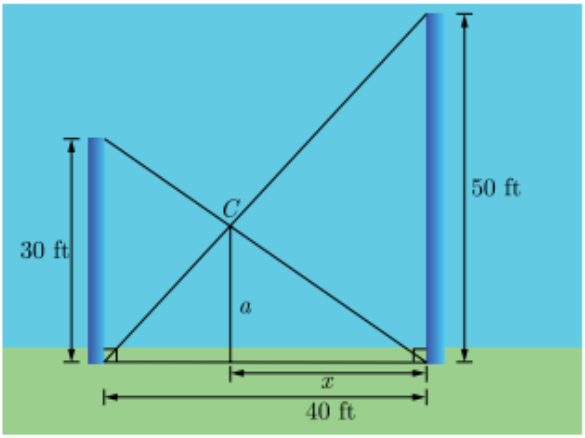
Prepared by: Sumeet Sahu
+91-8103405051
Visit: www.uniquestudyonline.com
YOUTUBE:UNIQUESTUDYPOINT

UNIQUE STUDY POINT
CASE STUDY BASED WORKSHEET: CLASS X
TRIANGLES

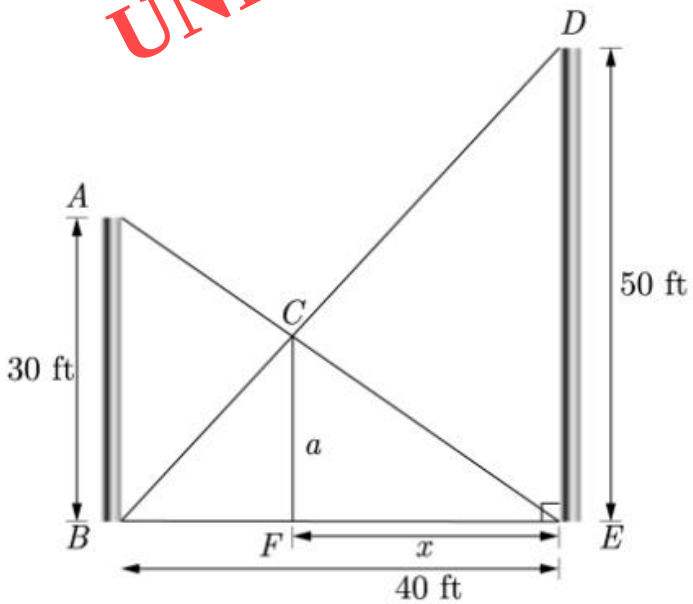
1.

Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



(i) What is the horizontal distance from C to the taller pole?
(ii) How high above the ground is the coupling ?
(iii) How far down the wire from the smaller pole is the coupling ?

The poles form parallel line segments and the wires are transversals cutting through the ends of the parallel segments.



Here $\angle CFE$ is a right angle.
By AA similarity $\triangle AEB \sim \triangle CEF$

Thus $$\frac{x}{40} = \frac{a}{30}$$

UNIQUESTUDYONLINE.COM

$$30x = 40a$$

$$\frac{30}{40}x = a$$

By AA similarity $\triangle CBF \sim \triangle DBE$

$$\frac{a}{50} = \frac{40-x}{40}$$

$$40a = 50(40-x)$$

Substituting value of a we have

$$40\left(\frac{30}{40}x\right) = 2000 - 50x$$

$$30x = 2000 - 50x$$

$$80x = 2000$$

$$x = 25$$

(i) So, the distance from C to the taller pole is 25 feet.

(ii) $$\frac{x}{a} = \frac{40}{30}$$

$$30x = 40a$$

$$30(25) = 40a$$

$$750 = 40a$$

$$18.75 = a$$

The coupling is 18.75 feet above the ground.

(iii) Let y represent the length of the wire from the smaller pole.

$$30^2 + 40^2 = y^2 \quad \Rightarrow \quad y = 50$$

Let z represent the length of the wire from the top of the smaller pole to the coupling.

$$\frac{z}{50-z} = \frac{40-25}{25}$$

$$25z = 15(50-z)$$

$$5z = 3(50-z)$$

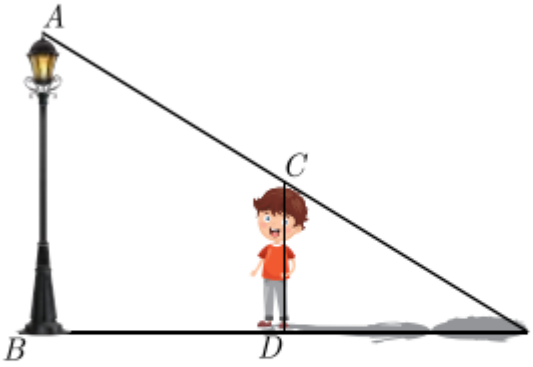
$$5z = 150 - 3z$$

$$8z = 150 \quad \Rightarrow \quad z = \frac{150}{8} = 18.75$$

The coupling is 18.75 feet down the wire from the top of the smaller pole.

UNIQUESTUDYONLINE.COM

Rohan is very intelligent in maths. He always try to relate the concept of maths in daily life. One day he is walking away from the base of a lamp post at a speed of 1 m/s. Lamp is 4.5 m above the ground.



(i) If after 2 second, length of shadow is 1 meter, what is the height of Rohan ?

(ii) What is the minimum time after which his shadow will become larger than his original height?

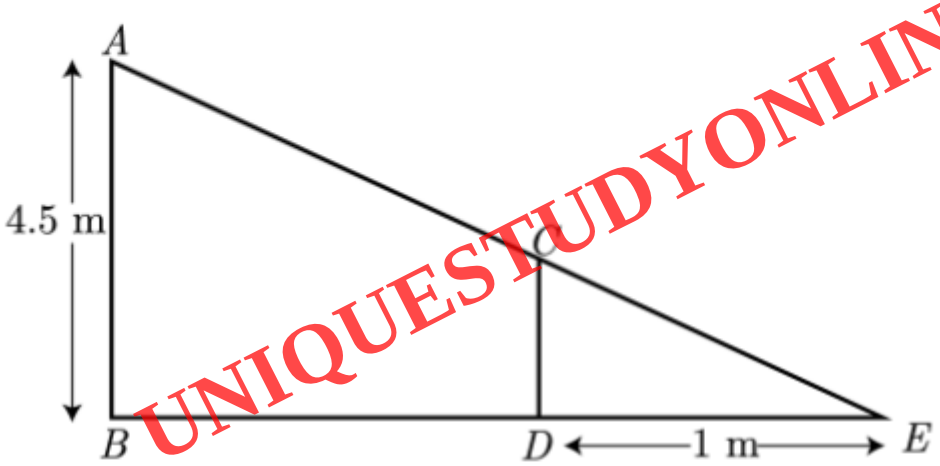
(iii) What is the distance of Rohan from pole at this point ?

(iv) What will be the length of his shadow after 4 seconds?

(v) Which similarity criterion is used in solving the above problem

Sol :

(i) As per question statement we make the diagram at following.



UNIQUESTUDYONLINE.COM

At $t = 2$ seconds, $BD = 2 \times 1 = 2\,\text{m}$

$DE = 1\ \text{m}$

Since, $ABE \sim \Delta\, CDE$

$$\frac{AB}{CD} = \frac{BE}{DE} = \frac{BD + DE}{DE}$$

or, $$\frac{4.5}{CD} = \frac{2+1}{1} = 3$$

$$CD = \frac{4.5}{3} = 1.5\ \text{m} = 150\ \text{cm}$$

(ii) At point where shadow is equal to her height,

$$CD = DE = 1.5 \text{ m}$$

$$\frac{AB}{CD} = \frac{BE}{DE} = \frac{BD+DE}{DE}$$

$$\frac{4.5}{1.5} = \frac{BD+1.5}{1.5}$$

$$4.5 = BD+1.5$$

$$BD = 4.5 - 1.5 = 3 \text{ m}$$

Time to reach at BD, $t = \frac{3}{1} = 3$ sec

(iii) As calculated in part (ii) we have $BD = 3$ m

(iv) After 4 sec, $BD = 1 \times 4 = 4$

$$\frac{AB}{CD} = \frac{BE}{DE} = \frac{BD+DE}{DE}$$

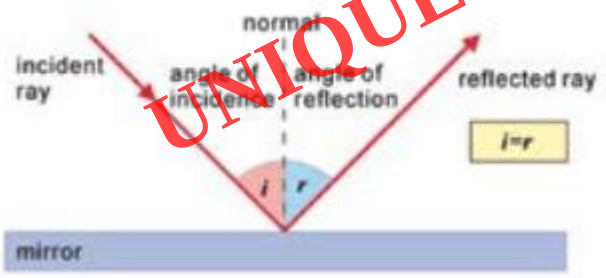
$$\frac{4.5}{1.5} = \frac{4+DE}{DE}$$

$$3DE = 4+DE \Rightarrow DE = 2 \text{ m}$$

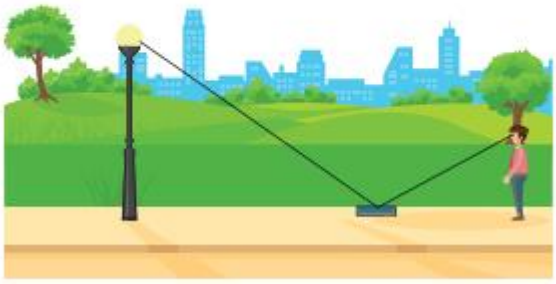
(v) We have used AA similarity criterion.

3.

The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.



Ramesh places a mirror on level ground to determine the height of a pole (with traffic light fired on it). He stands at a certain distance so that he can see the top of the pole reflected from the mirror. Ramesh's eye level is 1.5 m above the ground. The distance of Ramesh and the pole from the mirror are 1.8 m and 6 m respectively.



UNIQUESTUDYONLINE.COM

(i) Which criterion of similarity is applicable to similar triangles?

(ii) What is the height of the pole?

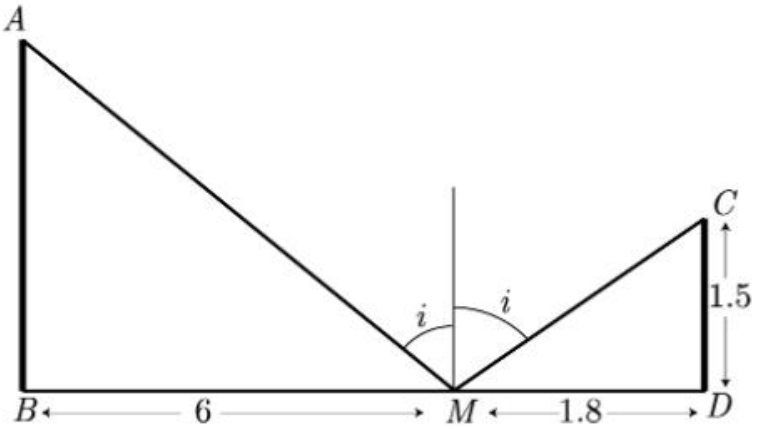
(iii) If angle of incidence is i , find tan i .

(iv) Now Ramesh move behind such that distance between pole and Ramesh is 13 meters. He place mirror between him and pole to see the reflection of light in right position. What is the distance between mirror and Ramesh ?

(v) What is the distance between mirror and pole?

Sol :

(i) Since angle of incidence and angle of reflection are the same, we draw the figure as given below.



Now $\angle AMB = \angle CMD$

Also, $\angle ABM = \angle CDM = 90^\circ$

So, by AA similarity criterion,

$$\Delta AMB \sim \Delta CDM$$

(ii) As $\Delta ABM \sim \Delta CDM$ we obtain,

$$\frac{AB}{CD} = \frac{BM}{DM}$$

$$\frac{AB}{1.8} = \frac{5}{1.5}$$

$$AB = \frac{6}{1.8} \times 1.5 = 5 \text{ m}$$

Thus, the height of the pole is 5 metres.

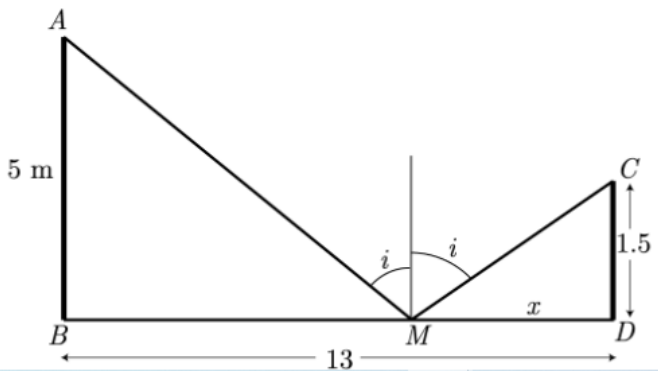
(iii) From the geometry of diagram we have

$$\angle MCD = i$$

$$\tan \angle MCD = \frac{MD}{CD}$$

$$\tan i = \frac{1.8}{1.5} = \frac{6}{5}$$

(iv) On the basis of given information we have drawn the figure as follows:



UNIQUESTUDYONLINE.COM

Once again due to AA similarity criterion,

$$\Delta AMB \sim \Delta CDM$$

$$\frac{5}{13-x} = \frac{1.5}{x}$$

$$\frac{1}{13-x} = \frac{0.3}{x}$$

$$x = 3.9 - 0.3x$$

$$1.3x = 3.9 \Rightarrow x = 3$$

(v) Distance between mirror and pole,

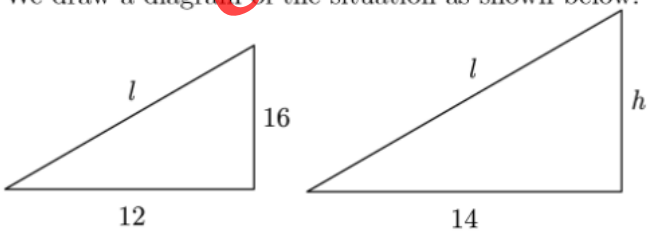
$$= 13 - x = 13 - 3 = 10 \text{ m}$$

4.

Wall Paint : A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbour's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?

UNIQUESTUDYONLINE.COM

We draw a diagram of the situation as shown below.



We use the Pythagorean Theorem to find the length of the ladder, represented by y.

$$12^2 + 16^2 = \ell^2$$

$$144 + 256 = \ell^2$$

$$400 = \ell^2$$

$$\Rightarrow l = \sqrt{400} = 20 \text{ feet}$$

The ladder is 20 feet long.

Now

$$(2+12)^2 + h^2 = \ell^2$$

$$(2+12)^2 + h^2 = 20^2$$

$$14^2 + h^2 = 20^2$$

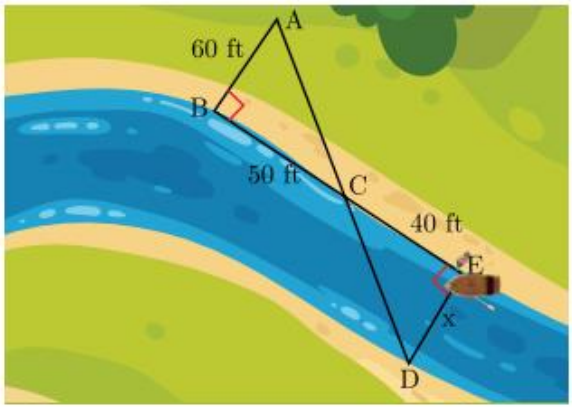
$$h = \sqrt{20^2 - 14^2} = \sqrt{204}$$

$$= 2\sqrt{51}$$

$$\approx 14.3$$

5.

Tania is very intelligent in maths. She always try to relate the concept of maths in daily life. One day she plans to cross a river and want to know how far it is to the other side. She takes measurements on her side of the river and make the drawing as shown below.



(i) Which similarity criterion is used in solving the above problem ?

(ii) Consider the following statement :

$$S_1 : \angle ACB = \angle DCE$$

$$S_2 : \angle BAC = \angle CDE$$

Which of the above statement is/are correct.

(a) S_1 and S_2 both (b) S_1

(c) S_2 (d) None

(iii) Consider the following statement :

$$S_3 : \frac{AB}{DE} = \frac{CA}{CD}$$

$$S_4 : \frac{BC}{CE} = \frac{AB}{DE}$$

$$S_5 : \frac{CA}{CD} = \frac{DE}{AB}$$

Which of the above statements are correct ?

(a) S_3 and S_5 (b) S_4 and S_5

(c) S_3 and S_4 (d) All three

(iv) What is the distance x across the river?

(v) What is the approximate length of AD shown in the figure?

Sol :

(i) We have used AA similarity criterion.

(ii) Here, $\angle ABC = \angle DEC$ (90° each)

Since vertical opposite angle are equal,

$$\angle ACB = \angle DCE$$

Thus due to AA similarity criterion,

$$\Delta ABC \sim \Delta DEC$$

and $\angle BAC = \angle CDE$

Therefore both are correct.

Thus (a) is correct option.

(iii) Since ΔABC and ΔDEC are similar triangle,

$$\frac{AB}{DE} = \frac{BC}{CE} = \frac{CA}{CD}$$

Here S_5 is not correct because $\frac{AB}{DE} = \frac{CA}{CD}$

Thus (c) is correct option.

(iv) We have $\frac{AB}{DE} = \frac{BC}{CE}$

$$\frac{60}{x} = \frac{50}{40} \Rightarrow x = 48 \text{ ft}$$

(v) $AC = \sqrt{60^2 + 50^2} = \sqrt{6100} = 71.8$

$CD = \sqrt{40^2 + 48^2} = \sqrt{3904} = 62.5$

UNIQUESTUDYONLINE.COM

$$AD = AC + CD = 71.8 + 62.5$$
$$= 140.6 \text{ (Approx)} \approx 140$$

Thus (c) is correct option.

UNIQUESTUDYONLINE.COM