LINEAR EQUATION IN TWO VARIABLES

The value of k for which the system of linear equations $x+2y=3$, $5x+ky+7=0$ is inconsistent is

(a) $-\frac{14}{3}$

(b) $\frac{2}{5}$

(c) 5

(d) 10

We have $x + 2y - 3 = 0$

and $5x + ky + 7 = 0$

If system is inconsistent, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

From first two orders, we have

$$\frac{1}{5} = \frac{2}{k} \Rightarrow k = 10$$

Thus (d) is correct option.

For which value(s) of p, will the lines represented by the following pair of linear equations be parallel

$$3x - y - 5 = 0$$

$$6x - 2y - p = 0$$

(a) all real values except 10 (b) 10

(c) 5/2 (d) 1/2

We have, $3x - y - 5 = 0$

and $6x - 2y - p = 0$

Here, $a_1 = 3,\ b_1 = -1,\ c_1 = -5$

and $a_2 = 6,\ b_2 = -2,\ c_2 = -p$

Since given lines are parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{-1}{-2} \neq \frac{-5}{-p}$$

$$p \neq 5 \times 2 \Rightarrow p \neq 10$$

If the lines given by $3x+2ky=2$ and $2x+5y+1=0$ are parallel, then the value of k is

(a) $-\frac{5}{4}$

(b) $\frac{2}{5}$

(c) $\frac{15}{4}$

(d) $\frac{3}{2}$

We have $3x + 2ky - 2 = 0$

and $2x + 5y + 1 = 0$

Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \ldots(\text{i})$$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = 1$

From equation (i), we have

$$\frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$

Considering, $\frac{3}{2} = \frac{2k}{5} \quad \left[\frac{3}{2} \neq \frac{-2}{1} \text{ in any case}\right]$

$$k = \frac{15}{4}$$

The pair of equations $x+2y+5=0$ and $-3x-6y+1=0$ has

(a) a unique solution

(b) exactly two solutions

(c) infinitely many solutions

(d) no solution

Given,equations are

$$x + 2y + 5 = 0$$

and $$-3x - 6y + 1 = 0$$

Here, $a_1 = 1, b_1 = 2, c_1 = 5$

and $a_2 = -3, b_2 = -6, c_2 = 1$

Now $$\frac{a_1}{a_2} = -\frac{1}{3}, \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$$

Now, we observe that

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.
Thus (d) is correct option.

One equation of a pair of dependent linear equations $-5x+7y=2$ The second equation can be

(a) $10x+14y+4=0$

(b) $-10x-14y+4=0$

(c) $-10x+14y+4=0$

(d) $10x-14y=-4$

For dependent linear equation,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Checking for option (a):

$$\frac{-5}{10} \neq \frac{7}{14}$$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ So, option (a) is rejected.

Checking for option (b):

$$\frac{-5}{-10} \neq \frac{7}{-14}$$

So, option (b) is also rejected.

Checking for option (c):

$$\frac{-5}{-10} = \frac{7}{14} \neq \frac{-2}{4}$$

So, option (b) is also rejected

Checking for option (d):

If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(a) 3 and 5

(b) 5 and 3

(c) 3 and 1

(d) -1 and -3

Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15

(b) 35 and 20

(c) 15 and 35

(d) 25 and 25

Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

Now, by given conditions,

$$x + y = 50 \quad \ldots(1)$$

Also, $x \times 1 + y \times 2 = 75$

$$x + 2y = 75 \quad \ldots(2)$$

Subtracting equation (1) form equation (2), we get

$$(x + 2y) - (x + y) = 75 - 50$$

$$y = 25$$

From equation (i), $x = 75 - 2x(25)$

Then, $x = 25$

Thus (d) is correct option.

The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively.

(a) 4 and 24

(b) 5 and 30

(c) 6 and 36

(d) 3 and 24

Let the present age of father $= x$ years

and the present age of son $= y$ years

Four years hence, it has relation by given condition

$$(x+4) = 4(y+4)$$

$$x - 4y = 12 \quad \ldots(1)$$

As the father's age is six times his son's age, so we have

$$x = 6y \quad \ldots(2)$$

Putting the value of x from equation (2) in equation (1), we get

$$6x - 4y = 12$$

$$2y = 12$$

$$y = 6$$

From equation (1), $x = 6 \times 6$

Then, $x = 36$

Hence, present age of father is 36 year and age of son is 6 year.

Assertion : Pair of linear equations : $9x+3y+12=0$, $8x+6y+24=0$ have infinitely many solutions.

Reason : Pair of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ have infinitely many solutions, if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

From the given equations, we have

$$\frac{9}{18} = \frac{3}{6} = \frac{12}{24}$$

$$\frac{1}{2} = \frac{1}{2} = \frac{1}{2} \text{ i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Assertion : $x+y-4=0$ and $2x+ky-3=0$ has no solution if $k=2$.

Reason : $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are consistent if $\frac{a_1}{a_2} \neq \frac{k_1}{k_2}$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

For assertion, given equation has no solution if

$$\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3} \text{ i.e. } \frac{4}{3}$$

$$k = 2\left[\frac{1}{2} \neq \frac{4}{3} \text{ holds}\right]$$

Assertion is true.

Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
Thus (b) is correct option.

In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. What is the number ?

Let x be units digit and y be tens digit, then number will be $10y + x$

Then, $$x = 2y \quad \ldots(1)$$

If 36 be added to the number, the digits are reversed, i.e number will be $10x + y$.

$$10y + x + 36 = 10x + y$$

$$9x - 9y = 36$$

$$x - y = 4 \quad \ldots(2)$$

Solving (1) and (2) we have $x = 8$ and $y = 4$.

Thus number is 48.

A fraction becomes 4 when 1 is added to both the numerator and denominator and it becomes 7 when 1 is subtracted from both the numerator and denominator. What is the numerator of the given fraction ?

Let the fraction be $\frac{x}{y}$,

$$\frac{x+1}{y+1} = 4 \Rightarrow x = 4y + 3 \qquad \ldots(1)$$

and

$$\frac{x-1}{y-1} = 7 \Rightarrow x = 7y - 6 \qquad \ldots(2)$$

Solving (1) and (2), we have $x = 15$, $y = 3$,

Solve : $99x + 101y = 499, \quad 101x + 99y = 501$

We have $99x + 101y = 499$ ...(1)

$101x + 99y = 501$ ...(2)

Adding equation (1) and (2), we have

$200x + 200y = 1000$

$x + y = 5$ ...(3)

Subtracting equation (2) from equation (1), we get

$-2x + 2y = -2$

$x - y = 1$ ...(4)

Adding equations (3) and (4), we have

$2x = 6 \Rightarrow x = 3$

Substituting the value of x in equation (3), we get

$3 + y = 5 \Rightarrow y = 2$

Solve for x and y :

$$\frac{x+1}{2}+\frac{y-1}{3}=9\ ;\ \frac{x-1}{3}+\frac{y+1}{2}=8.$$

We have $\frac{x+1}{2}+\frac{y-1}{3}=9$

$$3(x+1)+2(y-1)=54$$
$$3x+3+2y-2=54$$
$$3x+2y=53$$

and $\frac{x-1}{3}+\frac{y+1}{2}=8$

$$2(x-1)+3(y+1)=48$$
$$2x-2+3y+3=48$$
$$2x+3y=47$$

Multiplying equation (1) by 3 we have

$$9x+6y=159$$

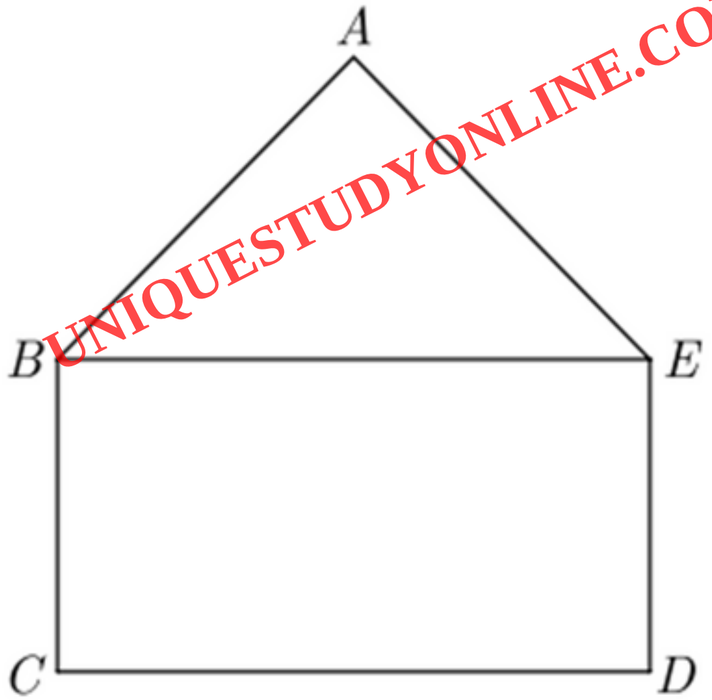
Multiplying equation (2) by 2 we have

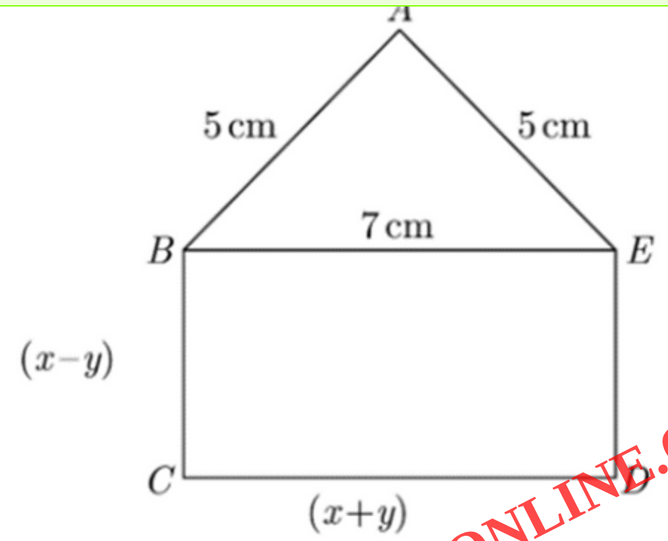
$$4x+6y=94$$

Subtracting equation (4) from (3) we have

$$5x=65$$

In the figure, $ABCDE$ is a pentagon with $BE \parallel CD$ and $BC \parallel DE$. BC is perpendicular to CD. $AB = 5$ cm, $AE = 5$ cm, $BE = 7$ cm, $BC = x - y$ and $CD = x + y$. If the perimeter of $ABCDE$ is 27 cm. Find the value of x and y, given x, $y \neq 0$.





We have $CD = BE$

$$x + y = 7 \qquad \ldots(1)$$

Also, perimeter of $ABCDE$ is 27 cm, thus

$$AB + BC + CD + DE + AE = 27$$

$$5 + (x - y) + (x + y) + (x - y) + 5 = 27$$

$$3x - y = 17 \qquad \ldots(2)$$

Adding equation (1) and (2) we have

$$4x = 24 \Rightarrow x = 6$$

Substituting $x = 6$ in equation (1) we obtain

$$y = 7 - x = 7 - 6 = 1$$

Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

Let the length of the garden be x m and its width be ym.
Perimeter of rectangular garden

$$p = 2(x+y)$$

Since half perimeter is given as 36 m,

$$x+y = 36 \quad \ldots(1)$$

Also, $x = y+4$

or $x-y = 4 \quad \ldots(2)$

Now adding eq (1) and (2) we have

$$2x = 40 \Rightarrow x = \frac{40}{2} = 20$$

Subtracting eq (2) from (1) we have

$$2y = 32 \Rightarrow x = \frac{32}{2} = 16$$

Hence, length is 20 m and width is 16 m.

A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay Rs. 3,000 as hostel charges whereas Mansi who takes food for 25 days Rs. 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

Let fixed charge be x and per day food cost be y

$$x + 20y = 3000 \quad \ldots(1)$$

$$x + 25y = 3500 \quad \ldots(2)$$

Subtracting (1) from (2) we have

$$5y = 500 \Rightarrow y = 100$$

Substituting this value of y in (1), we get

$$x + 20(100) = 3000$$

$$x = 1000$$

Thus $x = 1000$ and $y = 100$

Fixed charge and cost of food per day are Rs. 1,000 and Rs. 100.

The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

Let the incomes of two persons be $11x$ and $7x$.
Also the expenditures of two persons be $9y$ and $5y$.

$$11x - 9y = 400 \quad \ldots(1)$$

and
$$7x - 5y = 400 \quad \ldots(2)$$

Multiplying equation (1) by 5 and equation (2) by 9 we have

$$55x - 45y = 2000 \quad \ldots(3)$$

and
$$63x - 45y = 3600 \quad \ldots(4)$$

Subtracting, above equation we have

$$-8x = -1600$$

or,
$$x = \frac{-1,600}{-8} = 200$$

Hence Their monthly incomes are $11 \times 200 =$ Rs 2200 and $7 \times 200 =$ Rs 1400.

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Let the number of students in a row be x and the number of rows be y. Thus total will be xy.

Now
$$(x+3)(y-1) = xy$$
$$xy + 3y - x - 3 = xy$$
$$-x + 3y - 3 = 0 \quad \ldots(1)$$

and
$$(x-3)(y+2) = xy$$
$$xy - 3y + 2x - 6 = xy$$
$$2x - 3y - 6 = 0 \quad \ldots(2)$$

Multiply equation (1) 2 we have
$$-2x + 6y - 6 = 0 \quad \ldots(3)$$

Adding equation (2) and (3) we have
$$3y - 12 = 0$$

Production : Ridhima Electronics Pvt . Ltd is main supplier for CASIO for electronics component. They manufactures keyboards and screens for graphing calculators at plants in Bangalore and Bhiwadi. The hourly production rates at each plant are given in the table. How many hours should each plant be operated to fill an order for exactly 4,000 keyboards and exactly 4,000 screens?

Plant	Keyboards	Screens
Bangalore	40	32
Bhiwadi	20	32

Since we require 4000 keyboard, thus

$$40x + 20y = 4000$$

$$2x + y = 200 \quad (1)$$

Since we require 4000 screens, thus

$$32x + 32y = 4000$$

$$x + y = 125 \quad (2)$$

Subtracting eq (2) from (1) we have

$$x = 75 \text{ hours}$$

Substituting above value of x in (ii) we have

$$75 + y = 125$$

$$y = 125 - 75 = 50 \text{ hours}$$

Thus Bangalore plant should be operated for 75 hours and Bhiwadi plant should be operated for 50 hours.

Presale Order : A wireless store owner takes presale orders for a new smartphone and tablet. He gets 340 preorders for the smartphone and 250 preorders for the tablet. The combined value of the preorders is Rs 27,050,000. The price of a smartphone and tablet together is Rs 96500.

(i) How much does smartphone cost?

(ii) How much does tablet cost?