 2AE
⇒PC = AE
∴AE = PE = PC
∴AC = AE + EP + PC
AC = AE + AE + AE
⇒AE = $\frac{1}{3}$ AC
Hence Proved.

---

23. Prove that the bisectors of the angles of a Parallelogram enclose a rectangle. It is given that adjacent sides of the parallelogram are unequal.



Ans. ∵ABCD is a parallelogram

$\therefore \angle A + \angle D = 180°$

or $\frac{1}{2}(\angle A + \angle D) = 90°$

Or $\angle APD = 90°$ [Sum of angle of a $\Delta 180°$]

$\therefore \angle SPQ = \angle APD = 90°$

Similarly, $\angle QRS = 90°$ and $\angle PQR = 90°$

$\angle P + \angle Q + \angle R + \angle S = 360°$

$\therefore \angle PSR = 90°$. Thus each angle of quadrilateral PQRS is $90°$

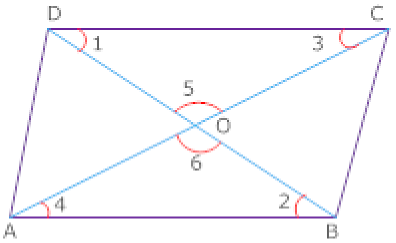
Hence PQRS is a rectangle.

---

24. Prove that a quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal

Ans. Given: ABCD is a quadrilateral in which AB||DC and BC||AD.

To Prove: ABCD is a parallelogram



Construction: Join AC and BD intersect each other at O.

Proof: $\Delta AOB \cong \Delta DOC$ [By AAA

Because $\angle 1 = \angle 2$

$\angle 3 = \angle 4$ and $\angle 5 = \angle 6$

∴AO=OC

And BO=OD

∴ ABCD is a parallelogram

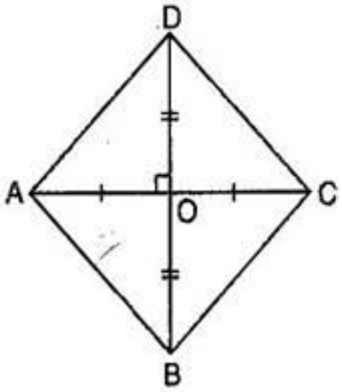
∵Diagonals of a parallelogram bisect each other.

3 Marks Questions

1. Show that is diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Ans. Given: Let ABCD is a quadrilateral.

Let its diagonal AC and BD bisect each other at right angle at point O.



∴OA = OC, OB = OD

And ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°

To prove: ABCD is a rhombus.

Proof: In △AOD and △BOC,

OA = OC[Given]

∠AOD = ∠BOC[Given]

OB = OD[Given]

∴△AOD≅△COB [By SAS congruency]

⇒AD = CB [By C.P.C.T.]……….(i)

Again, In △AOB and △COD,

OA = OC[Given]

∠AOB = ∠COD[Given]

OB = OD[Given]

∴△AOB≅△COD [By SAS congruency]

⇒AD = CB[By C.P.C.T.]……….(ii)

Now In △AOD and △BOC,

OA = OC[Given]

∠AOB = ∠BOC[Given]

OB = OB[Common]

∴△AOB≅△COB [By SAS congruency]

⇒AB = BC [By C.P.C.T.]……….(iii)

From eq. (i), (ii) and (iii),

AD = BC = CD = AB

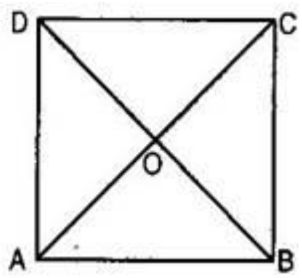
And the diagonals of quadrilateral ABCD bisect each other at right angle.

Therefore, ABCD is a rhombus.

---

2. Show that the diagonals of a square are equal and bisect each other at right angles.

Ans. Given: ABCD is a square. AC and BD are its diagonals bisect each other at point O.



To prove: AC = BD and AC $\perp$ BD at point O.

Proof: In triangles ABC and BAD,

AB = AB[Common]

$\angle$ABC = $\angle$BAD = 90°

BC = AD [Sides of a square]

$\therefore \Delta$ABC $\cong \Delta$BAD [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.] Hence proved.

Now in triangles AOB and AOD,

AO = AO[Common]

AB = AD[Sides of a square]

OB = OD[Diagonals of a square bisect each other]

$\therefore \Delta$AOB$\cong \Delta$AOD[By SSS congruency]

$\angle$AOB = $\angle$AOD[By C.P.C.T.]

But $\angle$AOB + $\angle$AOD = 180° [Linear pair]

$\therefore \angle$AOB = $\angle$AOD = 90°

$\Rightarrow$ OA $\perp$ BD or AC $\perp$ BD

Hence proved.

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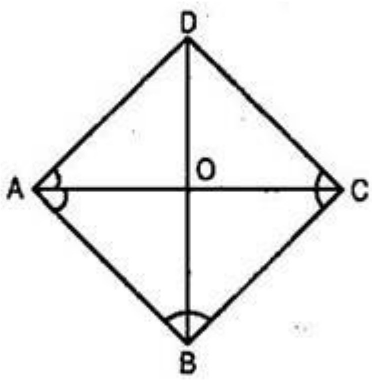
3. ABCD is a rhombus. Show that the diagonal AC bisects $\angle$A as well as $\angle$C and diagonal BD bisects $\angle$B as well as $\angle$D.



Ans. ABCD is a rhombus. Therefore, AB = BC = CD = AD

Let O be the point of bisection of diagonals.

$\therefore$OA = OC and OB = OD

In ΔAOB and ΔAOD,

OA = OA[Common]

AB = AD[Equal sides of rhombus]

OB = OD(diagonals of rhombus bisect each other]

$\therefore \Delta AOB \cong \Delta AOD$[By SSS congruency]

$\Rightarrow \angle$OAD = $\angle$OAB[By C.P.C.T.]

$\Rightarrow$OA bisects $\angle$A..........(i)

Similarly $\Delta BOC \cong \Delta DOC$[By SSS congruency]

$\Rightarrow \angle$OCB = $\angle$OCD[By C.P.C.T.]

$\Rightarrow$OC bisects $\angle$C..........(ii)

From eq. (i) and (ii), we can say that diagonal AC bisects $\angle$A and $\angle$C.

Now in ΔAOB and ΔBOC,

OB = OB[Common]

AB = BC[Equal sides of rhombus]

OA = OC(diagonals of rhombus bisect each other]

$\therefore \triangle AOB \cong \triangle COB$[By SSS congruency]

$\Rightarrow \angle OBA = \angle OBC$[By C.P.C.T.]

$\Rightarrow$ OB bisects $\angle B$……….(iii)

Similarly $\triangle AOD \cong \triangle COD$[By SSS congruency]

$\Rightarrow \angle ODA = \angle ODC$[By C.P.C.T.]

$\Rightarrow$ BD bisects $\angle D$……….(iv)

From eq. (iii) and (iv), we can say that diagonal BD bisects $\angle B$ and $\angle D$

---

4. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (See figure). Show that:

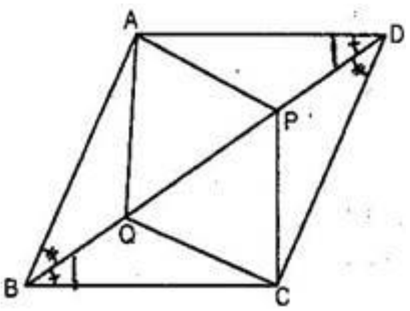
(i) $\triangle APD \cong \triangle CQB$

(ii) AP = CQ

(iii) $\triangle AQB \cong \triangle CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram.



Ans. (i) In $\triangle APD$ and $\triangle CQB$,

DP = BQ[Given]

$\angle ADP = \angle QBC$[Alternate angles (AD $\parallel$ BC and BD is transversal)]

AD = CB[Opposite sides of parallelogram]

$\therefore \triangle APD \cong \triangle CQB$[By SAS congruency]

(ii) Since $\triangle APD \cong \triangle CQB$

$\Rightarrow$ AP = CQ[By C.P.C.T.]

(iii) In ΔAQB and ΔCPD,

BQ = DP[Given]

$\angle$ABQ = $\angle$PDC[Alternate angles (AB $\parallel$ CD and BD is transversal)]

AB = CD[Opposite sides of parallelogram]

$\therefore \Delta$AQB $\cong \Delta$CPD[By SAS congruency]

(iv) Since ΔAQB $\cong \Delta$CPD

$\Rightarrow$ AQ = CP[By C.P.C.T.]

(v) In quadrilateral APCQ,

AP = CQ[proved in part (i)]

AQ = CP[proved in part (iv)]

Since opposite sides of quadrilateral APCQ are equal.

Hence APCQ is a parallelogram.

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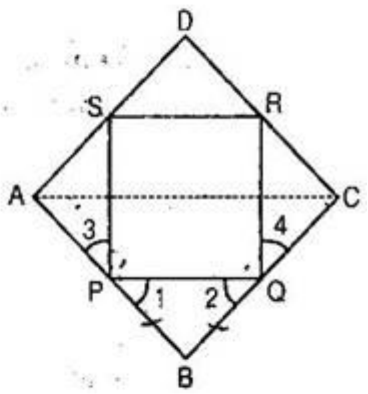
5. ABCD is a rhombus and P, Q, R, S are mid-points of AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rectangle.



Ans. Given: P, Q, R and S are the mid-points of respective sides AB, BC, CD and DA of rhombus. PQ, QR, RS and SP are joined.

To prove: PQRS is a rectangle.

Construction: Join A and C.

Proof: In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore$PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC ....(i)

In ΔADC, R is the mid-point of CD and S is the mid-point of AD.

$\therefore$SR $\parallel$ AC and SR = $\frac{1}{2}$ AC..........(ii)

From eq. (i) and (ii),PQ $\parallel$ SR and PQ = SR

$\therefore$PQRS is a parallelogram.

Now ABCD is a rhombus. [Given]

$\therefore$AB = BC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ BC$\Rightarrow$PB = BQ

$\therefore \angle$1 = $\angle$2[Angles opposite to equal sides are equal]

Now in triangles APS and CQR, we have,

AP = CQ[P and Q are the mid-points of AB and BC and AB = BC]

Similarly AS = CR and PS = QR[Opposite sides of a parallelogram]

$\therefore \Delta$APS $\cong \Delta$CQR[By SSS congreuancy]

$\Rightarrow \angle$3 = $\angle$4[By C.P.C.T.]

Now we have $\angle$1 + $\angle$SPQ + $\angle$3 = 180°

And $\angle$2 + $\angle$PQR + $\angle$4 = 180° [Linear pairs]

$\therefore \angle$1 + $\angle$SPQ + $\angle$3 = $\angle$2 + $\angle$PQR + $\angle$4

Since $\angle$1 = $\angle$2 and $\angle$3 = $\angle$4[Proved above]

$\therefore \angle$SPQ = $\angle$PQR..........(iii)

Now PQRS is a parallelogram [Proved above]

$\therefore \angle$SPQ + $\angle$PQR = 180° ....(iv)[Interior angles]

Using eq. (iii) and (iv),

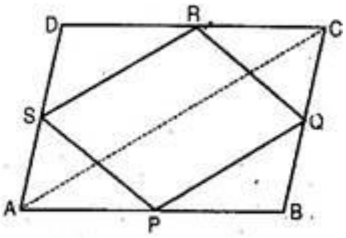
$\angle$SPQ + $\angle$SPQ = $180^\circ \Rightarrow 2\angle$SPQ = 180°

$\Rightarrow \angle$SPQ = 90°

Hence PQRS is a rectangle.

6. ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Ans. Given: A rectangle ABCD in which P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To prove: PQRS is a rhombus.

Construction: Join AC.

Proof: In $\triangle$ABC, P and Q are the mid-points of sides AB, BC respectively.

$\therefore$PQ $\parallel$ AC and PQ = $\frac{1}{2}$ AC……….(i)

In $\triangle$ADC, R and S are the mid-points of sides CD, AD respectively.

$\therefore$SR $\parallel$ AC and SR = $\frac{1}{2}$ AC……….(ii)

From eq. (i) and (ii), PQ $\parallel$ SR and PQ = SR……….(iii)

$\therefore$PQRS is a parallelogram.

Now ABCD is a rectangle.[Given]

$\therefore$AD = BC

$\Rightarrow \frac{1}{2}$ AD = $\frac{1}{2}$ BC$\Rightarrow$AS = BQ……….(iv)

In triangles APS and BPQ,

AP = BP[P is the mid-point of AB]

$\angle$PAS = $\angle$PBQ[Each 90°]

And AS = BQ[From eq. (iv)]

$\therefore\triangle$ APS $\cong\triangle$BPQ[By SAS congruency]

$\Rightarrow$PS = PQ[By C.P.C.T.]………(v)

From eq. (iii) and (v), we get that PQRS is a parallelogram.

$\Rightarrow$ PS = PQ

$\Rightarrow$ Two adjacent sides are equal.

Hence, PQRS is a rhombus.

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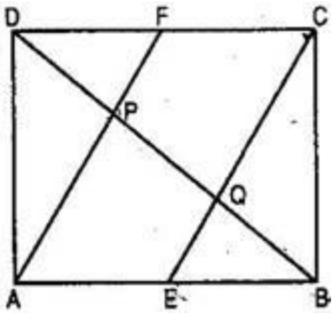
7. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (See figure). Show that the line segments AF and EC trisect the diagonal BD.



Ans. Since E and F are the mid-points of AB and CD respectively.

$\therefore$ AE = $\frac{1}{2}$ AB and CF = $\frac{1}{2}$ CD……….(i)

But ABCD is a parallelogram.

$\therefore$ AB = CD and AB $\parallel$ DC

$\Rightarrow \frac{1}{2}$ AB = $\frac{1}{2}$ CD and AB $\parallel$ DC

$\Rightarrow$ AE = FC and AE $\parallel$ FC[From eq. (i)]

$\therefore$ AECF is a parallelogram.

$\Rightarrow$ FA $\parallel$ CE $\Rightarrow$ FP $\parallel$ CQ[FP is a part of FA and CQ is a part of CE] ………(ii)

Since the segment drawn through the mid-point of one side of a triangle and parallel to the other side bisects the third side.

In ΔDCQ, F is the mid-point of CD and $\Rightarrow$ FP $\parallel$ CQ

$\therefore$ P is the mid-point of DQ.

$\Rightarrow$ DP = PQ……….(iii)

Similarly, In ΔABP, E is the mid-point of AB and $\Rightarrow$ EQ $\parallel$ AP

$\therefore$ Q is the mid-point of BP.

⇒ BQ = PQ……….(iv)

From eq. (iii) and (iv),

DP = PQ = BQ………(v)

Now BD = BQ + PQ + DP = BQ + BQ + BQ = 3BQ

⇒ BQ = $\frac{1}{3}$ BD……….(vi)

From eq. (v) and (vi),

DP = PQ = BQ = $\frac{1}{3}$ BD

⇒ Points P and Q trisects BD.

So AF and CE trisects BD.

---

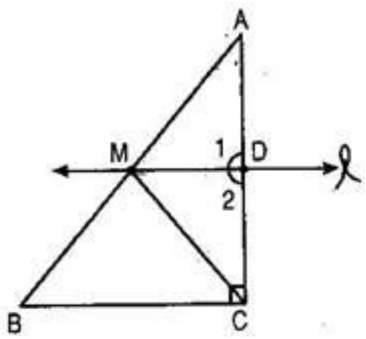
8. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D.

Ans. (i) In $\triangle$ABC, M is the mid-point of AB[Given]

MD $\parallel$ BC

$\therefore$ AD = DC[Converse of mid-point theorem]

Thus D is the mid-point of AC.



(ii) $l \parallel$ BC (given) consider AC as a transversal.

$\therefore \angle$1 = $\angle$C[Corresponding angles]

⇒ $\angle$1 = 90° [$\angle$C = 90°]

Thus MD $\perp$ AC.

(iii) In $\triangle$AMD and $\triangle$CMD, AD = DC[proved above]

$\angle 1 = \angle 2 = 90^\circ$ [proved above]

MD = MD[common]

$\therefore \Delta$ AMD $\cong \Delta$ CMD[By SAS congruency]

$\Rightarrow$ AM = CM[By C.P.C.T.]……….(i)

Given that M is the mid-point of AB.

$\therefore$ AM = $\frac{1}{2}$ AB……….(ii)

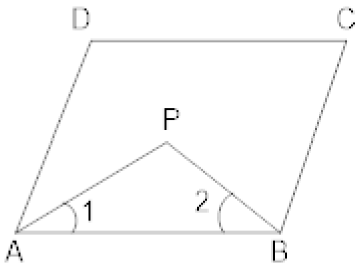
From eq. (i) and (ii),

CM = AM = $\frac{1}{2}$ AB

---

9. In a parallelogram ABCD, bisectors of adjacent angles A and B intersect each other at P. prove that $\angle APB = 90^0$

Ans. Given ABCD is a parallelogram is and bisectors of $\angle A$ and $\angle B$ intersect each other at P.



To prove $\angle APB = 90^0$

Proof:

$$\angle 1 + \angle 2 = \frac{1}{2}\angle A + \frac{1}{2}\angle B$$

$$= \frac{1}{2}(\angle A + \angle B) \quad \rightarrow (i)$$

But ABCD is a parallelogram and AD $\parallel$ BC

$$\therefore \angle A + \angle B = 180^0$$

$$\therefore \angle 1 + \angle 2 = \frac{1}{2} \times 180^0 = 90^0$$

In ΔAPB

$\angle 1+\angle 2+\angle APB=180^0$

$90^0+\angle APB=180^0$

$\angle APB=90^0$

Hence Proved

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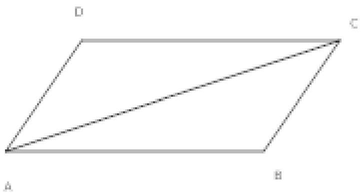
10. In figure diagonal AC of parallelogram ABCD bisects $\angle A$ show that

(i) if bisects $\angle C$

ABCD is a rhombus



Ans.(i) AB $\parallel$ DC and AC is transversal

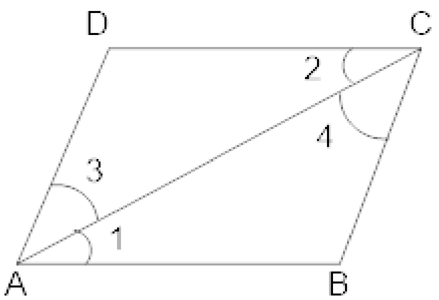
$\therefore \angle 1=\angle 2$ (Alternate angles)

And $\angle 3=\angle 4$ (Alternate angles)

But, $\angle 1=\angle 3$

$\therefore \angle 2=\angle 4$

$\therefore AC$ *bisects* $\angle C$



(ii) *In* ΔABC *and* ΔADC

AC=AC [common]

$\angle 1=\angle 3$ [given]

$\angle 2=\angle 4$ [proved]

$\therefore \Delta ABC \cong \Delta ADC$

$\therefore AB = AD$ [By CPCT]

$\therefore ABCD$ *is a* rhombus

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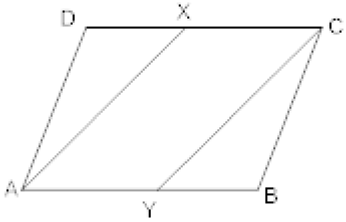
11. In figure ABCD is a parallelogram. AX and CY bisects angles A and C. prove that AYCX is a parallelogram.



Ans. Given in a parallelogram AX and CY bisects $\angle A$ *and* $\angle C$ respectively and we have to show that AYCX in a parallelogram.

In ΔADX *and* ΔCBY

$\angle D = \angle B$ ...(i) [opposite angles of parallelogram]

$\angle DAX = \frac{1}{2}\angle A$ [Given] ...(ii)

And $\angle BCY = \frac{1}{2}\angle C$ [give] .....(iii)

But $\angle A = \angle C$

$\therefore$By (2) and (3), we get

$\angle DAX = \angle BCY \rightarrow (iv)$

Also, $AD = BC$ [opposite sides of parallelogram] ....(v)

$\therefore$ From (i), (iv) and (v), we get

$\Delta ADX \cong \Delta CBY \quad [By\ ASA]$

$\therefore DX = BY \quad [CPCT]$

But, AB =CD [opposite sides of parallelogram]

AB-BY=CD-DX

Or

Ay=CX

But $AY \parallel XC$ $[\because ABCD \text{ is a } \parallel gm]$

$\therefore AYCX$ is a parallelogram

---

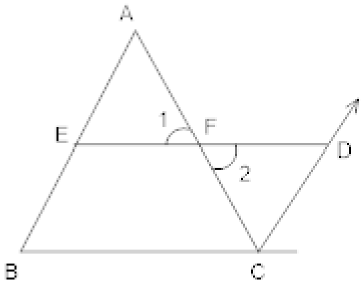
12. Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

Ans. Given ΔABC in which E and F are mid points of side AB and AC respectively.

To prove: EF||BC

Construction: Produce EF to D such that EF = FD. Join CD

Proof: $In\ \Delta AEF\ and\ \Delta CDF$



AF=FC$[\because F\ is\ mid-point\ of\ AC]$
$\angle 1 = \angle 2$ [vertically opposite angles]
EF=FD [By construction]
$\therefore \Delta AEF \cong \Delta CDF \quad [By\ SAS]$
And $\therefore AE = CD \quad [By\ CPCT]$
AE= BE[$\because E$ is the mid-point]
And $\therefore BE = cd$
$AB \parallel CD\ [\therefore \angle BAC = \angle ACD]$
$\therefore BCDE\ is\ a\ parallelogram$
$EF \parallel BC \quad Henceproved$

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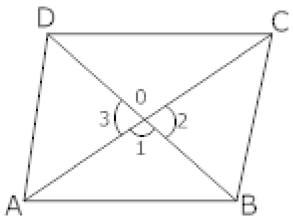
13. Prove that a quadrilateral is a rhombus if its diagonals bisect each other at right angles.



Ans. Given ABCD is a quadrilateral diagonals AC and BD bisect each other at O at right angles

<u>To Prove</u>: ABCD is a rhombus

<u>Proof:</u> $\because$diagonals AC and BD bisect each other at O

$\therefore OA = OC, OB = OD$ And $\angle 1 = \angle 2 = \angle 3 = 90°$

Now In ΔBOA And ΔBOC

$OA = OC$ Given

$OB = OB$ [Common]

And $\angle 1 = \angle 2 = 90^\circ$ (Given)

$\therefore \Delta BOA = \Delta BOC$ (SAS)

$\therefore BA = BC$ (C.P.C.T.)

Similarly, BC=CD, CD=DA and DA=AB,

Hence, ABCD is a rhombus.

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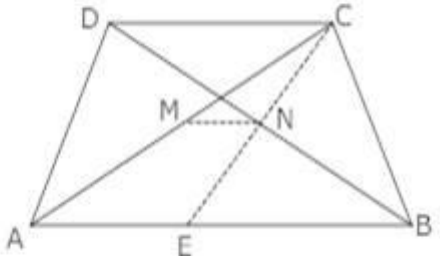
14. Prove that the straight line joining the mid points of the diagonals of a trapezium is parallel to the parallel sides.



Ans. Given a trapezium ABCD in which $AB \parallel DC$ and M,N are the mid Points of the diagonals AC and BD.

We need to prove that $MN \parallel AB \parallel DC$

Join CN and let it meet AB at E

Now in ΔCDN and ΔEBN

$\angle DCN = \angle BEN$ [Alternate angles]

$\angle CDN = \angle BEN$ [Alternate angles]

And $DN = BN$ [given]

$\therefore \Delta CDN \cong \Delta EBN$ [ASA]

$\therefore CN = EN$ [By C.P.C.T]

Now in $\Delta ACE, M$ and N are the mid points of the sides AC and CE respectively.

$\therefore MN \parallel AE$ Or $MN \parallel AB$

Also $AB \parallel DC$

$\therefore MN \parallel AB \parallel DC$

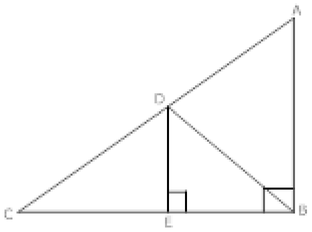
---

15. In fig $\angle B$ is a right angle in $\Delta ABC.D$ is the mid-point of $AC.DE \parallel AB$ intersects BC at E. show that

(i) E is the mid-point of BC

(ii) DE$\perp$ BC

(ii) BD = AD



Ans. Proof: $\because DE \parallel AB$ and D is mid points of AC

In ΔDCE and ΔDBE

CE=BE

DE= DE

And $\angle DEC = \angle DEB = 90°$

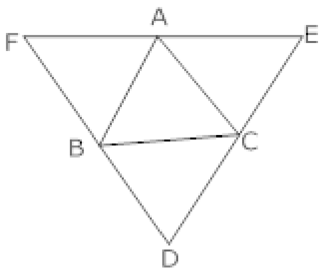
$\therefore \Delta DCE = \Delta DBE$

$\therefore \Delta DCE \cong \Delta DBE$

$\therefore CD = BD$

---

16. ABC is a triangle and through vertices A, B and C lines are drawn parallel to BC, AC and AB respectively intersecting at D, E and F. prove that perimeter of ΔDEF is double the perimeter of ΔABC.



Ans. $\because BCAF$ Is a parallelogram

$\therefore BC = AF$

$\because ABCE$ Is a parallelogram

$\therefore BC = AE$

$AF + AE = 2BC$

Or $EF = 2BC$

Similarly, ED = 2AB and FD = 2AC

$\therefore$Perimeter of $\Delta ABC = AB + BC + AC$

Perimeter of $\Delta DEF = DE + EF + DF$

= 2AB+2BC+2AC

= 2[AB+BC+AC]

= 2 Perimeter of ΔABC

Hence Proved.

---

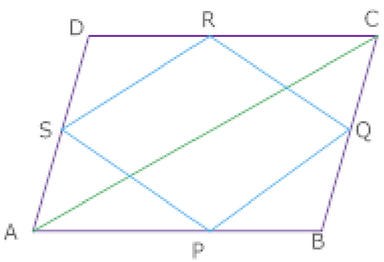
17. In fig ABCD is a quadrilateral P, Q, R and S are the mid Points of the sides AB, BC, CD and DA, AC is diagonal. Show that

(i) SR||AC

(ii) PQ=SR

(iii) PQRS is a parallelogram

(iv) PR and SQ bisect each other



Ans. In ΔABC, P and Q are the mid-points of the sides AB and BC respectively

(i) $\therefore$PQ||AC and PQ=$\frac{1}{2}$ AC

(ii) Similarly SR||AC and SR=$\frac{1}{2}$ AC

$\therefore$PQ||SR and PQ=SR

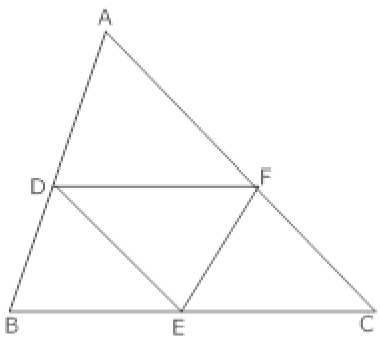
(iii) Hence PQRS is a Parallelogram.

(iv) PR and SQ bisect each other.

---

18. In $\Delta ABC, D, E, F$ are respectively the mid-Points of sides AB,DC and CA. show that ΔABC is divided into four congruent triangles by Joining D,E,F.

Ans. D and E are mid-Points of sides AB and BC of ΔABC



$\therefore$DE||AC {$\because$A line segment joining the mid-Point of any two sides of a triangle parallel to third side}

Similarly, DF||BC and EF||AB

$\therefore$ADEF, BDEF and DFCE are all Parallelograms.

DE is diagonal of Parallelogram BDFE

$\therefore \Delta BDE \cong \Delta FED$

Similarly, ΔDAF$\cong\Delta$FED

And ΔEFC$\cong\Delta$FED

So all triangles are congruent

---

19. ABCD is a Parallelogram is which P and Q are mid-points of opposite sides AB and CD. If AQ intersect DP at S BQ intersects CP at R, show that

(i) APCQ is a Parallelogram

(ii) DPBQ is a parallelogram

(iv) PSQR is a parallelogram

Ans. (i) In quadrilateral APCQ

AP||QC [∵AB||CD]........(i)

AP=$\frac{1}{2}$ AB, CQ=$\frac{1}{2}$ CD (Given)

Also AB= CD

So AP=QC..........(ii)

Therefore, APCQ is a parallelogram

[It any two sides of a quadrilateral equal and parallel then quad is a parallelogram]

(ii) Similarly, quadrilateral DPBQ is a Parallelogram because DQ||PB and DQ=PB

(iii) In quadrilateral PSQR,

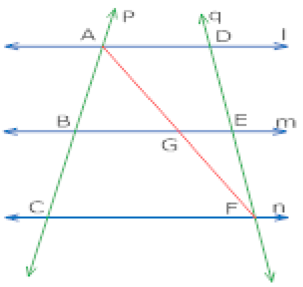
SP||QR [SP is a part of DP and QR is a Part of QB]

Similarly, SQ||PR

So. PSQR is also parallelogram.

---

20. l, m, n **are three parallel lines intersected by transversals P and q such that** l, m **and** n **cut off equal intercepts AB and BC on P In fig Show that** l, m, n **cut off equal intercepts DE and EF on q also.**



Ans. In fig l, m, n are 3 parallel lines intersected by two transversal P and Q.

To Prove DE=EF

Proof: In ΔACF

B is mid-point of AC

And BG||CF

$\therefore$G is mid-point of AF [By mid-point theorem]

Now In ΔAFD

G is mid-point of AF and GE || AD

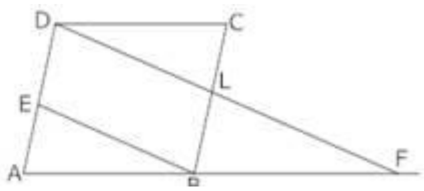
$\therefore$E is mid-point of FD [By mid-point theorem]

$\therefore$ DE=EF

Hence Proved.

---

21. ABCD is a parallelogram in which E is mid-point of AD. DF||EB meeting AB produced at F and BC at L prove that DF = 2DL



Ans. In ΔAFD

$\because E$ is mid-point of AD (Given)

BE||DF (Given)

$\therefore$By converse of mid-point theorem B is mid-point of AF

$\therefore AB = BF ....(i)$

ABCD is parallelogram

$\therefore AB = CD......(ii)$

From (i) and (ii)

CD = BF

Consider ΔDLC and ΔFLB

DC = FB [Proved above]

$\angle DCL = \angle FBL$ [Alternate angles]

$\angle DLC = \angle FLB$ [Vertically opposite angles]

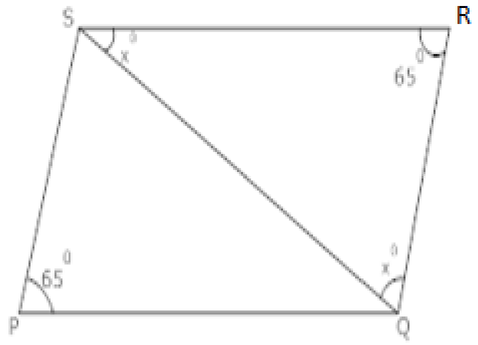
$\Delta DLC = \Delta FLB$ [ASA]

$\therefore$DL=LF

$\therefore$DF=2DL

---

22. PQRS is a rhombus if $\angle P = 65°$ find $\angle RSQ$



Ans. $\angle R = \angle P = 65°$ [opposite angles of a parallelogram are equal]

Let $\angle RSQ = x°$

In ΔRSQ we have RS=RQ

$\angle RQS = \angle RSQ = x°$ [opposite Sides of equal angles are equal]

In ΔRSQ

$\angle S + \angle Q + \angle R = 180°$ [By angle sum property]

$x° + x° + 65° = 180°$

$2x° = 180° - 65°$

$2x° = 115°$

$x = \frac{115}{2} = 57.5°$

$\therefore \angle RSQ = 57.5°$

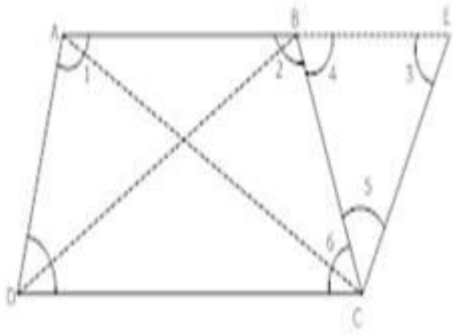
---

23. ABCD is a trapezium in which AB||CD and AD = BC show that

(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$



Ans. Produce AB and Draw a line Parallel to DA meeting at E

$\because$AD||EC

$\angle 1+\angle 3=180°$ ....(i) [Sum of interior angles on the some side of transversal is $180°$]

In ΔBEC

BC=CE (given)

$\therefore \angle 3=\angle 4$ .....(2) [in a Δequal side to opposite angles are equal]

$\angle 2+\angle 4=180°$ ......(3)

By (i) and (3)

$\angle 1+\angle 3=\angle 2+\angle 4$

$\angle 3=\angle 4$

$\therefore \angle 1=\angle 2$

(i) $\therefore \angle A=\angle B$

(ii) $\because AD \parallel EC$

$\angle D+\angle 6+\angle 5=180°......(i)$

$AE \parallel DC$

$\angle 6+\angle 5+\angle 3=180°........(ii)$

$\angle D+\angle 6+\angle 5=\angle 6+\angle 5+\angle 3$

$\angle D=\angle 3=\angle 4$

(iii) In ΔABC and ΔBAD

AB=AB [common]

$\angle 1=\angle 2$ [Proved above]

AD=BC [given]

$\therefore \Delta ABC \cong \Delta BAD$ [By SAS]

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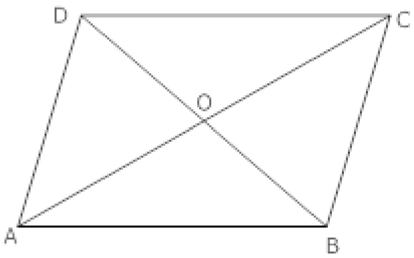
24. Show that diagonals of a rhombus are perpendicular to each other.

Ans. Given: A rhombus ABCD whose diagonals AC and BD intersect at a Point O



To Prove: $\angle BOC = \angle DOC = \angle AOD = \angle AOB = 90^\circ$

Proof: clearly ABCD is a Parallelogram in which

AB=BC=CD=DA

We know that diagonals of a Parallelogram bisect each other

$\therefore$OA=OC and OB=OD

Now in ΔBOC and ΔDOC, we have

OB=OD

BC=DC

OC=OC

$\therefore \Delta$BOC $\cong \Delta$DOC [By SSS]

$\therefore \angle BOC = \angle DOC$ [By C.P.C.T]

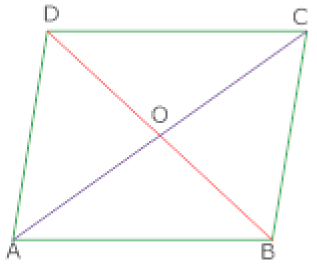
But $\angle BOC + \angle DOC = 180^\circ \therefore \angle BOC = \angle DOC = 90^\circ$

Similarly, $\angle AOB = \angle AOD = 90^\circ$

Hence diagonals of a rhombus bisect each other at 90°

---

25. Prove that the diagonals of a rhombus bisect each other at right angles



Ans. We are given a rhombus ABCD whose diagonals AC and BD intersect each other at O.

We need to prove that OA=OC, OB=OD and $\angle AOB = 90°$

In ΔAOB and ΔCOD

AB=CD [Sides of rhombus]

$\angle AOB = \angle COD$ [vertically opposite angles]

And $\angle ABO = \angle CDO$ [Alternate angles]

$\therefore \Delta AOB \cong \Delta COD$ [By ASA]

$\therefore$ OA=OC

And OB=OD [By C.P.C.T]

Also in ΔAOB and ΔCOB

OA=OC [Proved]

AB=CB [sides of rhombus]

And OB=OB [Common]

$\therefore \Delta AOB \cong \Delta COB$ [By SSS]

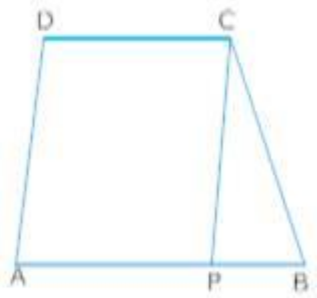
$\therefore \angle AOB = \angle COB$ [By C.P.C.T]

But $\angle AOB + \angle COB = 180°$ [linear pair]

$\therefore \angle AOB = \angle COB = 90°$

---

26. In fig ABCD is a trapezium in which AB||DC and AD=BC. Show that $\angle A = \angle B$



Ans. To show that $\angle A = \angle B$,

Draw CP||DA meeting AB at P

∵AP||DC and CP||DA

∴APCD is a parallelogram

Again in Δ CPB

CP=CB [∵BC=AD [Given]

$\angle CPB = \angle CBP ...(i)$ [Angles opposite to equal sides]

But $\angle CPA + \angle CPB = 180°$ [By linear pair]

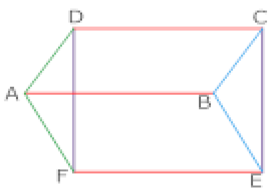
Also $\angle A + \angle CPA = 180°$ [∵APCD is a parallelogram]

∴ $\angle A + \angle CPA = \angle CPA + \angle CPB$ Or $\angle A = \angle CPB$

= $\angle$ CB

---

27. In fig ABCD and ABEF are Parallelogram, prove that CDFE is also a parallelogram.



Ans. ∵ABCD is a parallelogram

∴ AB=DC also AB||DC………….(i)

Also ABEF is a parallelogram

∴AB=FE and AB||FE……….(ii)

By (i) and (ii)

AB=DC=FE

∴AB=FE

And AB||DC||FE

∴ AB||FE

∴CDEF is a parallelogram.

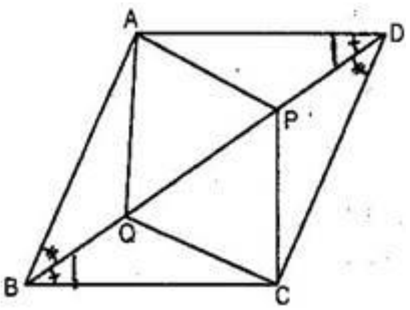
Hence Proved.

4 Marks Questions

1. ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that:

(i) ABCD is a square.

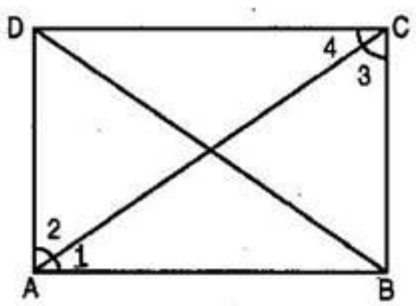
(ii) Diagonal BD bisects both ∠B as well as ∠D.



Ans. ABCD is a rectangle. Therefore AB = DC ……….(i)

And BC = AD

Also ∠A = ∠B = ∠C = ∠D = 90°



(i) In ΔABC and ΔADC

∠1 = ∠2 and ∠3 = ∠4

[AC bisects ∠A and ∠C (given)]

AC = AC [Common]

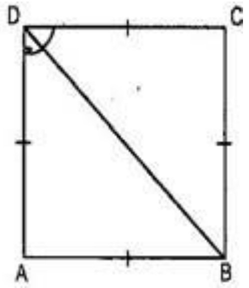
∴ ΔABC ≅ ΔADC [By ASA congruency]

⇒ AB = AD ……….(ii)

From eq. (i) and (ii), AB = BC = CD = AD

Hence ABCD is a square.

(ii) In $\triangle$ABC and $\triangle$ADC



AB = BA [Since ABCD is a square]

AD = DC [Since ABCD is a square]

BD = BD [Common]

$\therefore$ $\triangle$ABD $\cong$ $\triangle$CBD [By SSS congruency]

$\Rightarrow$ $\angle$ABD = $\angle$CBD [By C.P.C.T.] ....(iii)

And $\angle$ADB = $\angle$CDB [By C.P.C.T.] ....(iv)

From eq. (iii) and (iv), it is clear that diagonal BD bisects both $\angle$B and $\angle$D.

---

2. An $\triangle$ABC and $\triangle$DEF, AB = DE, AB $\parallel$ DE, BC = EF and BC $\parallel$ EF. Vertices A, B and C are joined to vertices D, E and F respectively (See figure). Show that:

(i) Quadrilateral ABED is a parallelogram.

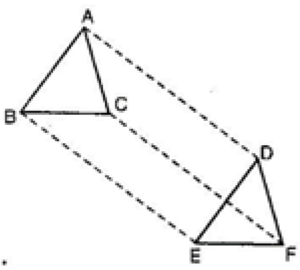
(ii) Quadrilateral BEFC is a parallelogram.

(iii) AD $\parallel$ CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram.

(v) AC = DF

(vi) $\triangle$ABC $\cong$ $\triangle$DEF



Ans. (i) In ΔABC and ΔDEF

AB = DE [Given]

And AB $\parallel$ DE [Given]

$\therefore$ ABED is a parallelogram.

(ii) In ΔABC and ΔDEF

BC = EF [Given]

And BC $\parallel$ EF [Given]

$\therefore$ BEFC is a parallelogram.

(iii) As ABED is a parallelogram.

$\therefore$ AD $\parallel$ BE and AD = BE ……….(i)

Also BEFC is a parallelogram.

$\therefore$ CF $\parallel$ BE and CF = BE ……….(ii)

From (i) and (ii), we get

$\therefore$ AD $\parallel$ CF and AD = CF

(iv) As AD $\parallel$ CF and AD = CF

$\Rightarrow$ ACFD is a parallelogram.

(v) As ACFD is a parallelogram.

$\therefore$ AC = DF

(vi) In ΔABC and ΔDEF,

AB = DE [Given]

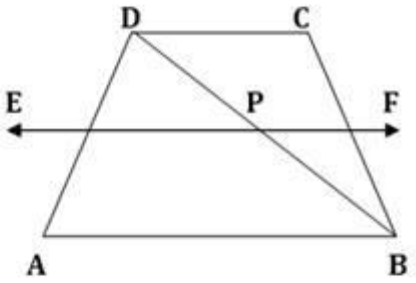
BC = EF [Given]

AC = DF [Proved]

$\therefore$ ΔABC $\cong$ ΔDEF [By SSS congruency]

---

3. ABCD is a trapezium, in which AB $\parallel$ DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E, parallel to AB intersecting BC at F (See figure). Show that F is the mid-point of BC.



Ans. Let diagonal BD intersect line EF at point P.

In $\triangle$DAB,

E is the mid-point of AD and EP $\parallel$ AB [$\because$EF$\parallel$ AB (given) P is the part of EF]

$\therefore$ P is the mid-point of other side, BD of $\triangle$DAB.

[A line drawn through the mid-point of one side of a triangle, parallel to another side intersects the third side at the mid-point]

Now in $\triangle$BCD,

P is the mid-point of BD and PF $\parallel$ DC [$\because$EF$\parallel$ AB (given) and AB $\parallel$ DC (given)]

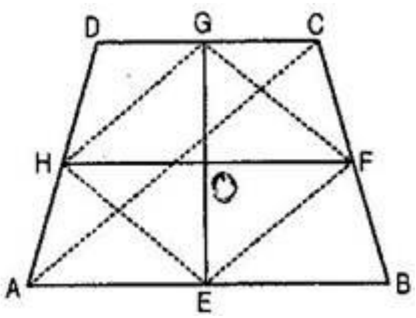
$\therefore$ EF$\parallel$ DC and PF is a part of EF.

$\therefore$ F is the mid-point of other side, BC of $\triangle$BCD. [Converse of mid-point of theorem]

---

4. Show that the line segments joining the mid-points of opposite sides of a quadrilateral bisect each other.

Ans. Given: A quadrilateral ABCD in which EG and FH are the line-segments joining the mid-points of opposite sides of a quadrilateral.



To prove: EG and FH bisect each other.

Construction: Join AC, EF, FG, GH and HE.

Proof: In $\triangle$ABC, E and F are the mid-points of respective sides AB and BC.

$\therefore$ EF $\parallel$ AC and EF $\frac{1}{2}$ AC ……….(i)

Similarly, in $\triangle$ADC,

G and H are the mid-points of respective sides CD and AD.

$\therefore$ HG $\parallel$ AC and HG $\frac{1}{2}$ AC ……….(ii)

From eq. (i) and (ii),

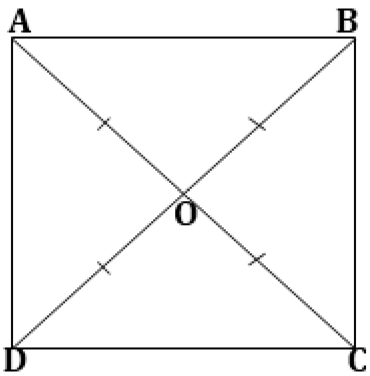
EF $\parallel$ HG and EF = HG

$\therefore$ EFGH is a parallelogram.

Since the diagonals of a parallelogram bisect each other, therefore line segments (i.e. diagonals) EG and FH (of parallelogram EFGH) bisect each other.

---

5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.



Ans. Let ABCD be a quadrilateral in which equal diagonals AC and BD bisect each other at right angle at point O.

We have AC = BD and OA = OC ……….(i)

And OB = OD ……….(ii)

Now OA + OC = OB + OD

$\Rightarrow$ OC + OC = OB + OB [Using (i) & (ii)]

$\Rightarrow$ 2OC = 2OB

$\Rightarrow$ OC = OB ……….(iii)

From eq. (i), (ii) and (iii), we get, OA = OB = OC = OD ……….(iv)

Now in ΔAOB and ΔCOD,

OA = OD [proved]

$\angle$AOB = $\angle$COD [vertically opposite angles]

OB = OC [proved]

$\therefore$ ΔAOB $\cong$ ΔDOC [By SAS congruency]

$\Rightarrow$ AB = DC [By C.P.C.T.] ……….(v)

Similarly, ΔBOC $\cong$ ΔAOD [By SAS congruency]

$\Rightarrow$ BC = AD [By C.P.C.T.] ……….(vi)

From eq. (v) and (vi), it is concluded that ABCD is a parallelogram because opposite sides of a quadrilateral are equal.

Now in ΔABC and ΔBAD,

AB = BA [Common]

BC = AD [proved above]

AC = BD [Given]

$\therefore$ ΔABC $\cong$ ΔBAD [By SSS congruency]

$\Rightarrow$ $\angle$ABC = $\angle$BAD [By C.P.C.T.] ……….(vii)

But $\angle$ABC + $\angle$BAD = 180° [ABCD is a parallelogram] ……….(viii)

$\therefore$ AD $\parallel$ BC and AB is a transversal.

$\Rightarrow$ $\angle$ABC + $\angle$ABC = 180° [Using eq. (vii) and (viii)]

$\Rightarrow$ 2$\angle$ABC = 180° $\Rightarrow$ $\angle$ABC = 90°

$\therefore$ $\angle$ABC = $\angle$BAD = 90° ……….(ix)

Opposite angles of a parallelogram are equal.

But $\angle$ABC = $\angle$BAD =

$\therefore$ $\angle$ABC = $\angle$ADC = 90° ……….(x)

$\therefore$ $\angle$BAD = $\angle$BDC = 90° ……….(xi)

From eq. (x) and (xi), we get

$\angle$ABC = $\angle$ADC = $\angle$BAD = $\angle$BDC = 90° ....(xii)

Now in ΔAOB and ΔBOC,

OA = OC [Given]

$\angle$AOB = $\angle$BOC = 90° [Given]

OB = OB [Common]

$\therefore$ ΔAOB $\cong$ ΔCOB [By SAS congruency]

$\Rightarrow$ AB = BC ....(xiii)

From eq. (v), (vi) and (xiii), we get,

AB = BC = CD = AD ....(xiv)

Now, from eq. (xii) and (xiv), we have a quadrilateral whose equal diagonals bisect each other at right angle.

Also sides are equal make an angle of 90° with each other.

$\therefore$ ABCD is a square.

---

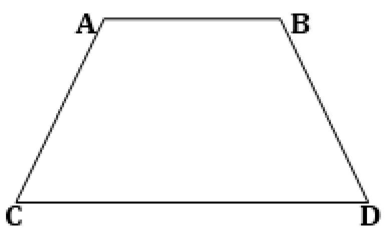
6. ABCD is a trapezium in which AB $\parallel$ CD and AD = BC (See figure). Show that:

(i) $\angle$A = $\angle$B

(ii) $\angle$C = $\angle$D

(iii) ΔABC $\cong$ ΔBAD

(iv) Diagonal AC = Diagonal BD



Ans. Given: ABCD is a trapezium.

AB $\parallel$ CD and AD = BC

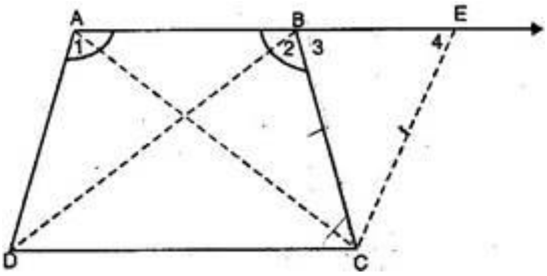
To prove: **(i)** $\angle$A = $\angle$B

(ii) $\angle$C = $\angle$D

(iii) ΔABC $\cong$ ΔBAD

(iv) Diag. AC = Diag. BD

Construction: Draw CE $\parallel$ AD and extend AB to intersect CE at E.



Proof: (i) As AECD is a parallelogram. [By construction]

$\therefore$ AD = EC

But AD = BC [Given]

$\therefore$ BC = EC

$\Rightarrow$ $\angle$3 = $\angle$4 [Angles opposite to equal sides are equal]

Now $\angle$1 + $\angle$4 = 180° [Interior angles]

And $\angle$2 + $\angle$3 = 180° [Linear pair]

$\Rightarrow$ $\angle$1 + $\angle$4 = $\angle$2 + $\angle$3

$\Rightarrow$ $\angle$1 = $\angle$2 [$\because$ $\angle$3 = $\angle$4]

$\Rightarrow$ $\angle$A = $\angle$B

(ii) $\angle$3 = $\angle$C [Alternate interior angles]

And $\angle$D = $\angle$4 [Opposite angles of a parallelogram]

But $\angle$3 = $\angle$4 [ΔBCE is an isosceles triangle]

$\therefore$ $\angle$C = $\angle$D

(iii) In ΔABC and ΔBAD,

AB = AB [Common]

$\angle$1 = $\angle$2 [Proved]

AD = BC [Given]

$\therefore$ ΔABC $\cong$ ΔBAD [By SAS congruency]

$\Rightarrow$ AC = BD [By C.P.C.T.]

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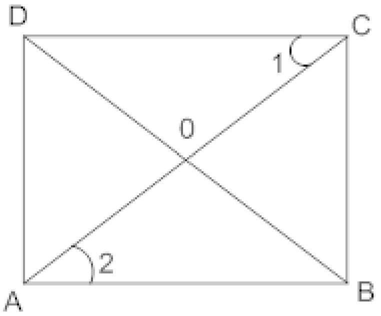
7. Prove that if the diagonals of a quadrilateral are equal and bisect each other at right angles then it is a square.



Ans. Given in a quadrilateral ABCD, AC = BD, AO = OC and BO = OD and $\angle AOB = 90^0$

To prove: ABCD is a square.

Proof: $In\ \Delta AOB\ and\ \Delta COD$

OA=OC

OB=OD [given]

And

$\angle AOB = \angle COD$ [vertically opposite angles]

$\therefore \Delta AOB \cong \Delta COD\ \ [By\ SAS]$

$\therefore AB = CD\ \ [By\ CPCT]$

$\angle 1 = \angle 2\ \ [By\ CPCT]$ But these are alternate angles $\therefore AB \parallel CD$

ABCD is a parallelogram whose diagonals bisects each other at right angles

$\therefore ABCD\ is\ a$ rhombus

Again in $\Delta ABD\ and\ \Delta BCA$

AB=BC [Sides of a rhombus]

AD=AB [Sides of a rhombus]

And BD=CA [Given]

$\therefore \Delta ABD \cong \Delta BCA$

$\therefore \angle BAD = \angle CBA$ [By CPCT]

These are alternate angles of these same side of transversal

$\therefore \angle BAD + \angle CBA = 180^0 \text{ or } \angle BAD = \angle CBA = 90^0$

Hence ABCD is a square.

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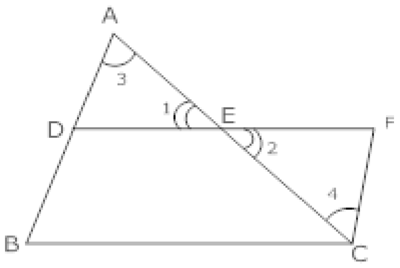
8. Prove that in a triangle, the line segment joining the mid points of any two sides is parallel to the third side.



Ans. Given: A ΔABC in which D and E are mid-points of the side AB and AC respectively

To Prove: $DE \parallel BC$

Construction: Draw $CF \parallel BA$

Proof: In ΔADE and ΔCFE

$\angle 1 = \angle 2$ [Vertically opposite angles]

AE=CE [Given]

And $\angle 3 = \angle 4$ [Alternate interior angles]

$\therefore \Delta ADE \cong \Delta CFE$ [By ASA]

$\therefore$ DE=FE [By C.P.C.T]

But DA = DB

$\therefore$ DB = FC

Now DB $\parallel$ FC

$\therefore$ DBCF is a parallelogram

$\therefore$ DE $\parallel$ BC

Also DE = EF = $\frac{1}{2}$ BC

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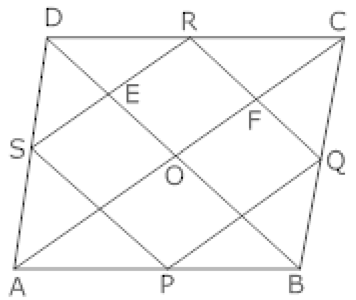
9. ABCD is a rhombus and P, Q, R, and S are the mid-Points of the sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS is a rectangle.

Ans. Join AC and BD which intersect at O let BD intersect RS at E and AC intersect RQ at F

IN ΔABD P and S are mid-points of sides AB and AD.



$\therefore$ PS||BD and PS= $\frac{1}{2}BD$

Similarly, RQ||DB and RQ= $\frac{1}{2}$ BD

$\therefore$ RS||BD||RQ and PS = $\frac{1}{2}BD = RQ$

PS=RQ and PS||RQ

$\therefore$ PQRS is a parallelogram

Now RF||EO and RE||FO

$\therefore$ OFRE is also a parallelogram.

Again, we know that diagonals of a rhombus bisect each other at right angles.

$\because \angle EOF = 90°$

$\because \angle EOF = \angle ERF$ [opposite angles of a parallelogram]

$\therefore \angle ERF = 90°$

$\therefore$ Each angle of the parallelogram PQRS is $90°$

Hence PQRS is a rectangle.

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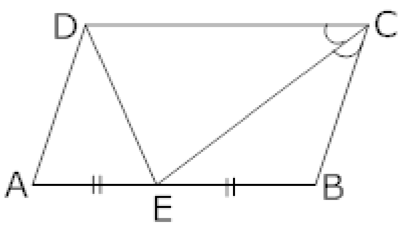
10. In the given Fig ABCD is a parallelogram E is mid-point of AB and CE bisects $\angle BCD$ Prove that:

(i) AE = AD

(ii) DE bisects $\angle ADC$

(iii) $\angle DEC = 90°$



Ans. ABCD is a parallelogram

$\therefore AB \parallel CD$ And EC cuts them

$\Rightarrow \angle BEC = \angle ECD$ [Alternate interior angle]

$\Rightarrow \angle BEC = \angle ECB$ $[\angle ECD = \angle ECB]$

$\Rightarrow EB = BC$

$\Rightarrow AE = AD$

(i) Now AE=AD

$\Rightarrow \angle ADE = \angle AED$

$\Rightarrow \angle ADE = \angle EAC$ [$\therefore \angle AED = \angle EDC$ Alternate interior angles]

(ii) $\therefore$ DE bisects $\angle ADC$

(iii) Now $\angle ADC + \angle BCD = 180°$

$\Rightarrow \frac{1}{2}\angle ADC + \frac{1}{2}\angle BCD = 90°$

$\Rightarrow \angle EDC + \angle DCE = 90°$

But, the sum of all the angles of the triangle is $180°$

$\Rightarrow 90° + \angle DEC = 180°$
$\Rightarrow \angle DEC = 90°$

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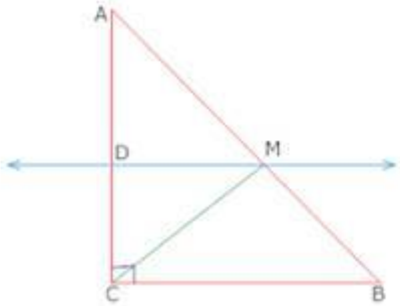
11. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. show that
(i) D is mid-point of AC
(ii) MD $\perp$ **AC**
(iii) CM = MA = $\frac{1}{2}$ **AB**



Ans. Given ABC is a Δ right angle at C

(i) M is mid-point of AB

And MD||BC

$\therefore$ D is mid-Point of AC [a line through midpoint of one side of a Δ parallel to another side bisect the third side.

(ii). $\because$ MD||BC

$\angle ADM = \angle DCB$ [Corresponding angles]

$\angle ADM = 90^\circ$

(iii) In Δ ADM and Δ CDM

AD=DC [$\because$ D is mid-point of AC]

DM=DM [Common]

$\therefore$ Δ ADM $\cong$ Δ CDM [By SAS]

$\therefore$ AM=CM [By C.P.C.T]

AM=CM=MB [$\because$ M is mid-point of AB]

$\therefore$ CM=MA= $\frac{1}{2}$ AB.