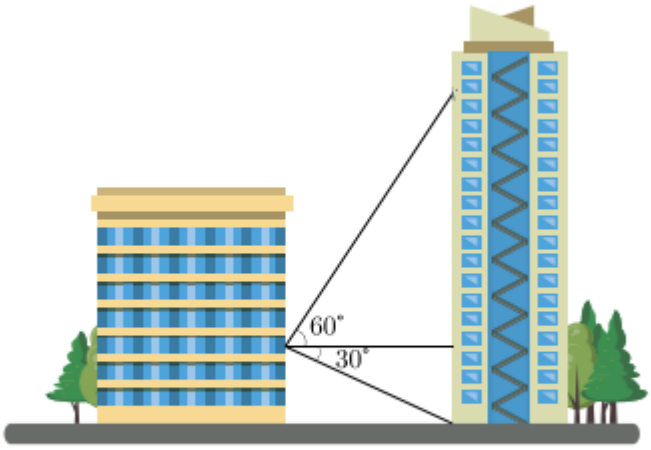
Prepared by: Sumeet Sahu
+91-8103405051

UNIQUE STUDY POINT
CASE STUDY BASED WORKSHEET: CLASS X
APPLICTIONS OF TRIGONOMETRY

1.

From his hotel room window on the fourth floor, Ranjan notices some window washers high above him on the hotel across the street.



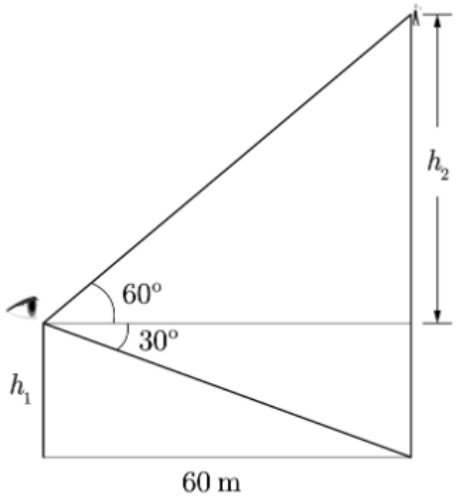
Curious as to their height above ground, he quickly estimates the buildings are 60 m apart, the angle of elevation to the workers is about 60°, and the angle of depression to the base of the hotel is about 30°.

(i) How high above ground is the window of Ranjan's hotel room?

(ii) How high above ground are the workers?

Sol :

Let h_1 be the height of Ranjan window from ground and h_2 be height of window washers from Ranjan. We draw a diagram of the situation as shown below.



Here $\tan 30^\circ = \frac{h_1}{60}$

$$\frac{1}{\sqrt{3}} = \frac{h_1}{60}$$

$$h_1 = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

$$= 20 \times 1.732 \ = 34.64 \ \text{m}$$

Now $\tan 60^\circ = \frac{h_2}{60}$

$$\sqrt{3} = \frac{h_2}{60}$$

$$h_2 = 60\sqrt{3}$$

$$= 60 \times 1.732 \ = 103.92 \ \text{m}$$

Height of workers from ground,

$$h_1 + h_2 = 34.64 + 103.92$$

$$= 138.56 \ \text{m}$$

(i) Window of Ranjan hotel is 34.64 meter above ground.
(ii) Workers are 138.64 meter above the ground.

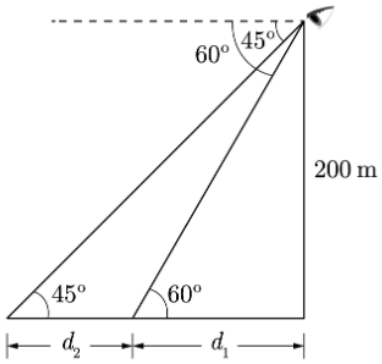
2.

From the observation deck of a seaside building 200 m high, Jignesh sees two fishing boats in the distance. The angle of depression to the nearer boat is 60° while for the boat farther away the angle is 45°.

(i) How far out to sea is the nearer boat?
(ii) How far apart are the two boats?

Sol :

Let d_1 be the distance of nearer boat from sea and d_2 be the distance between two boat. We draw a diagram of the situation as shown below.



Now
$$\tan 60^\circ = \frac{150}{d_1}$$
$$\sqrt{3} = \frac{150}{d_1}$$
$$d_1 = \frac{150}{\sqrt{3}} = 50\sqrt{3}$$
$$= 50 \times 1.732 = 86.6 \text{ m}$$

Now
$$\tan 45^\circ = \frac{150}{d_1 + d_2}$$
$$1 = \frac{150}{d_1 + d_2}$$
$$d_1 + d_2 = 150$$

Substituting value of d_1 we have
$$86.6 + d_2 = 150$$
$$d_2 = 150 - 86.6 = 63.4 \text{ m}$$

(i) Thus distance of nearer boat from seaside is 86.6 m
(ii) Both boat are 63.4 m apart.

2.

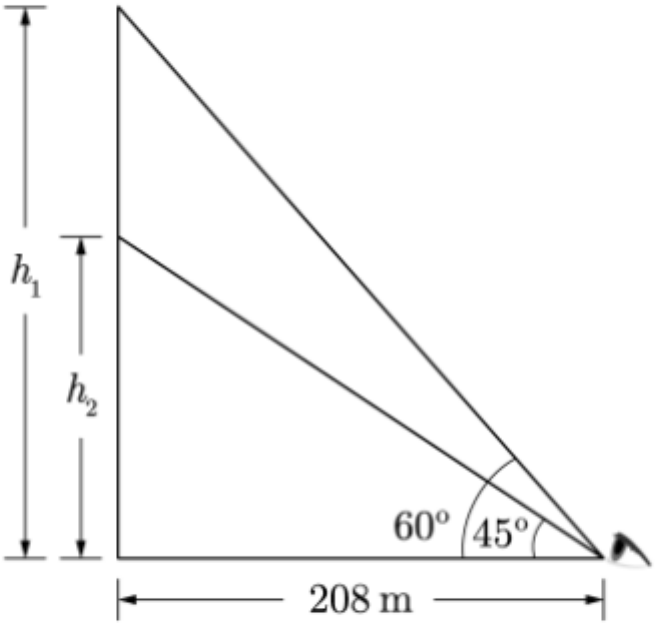
CN Tower : The tallest free-standing tower in the world is the CN Tower in Toronto, Canada. The tower includes a rotating restaurant high above the ground.

From a distance of 208 meter the angle of elevation to the pinnacle of the tower is 60°. The angle of elevation to the restaurant from the same vantage point is 45°.

(i) How tall is the CN Tower?
(ii) How far below the pinnacle of the tower is the restaurant located?

Sol :

Let h_1 be the height of tower from ground and h_2 be height of restaurant from ground. We draw a diagram of the situation as shown below.



Now $$\tan 60^\circ = \frac{h_1}{208}$$

$$\sqrt{3} = \frac{h_1}{208}$$

$$h_1 = 208 \times \sqrt{3}$$

$$= 208 \times 1.73 = 360 \text{ m}$$

Now, $$\tan 45^\circ = \frac{h_2}{208}$$

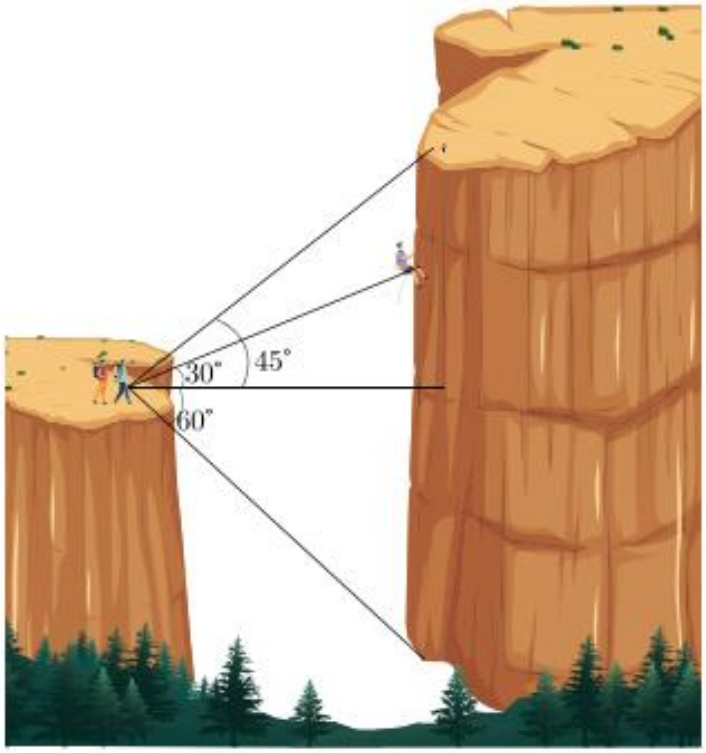
$$1 = \frac{h_2}{208}$$

$$h_2 = 208 \text{ m}$$

(i) Height of CN Tower is 360 meter

(ii) Restaurant is $360 - 208 = 152$ m below from top of tower.

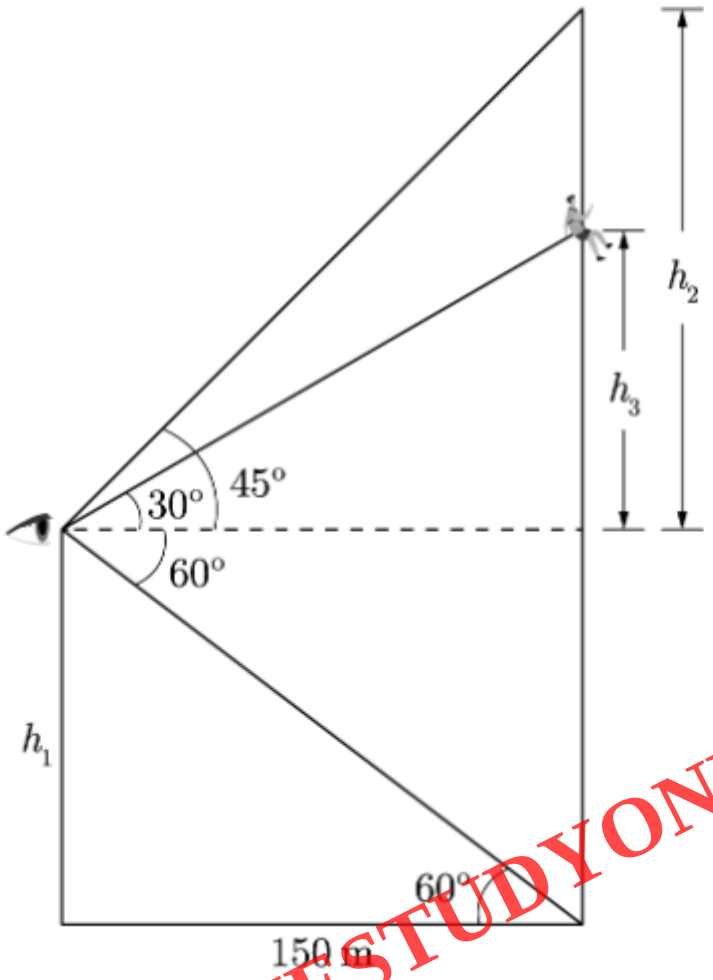
Height of a Climber : Himalayan Trekking Club has just hiked to the south rim of a large canyon, when they spot a climber attempting to scale the taller northern face. Knowing the distance between the sheer walls of the northern and southern faces of the canyon is approximately 150 meter, they attempt to compute the distance remaining for the climbers to reach the top of the northern rim.



Using a homemade transit, they sight an angle of depression of 60° to the bottom of the north face, and angles of elevation of 30° and 45° to the climbers and top of the northern rim respectively.

(i) How high is the southern rim of the canyon?
(ii) How high is the northern rim?
(iii) How much farther until the climber reaches the top?

Let h_1 be the height of southern rim of the canyon from ground and h_2 be the height of the northern rim from southern rim. Let h_3 be the height of climbers on northern rim from southern rim. We draw a diagram of the situation as shown below.



For h_1,

$$\tan 60^\circ = \frac{h_1}{150}$$
$$\sqrt{3} = \frac{h_1}{150}$$
$$h_1 = 150 \times \sqrt{3}$$
$$= 150 \times 1.732 = 259.8 \text{ m}$$

For h_2,

$$\tan 45^\circ = \frac{h_2}{150}$$
$$1 = \frac{h_2}{150}$$
$$h_2 = 150 \text{ m}$$

For h_3,

$$\tan 30^\circ = \frac{h_3}{150}$$
$$\frac{1}{\sqrt{3}} = \frac{h_3}{150}$$
$$h_3 = \frac{150}{\sqrt{3}} = 50\sqrt{3}$$
$$= 50 \times 1.732 = 86.6 \text{ m}$$
$$h_2 - h_3 = 150 - 86.6 = 63.4 \text{ m}$$

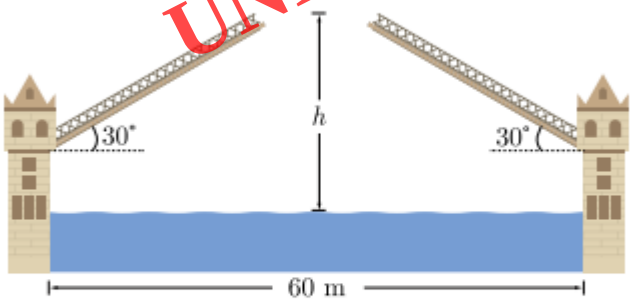
(i) $h_1 = 259.8$ metre is the height of the south rim.

(ii) $h_1 + h_2 = 259.8 + 150 = 409.8$ metre is the height of the north rim.

(iii) Climbers have to go $h_2 - h_3 = 63.4$ m to the top.

5.

Drawbridge : A drawbridge is a bridge that can be moved in order to stop or allow passage across it. Modern drawbridges are often built across large, busy waterways. They can be lifted to allow large ships to pass or lowered to allow land vehicles or pedestrians to cross.

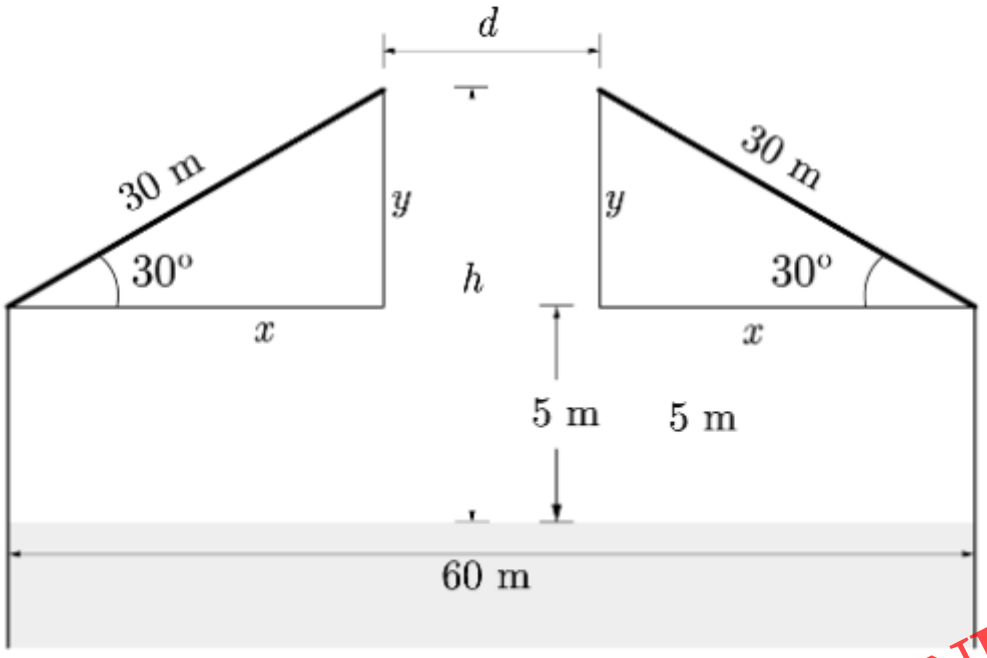


A drawbridge is 60 metre long when stretched across a river. As shown in the figure, the two sections of the bridge can be rotated upward through an angle of 30°.

(i) If the water level is 5 metre below the closed bridge, find the height h between the end of a section and the water level when the bridge is fully open.

(ii) How far apart are the ends of the two sections when the bridge is fully opened, as shown in the figure?

It may be easily seen that length of each section of bridge is $\frac{60}{2} = 30$ m. We draw a diagram of the situation as shown below. Let h be height between the end of a section and the water level, when bridge is fully opened. Let d be distance between the end of a section, when bridge is fully opened.



Now $\sin 30^\circ = \frac{y}{30}$

$$\frac{1}{2} = \frac{y}{30}$$

$$y = \frac{30}{2} = 15 \text{ m}$$

$$h = y + 5 = 15 + 5 = 20 \text{ m}$$

Now $\cos 30^\circ = \frac{x}{30}$

$$\frac{\sqrt{3}}{2} = \frac{x}{30}$$

$$x = \frac{30\sqrt{3}}{2} = 15\sqrt{3} \text{ m}$$

$$d = 60 - x - x$$

$$= 60 - 15\sqrt{3} - 15\sqrt{3}$$

$$= 60 - 30\sqrt{3} = 30(2 - \sqrt{3})$$

$$= 30(2 - 1.732)$$

$$= 30 \times 0.268 = 8.04 \text{ m}$$

(i) When the bridge is fully open, height is 20 meter between the end of a section and the water level.

(ii) When the bridge is fully opened the ends of the two sections are 8.04 metre apart.