CBSE Class 10 Mathematics
Important Questions
Chapter 8
Introduction to Trigonometry

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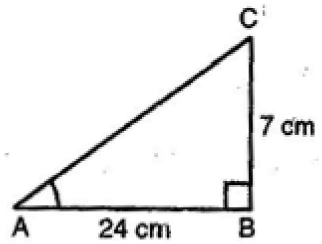
2 Marks Questions

1. In Δ ABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$



Ans. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$= (24)^2 + (7)^2$ = 576 + 49 = 625

$\Rightarrow$ AC = 25 cm

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{BC}{AC} = \frac{7}{25}$

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2. In adjoining figure, find $\tan P - \cot R$:

Ans. Using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow$ QR = 5 cm

$\therefore \tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{13} - \frac{5}{13} = 0$

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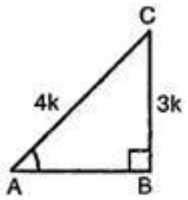
3. **If** $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.



Ans. Given: A triangle ABC in which $\angle$ B = 90°

Let BC = $3k$ and AC = $4k$

Then, Using Pythagoras theorem,

AB = $\sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$

$= \sqrt{16k^2 - 9k^2} = k\sqrt{7}$

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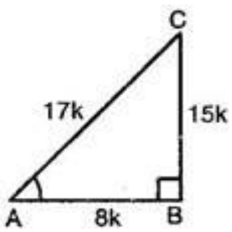
4. Given $15 \cot A = 8$, **find** $\sin A$ **and** $\sec A$

Ans. Given: A triangle ABC in which $\angle$ B = 90°

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$

Let AB = $8k$ and BC = $15k$



Then using Pythagoras theorem,

AC = $\sqrt{(AB)^2 + (BC)^2} = \sqrt{(8k)^2 + (15k)^2}$

$= \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

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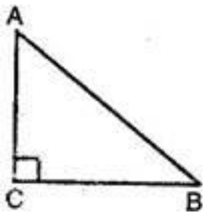
5. If $\angle$ And $\angle$ B are acute angles such that $\cos A = \cos B$, then show that $\angle$ A = $\angle$ B.



Ans. In right triangle ABC,

$\cos A = \frac{AC}{AB}$ and $\cos B = \frac{BC}{AB}$

But $\cos A = \cos B$ [Given]

$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB} \Rightarrow$ AC = BC

$\Rightarrow \angle$A = $\angle$B

[Angles opposite to equal sides are equal]

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6. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin \theta = \frac{4}{3}$ for some angle θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

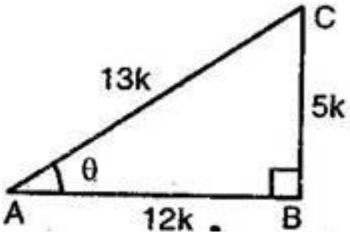
(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1

3 Marks Questions

1. Given $\sec\theta = \frac{13}{12}$, calculate all other trigonometric ratios.



Ans. Consider a triangle ABC in which $\angle$A = θ and $\angle$B = 90°

Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2-(AB)^2}$

= $\sqrt{(13k)^2-(12k)^2}$

= $\sqrt{169k^2-144k^2}$

= $\sqrt{25k^2}$ = $5k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$

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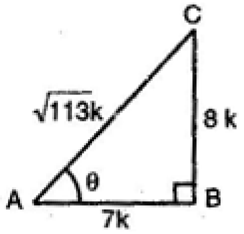
2. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$



Ans. Consider a triangle ABC in which ∠A = θ and ∠B = 90°
Let AB = $7k$ and BC = $8k$
Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2+(AB)^2}$ = $\sqrt{(8k)^2+(7k)^2}$
= $\sqrt{64k^2+49k^2}$ = $\sqrt{113k^2}$ = $\sqrt{113}k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$

$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$
= $\frac{1-\sin^2\theta}{1-\cos^2\theta}$
= $\frac{1-\frac{64}{113}}{1-\frac{49}{113}}$
= $\frac{113-64}{113-49}$ = $\frac{49}{64}$

(ii) $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$
= $\frac{49/113}{64/113}$ = $\frac{49}{64}$

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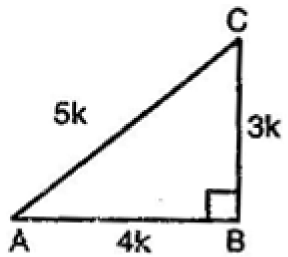
3. If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.



Ans. Consider a triangle ABC in which ∠B = 90°.

And $3\cot A = 4$ $\Rightarrow$ $\cot A = \frac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC $= \sqrt{(BC)^2 + (AB)^2}$

$= \sqrt{(3k)^2 + (4k)^2}$

$= \sqrt{16k^2 + 9k^2} = \sqrt{25k^2} = 5k$

$\therefore \sin A = \frac{BC}{AC} = \frac{3k}{5k} = \frac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

And $\tan A = \frac{BC}{AB} = \frac{3k}{4k} = \frac{3}{4}$

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A}$

$= \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

$= \frac{16-9}{16+9} = \frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$\because$ L.H.S. = R.H.S.

$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

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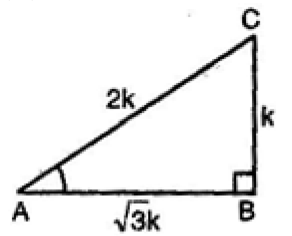
4. In Δ ABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, find value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$



Ans. Consider a triangle ABC in which $\angle$ B $= 90^\circ$.

Let BC $= k$ and AB $= \sqrt{3}k$

Then, using Pythagoras theorem,

AC $= \sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$

= $\sqrt{k^2 + 3k^2}$ = $\sqrt{4k^2}$ = $2k$

$\therefore$ $\sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle$ C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$\therefore$ $\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C$

= $\frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

= $\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$ = 1

(ii) $\cos A \cos C - \sin A \sin C$

= $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

= $\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

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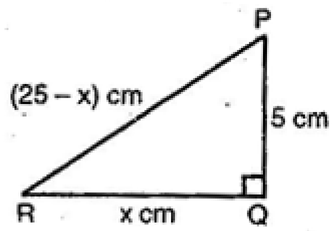
5. In Δ PQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.



Ans. In ΔPQR, right angled at Q.
PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm and PR = $(25 - x)$ cm
Using Pythagoras theorem,
$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25 - x)^2 = (x)^2 + (5)^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = 12$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$\therefore \quad \sin P = \frac{RQ}{RP} = \frac{12}{13}$

$\cos P = \frac{PQ}{RP} = \frac{5}{13}$

And $\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

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6. If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}};\ 0^\circ < A+B \le 90^\circ;\ A > B,$ **find A and B.**

Ans. (i) False, because $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ$ = 1

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$ = $\frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \ne \sin A + \sin B$

(ii) True, because

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

It is clear, the value of $\sin\theta$ increases as θ increases.

(iii) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$

True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$ = $\frac{1}{0}$ i.e. undefined

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7. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$ =

(A) 0 (B) 1 (C) 2 (D) none of these

(iii) $(\sec A + \tan A)(1-\sin A)$ =

(A) $\sec A$ **(B)** $\sin A$ **(C)** $\operatorname{cosec} A$ **(D)** $\cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$ **=**

(A) $\sec^2 A$ **(B)** -1 **(C)** $\cot^2 A$ **(D) none of these**

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$ = $9\left(\sec^2 A - \tan^2 A\right)$ = 9 x 1 = 9

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\cos ec\theta)$

= $\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

= $\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

= $\frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$ = 2

(iii) (D) $(\sec A+\tan A)(1-\sin A)$

= $\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

= $\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

= $\frac{1-\sin^2 A}{\cos A}$ = $\frac{\cos^2 A}{\cos A}$ = $\cos A$

$\left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv) (D) $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\frac{\sec^2 A-\tan^2 A+\tan^2 A}{\cos ec^2 A-\cot^2 A+\cot^2 A}$

= $\frac{\sec^2 A}{\cos ec^2 A}$ = $\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

= $\frac{\sin^2 A}{\cos^2 A}$ = $\tan^2 A$

4 Marks Questions

1. Express the trigonometric ratios $\sin A, \sec A$ **and** $\tan A$ **in terms of** $\cot A$

Ans. For $\sin A,$

By using identity $\cos ec^2 A-\cot^2 A=1$

$\Rightarrow \ cosec^2 A = 1 + \cot^2 A$

$\Rightarrow \ \frac{1}{\sin^2 A} = 1 + \cot^2 A$

$\Rightarrow \ \sin^2 A = \frac{1}{1 + \cot^2 A}$

$\Rightarrow \ \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$

For $\sec A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \ \sec^2 A = 1 + \tan^2 A$

$\Rightarrow \ \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$

$\Rightarrow \ \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$

$\Rightarrow \ \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$

For $\tan A$,

$\tan A = \frac{1}{\cot A}$

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2. Write the other trigonometric ratios of A in terms of $\sec A$

Ans. For $\sin A$,

By using identity, $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \ \sin^2 A = 1 - \cos^2 A$

$\Rightarrow \ \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$

$\Rightarrow \ \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$

For $\cos A$,

$$\cos A = \frac{1}{\sec A}$$

For $\tan A$,

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\cos ecA$,

$$\cos ecA = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$\Rightarrow \cos ecA = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

For $\cot A$,

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A$ **=**

(A) 1 (B) 9 (C) 8 (D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \cos ec\theta)$ **=**

(A) 0 (B) 1 (C) 2 (D) none of these

(iii) $(\sec A + \tan A)(1 - \sin A)$ **=**

(A) $\sec A$ **(B)** $\sin A$ **(C)** $\cos ecA$ **(D)** $\cos A$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$ **=**

(A) $\sec^2 A$ **(B)** -1 **(C)** $\cot^2 A$ **(D) none of these**

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$

= $9\left(\sec^2 A - \tan^2 A\right)$ = 9 x 1 = 9

(ii) (C) $\left(1+\tan\theta+\sec\theta\right)\left(1+\cot\theta-\cos ec\theta\right)$

= $\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

= $\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\frac{\left(\cos\theta+\sin\theta\right)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

= $\frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$ = 2

(iii) (D) $\left(\sec A+\tan A\right)\left(1-\sin A\right)$

= $\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)\left(1-\sin A\right)$

= $\left(\frac{1+\sin A}{\cos A}\right)\left(1-\sin A\right)$

= $\frac{1-\sin^2 A}{\cos A}$ = $\frac{\cos^2 A}{\cos A}$ = $\cos A$

$\left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv) (D) $\frac{1+\tan^2 A}{1+\cot^2 A}$

$= \dfrac{\sec^2 A - \tan^2 A + \tan^2 A}{\operatorname{cosec}^2 A - \cot^2 A + \cot^2 A} = \dfrac{\sec^2 A}{\operatorname{cosec}^2 A}$

$= \dfrac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$

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5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$

(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{\sin^2 A}{1-\cos A}$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$ **, using the identity** $\operatorname{cosec}^2 A = 1 + \cot^2 A$

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Ans. Proof:

(i) L.H.S. $(\operatorname{cosec}\theta - \cot\theta)^2$

= $\operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta$

= $\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - 2\times\frac{1}{\sin\theta}.\frac{\cos\theta}{\sin\theta}$

= $\frac{1+\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}$

= $\frac{1+\cos^2\theta - 2\cos\theta}{\sin^2\theta}$

= $\frac{(1-\cos\theta)^2}{\sin^2\theta}$

$\left[\because a^2 + b^2 - 2ab = (a-b)^2\right]$

= $\frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

= $\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

= $\frac{1-\cos\theta}{1+\cos\theta}$ = R.H.S.

(ii) L.H.S. $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

= $\frac{\cos^2\theta + 1 + \sin^2\theta + 2\sin A}{(1+\sin A)\cos A}$

= $\frac{\cos^2\theta + \sin^2\theta + 1 + 2\sin A}{(1+\sin A)\cos A}$

= $\frac{1+1+2\sin A}{(1+\sin A)\cos A}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

= $\frac{2+2\sin A}{(1+\sin A)\cos A}$ = $\frac{2(1+\sin A)}{(1+\sin A)\cos A}$

= $\frac{2}{\cos A}$ = $2\sec A$ = R.H.S

(iii) L.H.S. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta-\cos\theta}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{\cos\theta-\sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$\left[\because a^3-b^3=(a-b)(a^2+b^2+ab)\right]$

$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \frac{1}{\sin\theta\cos\theta}+1 = 1+\frac{1}{\sin\theta\cos\theta}$

$= 1+\sec\theta\cos ec\theta$

(iv) L.H.S. $\frac{1+\sec A}{\sec A}=\frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A}\times\frac{\cos A}{1} = 1+\cos A$

$= 1+\cos A\times\frac{1-\cos A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos A}$

$= \dfrac{\sin^2 A}{1-\cos A}$ = R.H.S.

(v) L.H.S. $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing all terms by $\sin A$,

$= \dfrac{\cot A - 1 + \cos ecA}{\cot A + 1 - \cos ecA} = \dfrac{\cot A + \cos ecA - 1}{\cot A - \cos ecA + 1}$

$= \dfrac{(\cot A + \cos ecA) - (\cos ec^2 A - \cot^2 A)}{(1 + \cot A - \cos ecA)}$

$= \dfrac{(\cot A + \cos ecA) + (\cot^2 A - \cos ec^2 A)}{(1 + \cot A - \cos ecA)}$

$= \dfrac{(\cot A + \cos ecA)(1 + \cot A - \cos ecA)}{(1 + \cot A - \cos ecA)}$

$= \cot A + \cos ecA$ = R.H.S.

(vi) L.H.S. $\sqrt{\dfrac{1+\sin A}{1-\sin A}}$

$= \sqrt{\dfrac{1+\sin A}{1-\sin A}} \times \sqrt{\dfrac{1+\sin A}{1+\sin A}}$

$= \sqrt{\dfrac{(1+\sin A)^2}{1-\sin^2 A}}$

$[\because (a+b)(a-b) = a^2 - b^2]$

$= \sqrt{\dfrac{(1+\sin A)^2}{\cos^2 A}}$ $[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \dfrac{1+\sin A}{\cos A} = \dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}$

$= \sec A + \tan A$ = R.H.S.

(vii) L.H.S. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \dfrac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta[2(1-\sin^2\theta)-1]}$ $[\because 1-\sin^2\theta=\cos^2\theta]$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2-2\sin^2\theta-1)}$

$= \dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(1-2\sin^2\theta)}$ $= \dfrac{\sin\theta}{\cos\theta}$ $=$ $\tan\theta$ = R.H.S

(viii) L.H.S. $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2$

$= \left(\sin A+\dfrac{1}{\sin A}\right)^2+\left(\cos A+\dfrac{1}{\cos A}\right)^2$

$= \sin^2 A+\dfrac{1}{\sin^2 A}+2.\sin A.\dfrac{1}{\sin A}+\cos^2 A+\dfrac{1}{\cos^2 A}+2.\cos A.\dfrac{1}{\cos A}$

$= 2+2+\sin^2 A+\cos^2 A+\dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 4+1+\dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 5+\operatorname{cosec}^2 A+\sec^2 A$

$= 5+1+\cot^2 A+1+\tan^2 A$

$[\because \operatorname{cosec}^2\theta=1+\cot^2\theta, \sec^2\theta=1+\tan^2\theta]$

$= 7+\tan^2 A+\cot^2 A$

= R.H.S.

(ix) L.H.S. $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)$

$= \left(\dfrac{1}{\sin A}-\sin A\right)\left(\dfrac{1}{\cos A}-\cos A\right)$

$= \left(\dfrac{1-\sin^2 A}{\sin A}\right)\left(\dfrac{1-\cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A}{\sin A}\times\dfrac{\sin^2 A}{\cos A}$ $=$ $\sin A.\cos A$

$= \frac{\sin A.\cos A}{\sin^2 A + \cos^2 A}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

Dividing all the terms by $\sin A.\cos A$,

$= \frac{\frac{\sin A.\cos A}{\sin A.\cos A}}{\frac{\sin^2 A}{\sin A.\cos A} + \frac{\cos^2 A}{\sin A.\cos A}}$ $= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{1}{\tan A + \cot A}$ = R.H.S.

(x) L.H.S. $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$

$= \frac{\sec^2 A}{\cos ec^2 A}$ $[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta]$

$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$ $= \tan^2 A$ = R.H.S.

Now, Middle side = $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ $= \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{-(1-\tan A)}{\tan A}}\right)$ $= (-\tan A)^2$ $= \tan^2 A$ = R.H.S.