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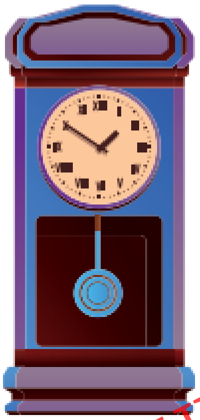
UNIQUE STUDY POINT
CASE STUDY BASED WORKSHEET: CLASS X
AREA REALTED TO CIRCLES

1.

Pendulum Clock : It is a clock that uses a pendulum, a swinging weight, as its timekeeping element. From its invention in 1656 by Christiaan Huygens, the pendulum clock was the world's most precise timekeeper, accounting for its widespread use. Their greater accuracy allowed for the faster pace of life which was necessary for the Industrial Revolution. The home pendulum clock was replaced by less-expensive, synchronous, electric clocks in the 1930s and 40s. Pendulum clocks are now kept mostly for their decorative and antique value.



Dhriti bought a pendulum clock for her living room. the clock contains a small pendulam of lenght 45 cm. the minute hand and hour hand of the clock are 9 cm and 6 cm long respectively.

(i) Find the area swept by the minute hand in 14 minutes.

(ii) Find the angle described by hour hand in 10 minutes.

(iii) Find the distance covered by the tip of hour hand in 3.5 hours.

(iv) If the tip of pendulum covers a distance of 66 cm in complete oscillation, then find the angle described by pendulum at the centre.

Let $r_m = 9$ cm, $r_h = 9$ cm and $r_p = 45$ cm

(i) Area swept by the minute hand in 60 minutes is πr^2, thus in 14 minutes.

$$= \pi r_m^2 \times \frac{14}{60}$$

$$= \frac{22}{7} \times 9 \times 9 \times \frac{14}{60}$$

$$= 22 \times 9 \times 9 \times \frac{1}{30}$$

$$= \frac{22 \times 9 \times 3}{10} = 59.4 \text{ cm}^2$$

(ii) Angle made by hour hand in 12 hours is 360°. Thus angle made in 10 minute.

$$\theta = \frac{360^\circ}{12} \times \frac{10}{60} = 5^\circ$$

(iii) Distance cover by tip of hour hand is $2\pi r$ in 12 hour. Distance covered in 3.5 hours,

$$= 2\pi r_h \times \frac{3.5}{12}$$

$$= 2 \times \frac{22}{7} \times 6 \times \frac{3.5}{12}$$

$$= \frac{22}{2} = 11 \text{ cm}$$

(iv) If the tip of pendulum covers a distance of 66 cm in complete oscillation, distance covered in half oscillation is 33 cm.

If θ is the angle described by pendulum at the centre to cover distance 33 cm, then,

$$l = 2\pi r_p \times \frac{\theta}{360^\circ}$$

$$33 = 2 \times \frac{22}{7} \times 45 \times \frac{\theta}{360^\circ}$$

$$3 = \frac{4}{7} \times \frac{\theta}{8}$$

$$\theta = \frac{3 \times 7 \times 8}{4} = 42^\circ$$

2.

Motion Detector : A motion sensor (or motion detector) is an electronic device that is designed to detect and measure movement. Motion sensors are used primarily in home and business security systems, but they can also be found in phones, paper towel dispensers, game consoles, and virtual reality systems.

A motion detector can detect movement up to 24 m away through an angle of $\theta = 70^\circ$.

(i) What area can the motion detector monitor?

(ii) What angle is required to monitor 50% more area?

(iii) For $\theta = 91^\circ$ what range is required for the detector to monitor 30% more area?

(i) Let $r_1 = 24$ m and $\theta_1 = 70^\circ$

Area monitor by motion detector,

$$A_1 = \pi r_1^2 \frac{\theta_1}{360^\circ}$$

$$= \frac{22}{7} \times (24)^2 \times \frac{70}{360^\circ}$$

$$= 22 \times 24 \times 24 \times \frac{1}{36}$$

$$= 22 \times 24 \times 2 \times \frac{1}{3}$$

$$= 22 \times 8 \times 2 = 352 \text{ m}^2$$

(ii) Let θ_2 be angle to monitor 50% more area. Radius is equal to $r_1 = 24$ in this case.

$$A_2 = 1.5 A_1$$

$$\pi r_1^2 \frac{\theta_2}{360^\circ} = 1.5 \pi r_1^2 \frac{\theta_1}{360^\circ}$$

$$\theta_2 = 1.5 \theta_1$$

$$= 70^\circ \times 1.5 = 105^\circ$$

(iii) Let r_3 be range required for the detector to monitor 30% more area where $\theta = 91^\circ$.

$$A_3 = 1.3 A_1$$

$$\pi r_3^2 \frac{\theta}{360^\circ} = 1.3 \pi r_1^2 \frac{\theta_1}{360^\circ}$$

$$r_3^2 \theta = 1.3 r_1^2 \theta_1$$

Substituting all values we have

$$r_3^2 \times 91^\circ = 1.3 \times 24^2 \times 70^\circ$$

$$r_3^2 = 24^2 \Rightarrow r_3 = 24 \text{ m}$$

3.

Laying New Sod : Sod, also known as turf, is grass. When harvested into rolls it is held together by its roots and a thin layer of soil. Sod is typically used for lawns, golf courses, and sports stadiums around the world.

When new sod is laid, a heavy roller is used to press the sod down to ensure good contact with the ground beneath. The radius of the roller is 28 cm.

(i) Through what angle has the roller turned after being pulled across 11 meter of yard?

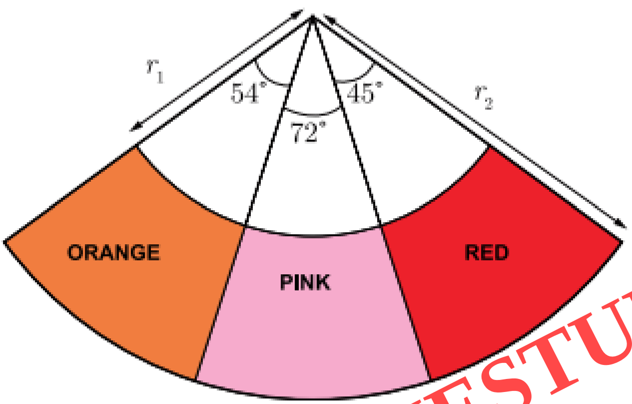
(ii) If it is turned through by 3 revolution and 45°, find the length, that the roller will press.

(i)
$$P = 2\pi r \frac{\theta}{360^\circ}$$
$$1100 = 2 \times \frac{22}{7} \times 28 \times \frac{\theta}{360^\circ}$$
$$100 = 2 \times 2 \times 4 \times \frac{\theta}{360}$$
$$100 = 4 \times \frac{\theta}{90^\circ}$$
$$\theta = \frac{100 \times 900}{4} = 2250^\circ$$
$$= 360^\circ \times 6 + 90^\circ$$

(ii)
$$P = 2\pi r \frac{\theta}{360^\circ}$$
$$= 2 \times \frac{22}{7} \times 28 \times \frac{1125}{360^\circ}$$
$$= 2 \times 22 \times 4 \times \frac{125}{40}$$
$$= 22 \times 8 \times \frac{25}{8}$$
$$= 22 \times 25 = 550 \text{ cm} = 5.5 \text{ m}$$

Traditional Japanese Fans : Japanese fans are made of paper on a bamboo frame, usually with a design painted on them. A Japanese Fan symbolises friendship, respect and good wishes and are given on special occasions, as well as to help cool you down in hot weather. The fan is an immediately recognizable icon of Japanese culture. Today they remain an important artistic medium and stylish fashion accessory.

Lavanya hold a Japanease folding fan in her hand as shown in figure. It is shapded like a sector of a circle. The inner and outer raddi are 14 cm and 21 cm. The fan has three colour material.



(i) If the region containing the pink colour makes an angle of $\theta_2 = 72^\circ$ at the centre, then find the area of the region having pink colour.

(ii) If the region containing the orange colour makes an angle of $\theta_1 = 54^\circ$ at the centre, then find the area of the region having orange colour.

(iii) If the region containing the red colour makes an angle of $\theta_1 = 45^\circ$ at the centre, then find the perimeter of the region having red colour.

(iv) Find the area of the region having radius 14 cm.

Sol :

Here $r_1 = 14$ cm and $r_2 = 21$ cm.

(i) Area of region having pink colour,

$$A_p = \pi r_2^2 \frac{\theta_2}{360^\circ} - \pi r_1^2 \frac{\theta_2}{360^\circ}$$

$$= \frac{\pi \theta_2}{360^\circ}(r_2^2 - r_1^2)$$

$$= \frac{22}{7} \times \frac{72^\circ}{360^\circ}(21^2 - 14^2)$$

$$= \frac{22}{7} \times \frac{1}{5}(21 + 14)(21 - 14)$$

$$= \frac{22}{7} \times \frac{1}{5} \times 35 \times 7$$

$$= 22 \times 7 = 154 \text{ cm}^2$$

(ii) Area of region having orange colour,

$$A_o = \frac{\pi \theta_1}{360^\circ}(r_2^2 - r_1^2)$$

$$= \frac{22}{7} \times \frac{54^\circ}{360^\circ}(21^2 - 14^2)$$

$$= \frac{22}{7} \times \frac{3}{20}(21 + 14)(21 - 14)$$

$$= \frac{22}{7} \times \frac{3}{20} \times 35 \times 7$$

$$= \frac{22 \times 3 \times 7}{4} = 115.5 \text{ cm}^2$$

(iii) Perimeter of the region having red colour,

$$p_r = 2(r_2 - r_1) + 2\pi r_1 \frac{\theta_3}{360^\circ} + 2\pi r_2 \frac{\theta_3}{360^\circ}$$

$$= 2(r_2 - r_1) + 2\pi \frac{\theta_3}{360^\circ}(r_1 + r_2)$$

$$= 2(21 - 14) + 2 \times \frac{22}{7} \times \frac{45^\circ}{360^\circ}(21 + 14)$$

$$= 2 \times 7 + 2 \times \frac{22}{7} \times \frac{1}{8} \times 35$$

$$= 14 + \frac{11 \times 5}{2}$$

$$= 14 + 27.5 = 41.5 \text{ cm}$$

(iv) Area of the region having radius 14 cm

$$A_s = \pi r_1^2 \frac{(\theta_1 + \theta_2 + \theta_3)}{360^\circ}$$

$$= \frac{22}{7} \times (14)^2 \times \frac{(54^\circ + 72^\circ + 45^\circ)}{360^\circ}$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{171^\circ}{360^\circ}$$

$$= 22 \times 2 \times 14 \times \frac{19}{40}$$

$$= \frac{22 \times 7 \times 19}{10} = 292.6 \text{ cm}^2$$

5.

Swimming Pool : A swimming pool or simply pool is a structure designed to hold water to enable swimming or other leisure activities. Pools can be built into the ground (in-ground pools) or built above ground. In-ground pools are most commonly constructed from materials such as concrete, natural stone, metal, plastic, or fiberglass, and can be of a custom size and shape or built to a standardized size, the largest of which is the Olympic-size swimming pool.

The area of a circular pool is 616 square metre. The owner wants to replace the tiling at the edge of the pool.

(i) The edging is 25 cm wide, so she plans to use 6-inch square tiles to form a continuous inner edge. How many tiles will she need to purchase?

(ii) Once the square tiles are in place around the pool, there will be extra space between the tiles. What shape of tile will best fill this space? How many tiles of this shape should she purchase?

Sol :

Let r be radius of pool.

(i) Area of pool, $A = \pi r^2$

$$616 = \frac{22}{7} r^2$$

$$r^2 = \frac{616 \times 7}{22} = 28 \times 7 = 196$$

$$r = \sqrt{196} = 14$$

Circumference of pool,

$$C = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 14 = 176 \text{ m}$$

If tile 25 cm wide, we have to place 4 tile per meter. Therefore total $176 \times 4 = 704$ tiles for whole circumference.

(ii) The square tiles will touch along the inner edge of the border, but there will be gaps along the outer edge. The tiles used to fill the gaps should be triangles. There will be 704 gaps between the 704 square tiles, so 704 triangular tiles will be needed.