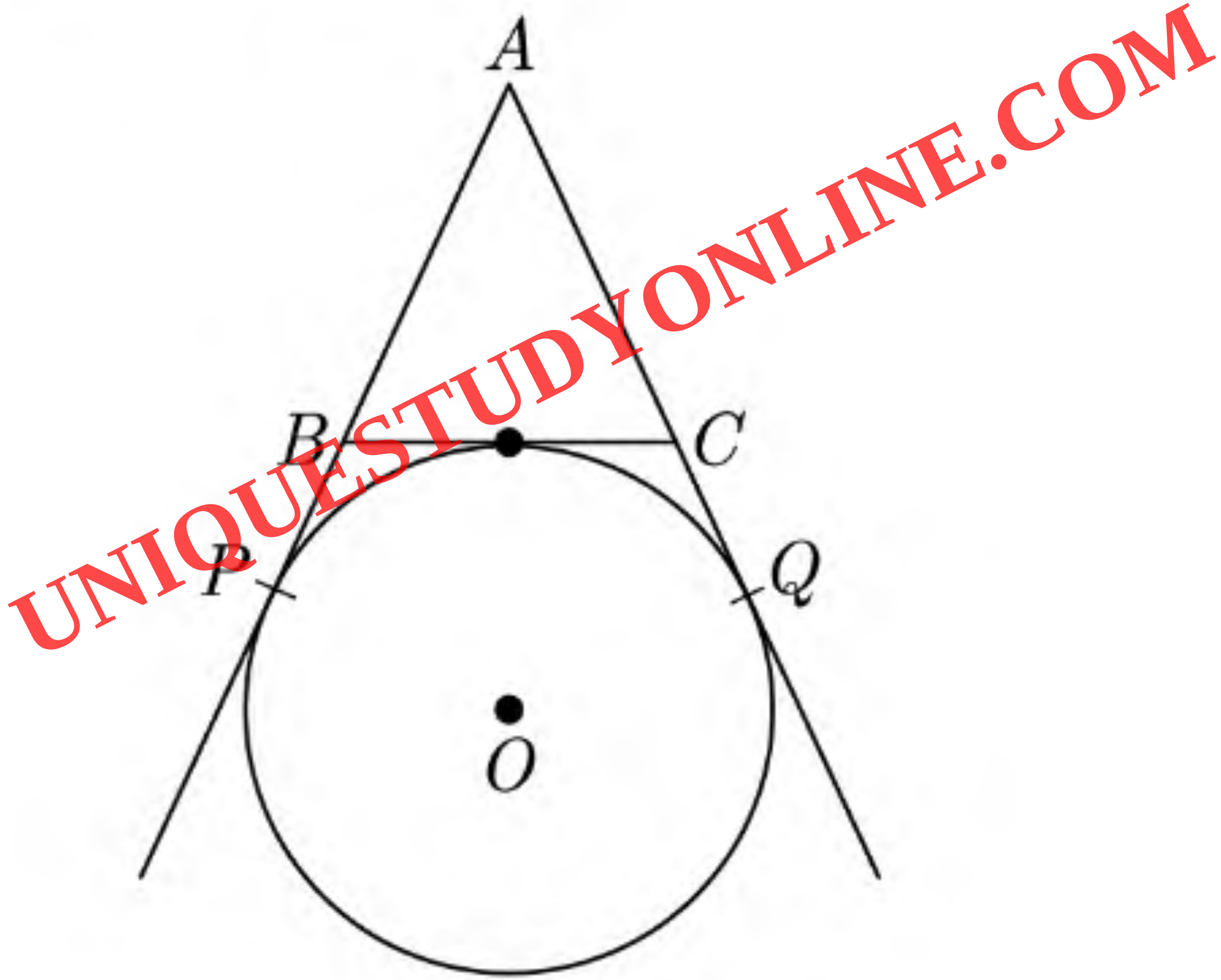
In figure, AP, AQ and BC are tangents of the circle with centre O. If $AB = 5$ cm, $AC = 6$ cm and $BC = 4$ cm, then the length of AP (in cm) is



Due to tangents from external points,

$BP = BR, CR = CQ$, and $AP = AQ$

Perimeter of $\Delta\, ABC$,

$$AB + BC + AC = AB + BR + RC + AC$$

$$5 + 4 + 6 = AB + BP + CQ + AC$$

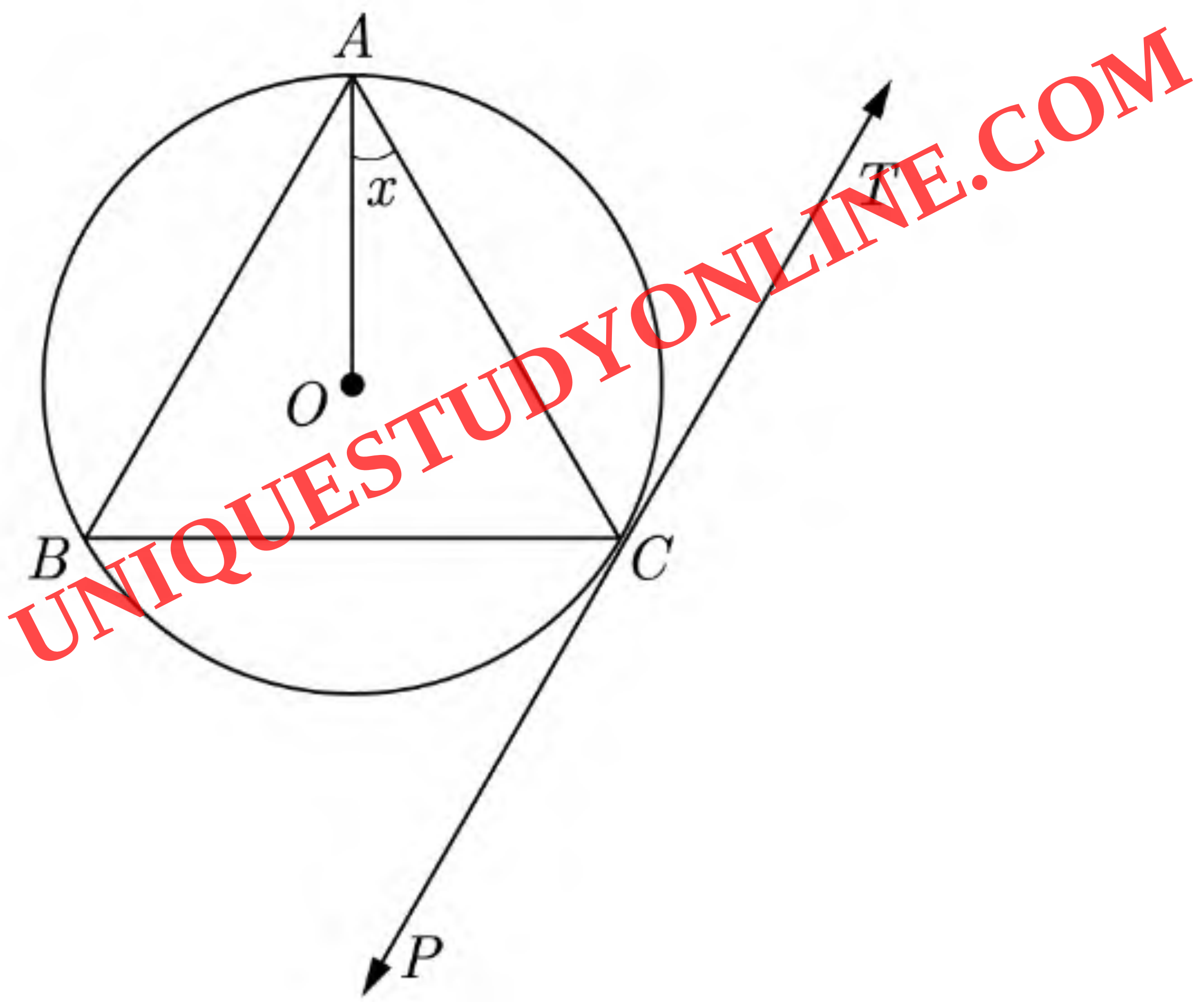
$$15 = AP + AQ$$

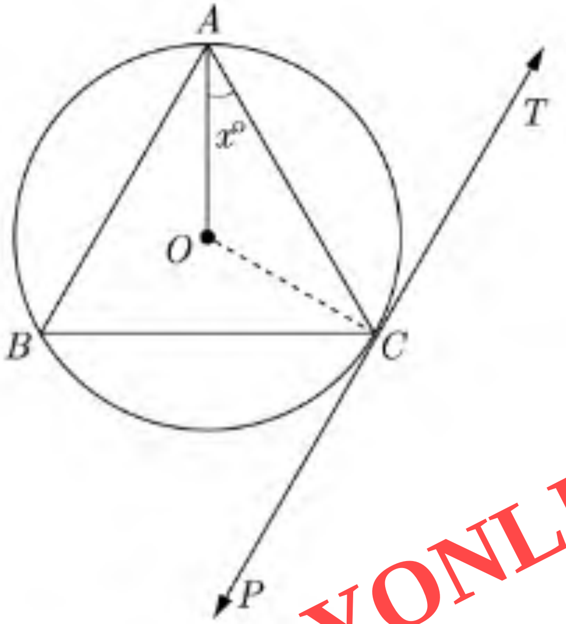
$$15 = 2AP$$

Thus $$AP = \frac{15}{2} = 7.5 \text{ cm}$$

Thus (d) is correct option.

In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118°$, then the measure of $\angle x$ is





Thus $OC \perp PT$

$$\angle OCP = 90^\circ$$

Given, $\angle ACP = 118^\circ$

$$\angle ACO = \angle ACP - \angle OCP$$

$$= 118^\circ - 90^\circ = 28^\circ$$

$$\angle ACO = 28^\circ$$

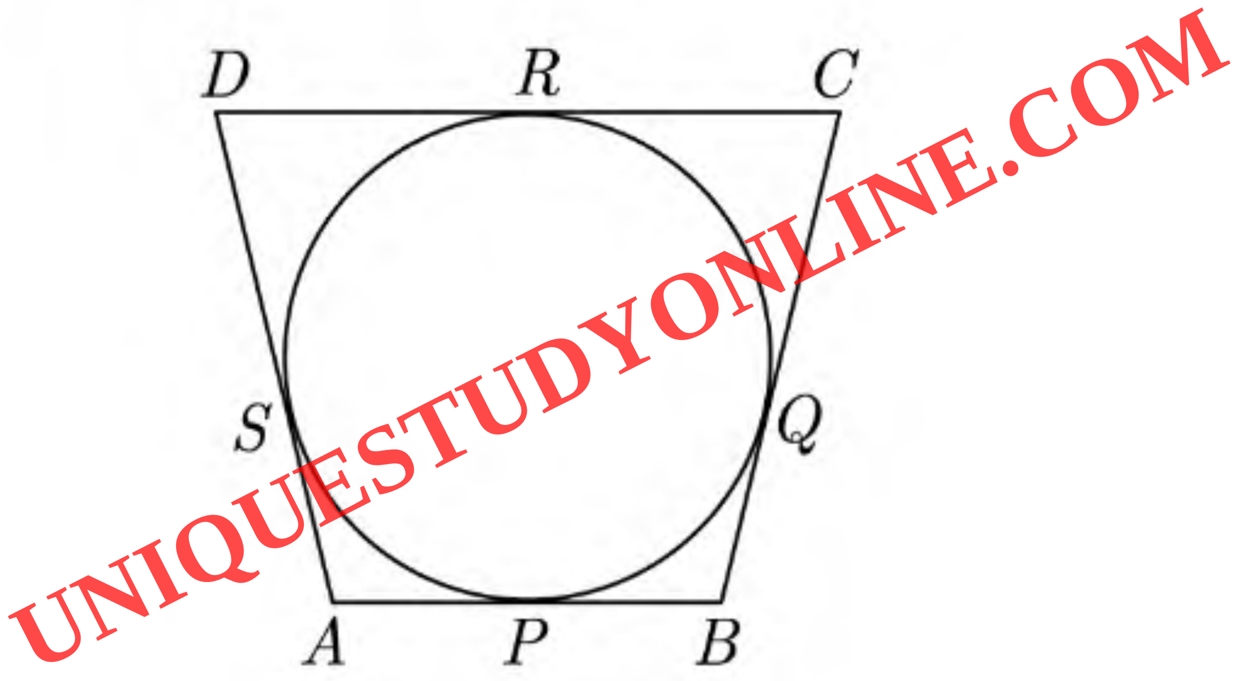
Since O is the circumcentre, thus $OA = OC$ (radius)

$$\angle OAC = \angle ACO$$

$$x = 28^\circ$$

Thus (a) is correct option.

In the given figure, a circle touches all the four sides of quadrilateral $ABCD$ with $AB = 6$ cm, $BC = 7$ cm and $CD = 4$ cm, then length of AD is



(a) 3 cm

(b) 4 cm

(c) 5 cm

(d) 6 cm

Four sides of a quadrilateral $ABCD$ are tangent to a circle.

$$AB + CD = BC + AD$$

$$6 + 4 = 7 + AD$$

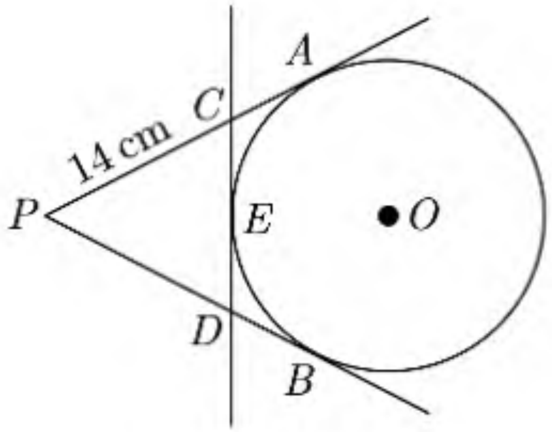
$$AD = 10 - 7 = 3 \text{ cm}$$

Thus (a) is correct option.

From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and $PA = 14$ cm. The perimeter of ΔPCD is

(a) 14 cm (b) 21 cm

(c) 28 cm (d) 35 cm



Here $PA = PB = 14$ cm

Also, CD is tangent at point E on the circle.

So, CA and CE are tangent to the circle from point C.

Therefore, $CA = CE$,

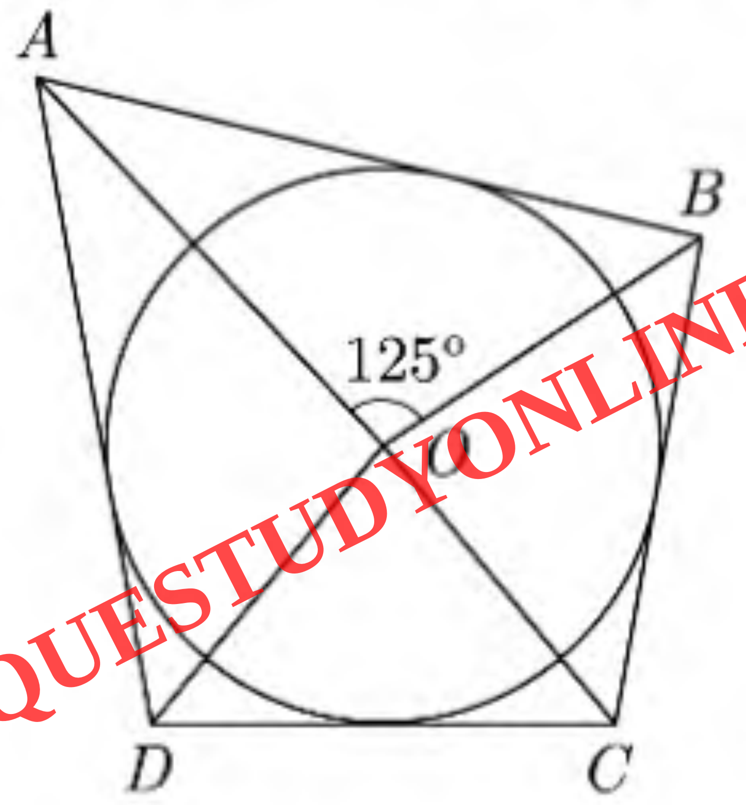
Similarly, $DB = DE$

Now, perimeter of ΔPCD,

$$\begin{aligned} PC + CD + PD &= PC + CE + ED + PD \\ &= PC + CA + PD + DB \\ &= PA + PB \\ &= 14 + 14 \\ &= 28 \text{ cm} \end{aligned}$$

Thus (c) is correct option.

In figure, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to



(a) 62.5°
(b) 45°
(c) 35°
(d) 55°

We know that, a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

i.e.

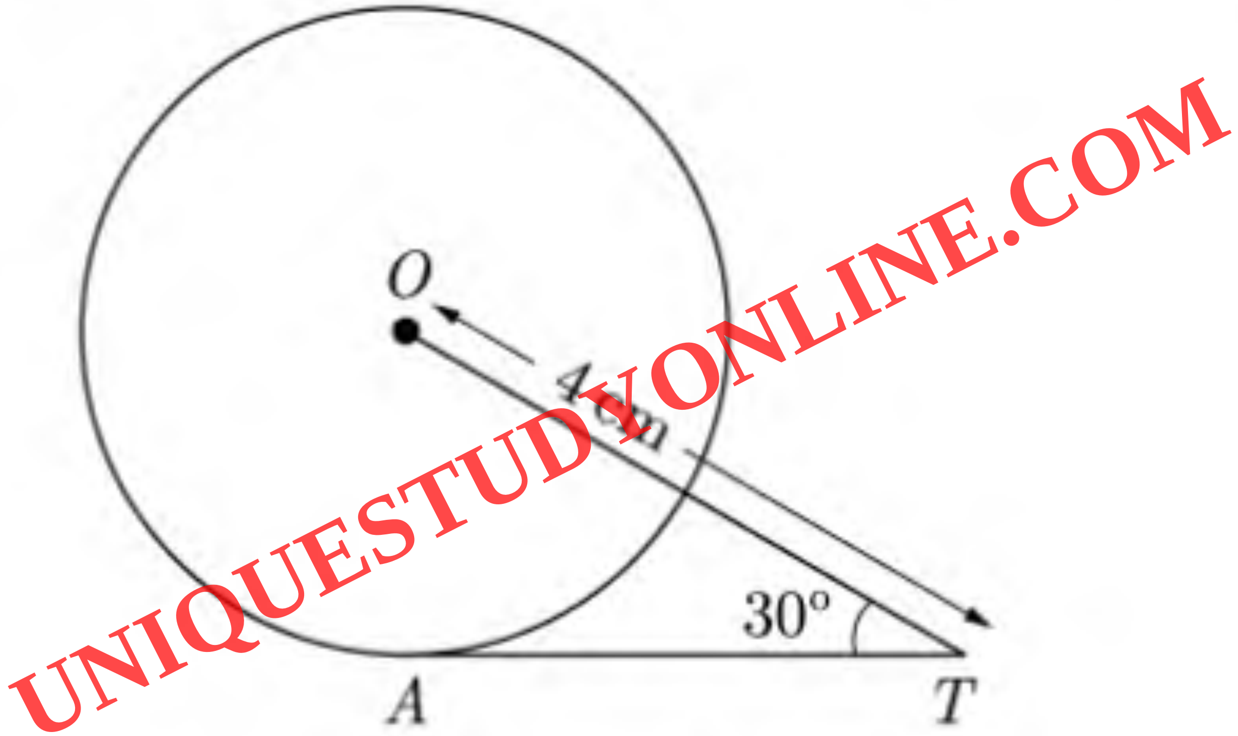
$$\angle AOB + \angle COD = 180^\circ$$

$$125^\circ + \angle COD = 180^\circ$$

$$\angle COD = 180^\circ - 125^\circ = 55^\circ$$

Thus (d) is correct option.

In figure, AT is a tangent to the circle with centre O such that $OT = 4$ cm and $\angle OTA = 30^\circ$. Then, AT is equal to



(a) 4 cm

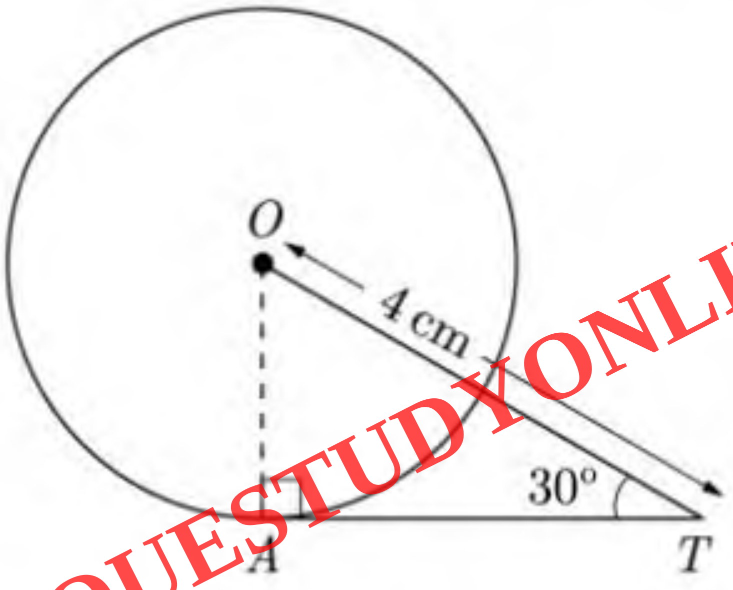
(b) 2 cm

(c) $2\sqrt{3}$ cm

(d) $4\sqrt{3}$ cm

First we joint OA. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OAT = 90^\circ \text{ and } OT = 4\text{ cm (given)}$$



In $\Delta\, OAT$,

$$\cos 30^\circ = \frac{AT}{OT}$$

$$\frac{AT}{4} = \frac{\sqrt{3}}{2}$$

$$AT = 2\sqrt{3}\text{ cm}$$

Thus (c) is correct option.

Assertion : If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

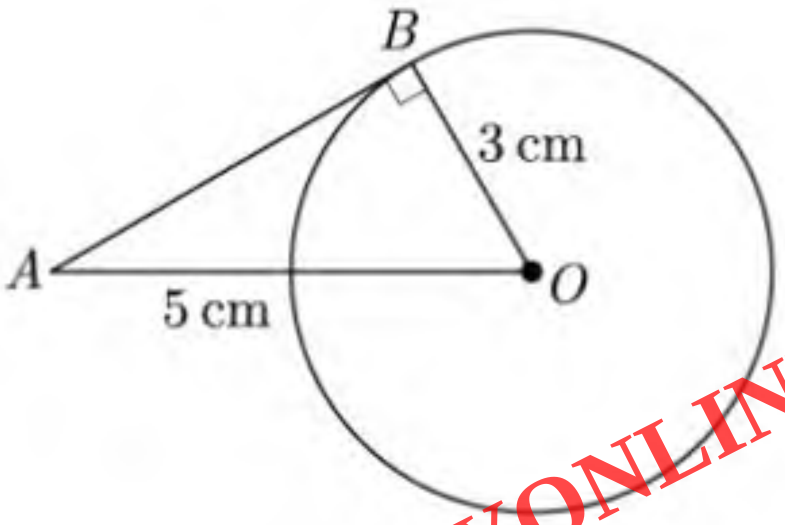
Reason : $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.



$$OA^2 = AB^2 + OB^2$$

$$5^2 = AB^2 + 3^2$$

$$AB = \sqrt{25-9} = 4\text{ cm}$$

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

Assertion : The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre.

Reason : A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

From an external point the two tangents drawn subtend equal angles at the centre.
So assertion is true.
Also, a parallelogram circumscribing a circle is a rhombus, so reason is also true but R is not correct explanation of A.

Thus (b) is correct option.

Assertion : PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110^\circ$, then $\angle APB = 90^\circ$.

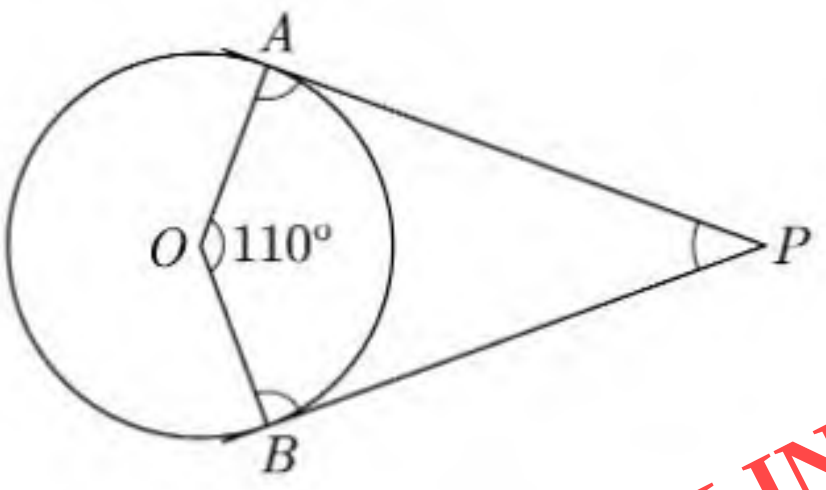
Reason : The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.



Radius is perpendicular to the tangent at point of contact.

Thus, $OA \perp AP$ and $OB \perp PB$.

In quadrilateral, $OAPB$, we have

$$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360^\circ$$

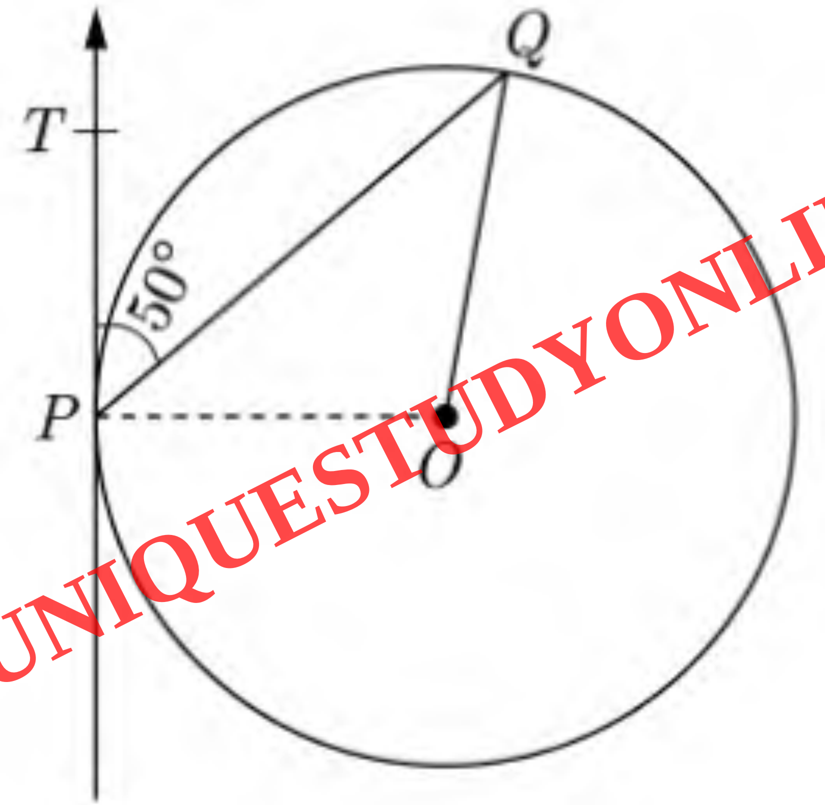
$$90^\circ + \angle APB + 90^\circ + 110^\circ = 360^\circ$$

$$\angle APB = 70^\circ$$

Assertion (A) is false but reason (R) is true.

Thus (d) is correct option.

In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with PQ. Find the angle $\angle POQ$.



Due to angle between radius and tangent,

$$\angle OPT = 90^\circ$$

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Also, $OP = OQ$ [Radii of a circle]

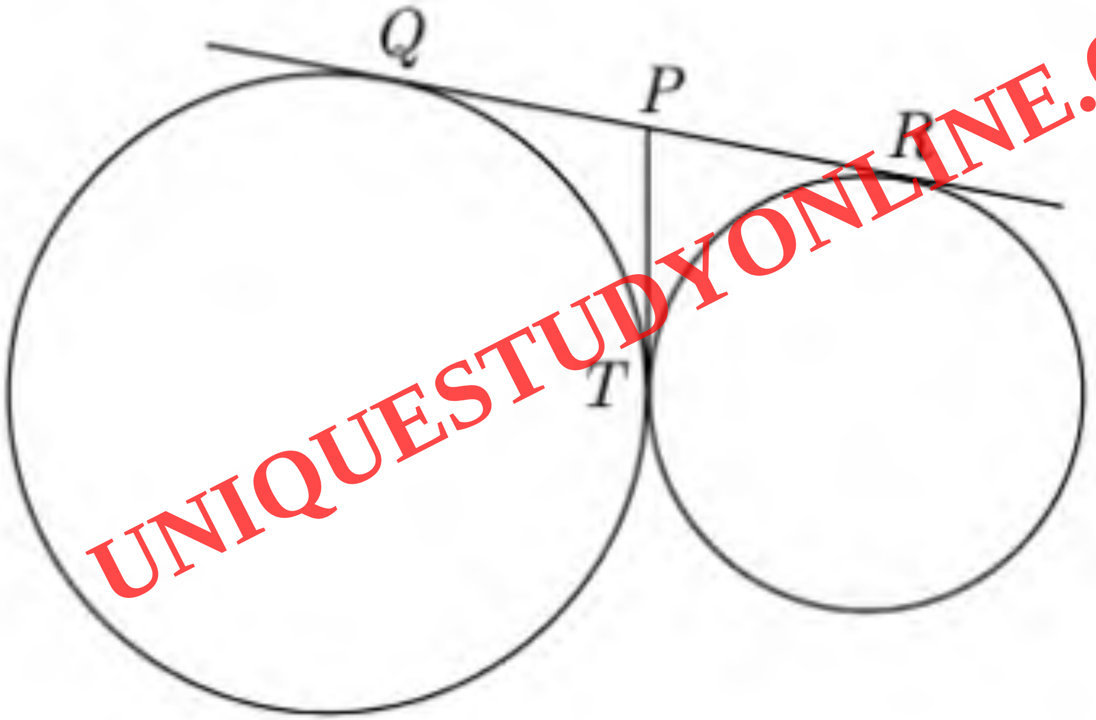
Since equal opposite sides have equal opposite angles,

$$\angle OPQ = \angle OQP = 40^\circ$$

$$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$$

$$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

In the figure, QR is a common tangent to given circle which meet at T. Tangent at T meets QR at P. If $QP = 3.8$ cm, then find length of QR.



Sol :

Let us first consider large circle. Since length of tangents from external points are equal, we can write

$$QP = PT$$

Thus $$QP = PT = 3.8 \quad \text{....(1)}$$

Now consider the small circle. For this circle we can also write using same logic,

$$PR = PT$$

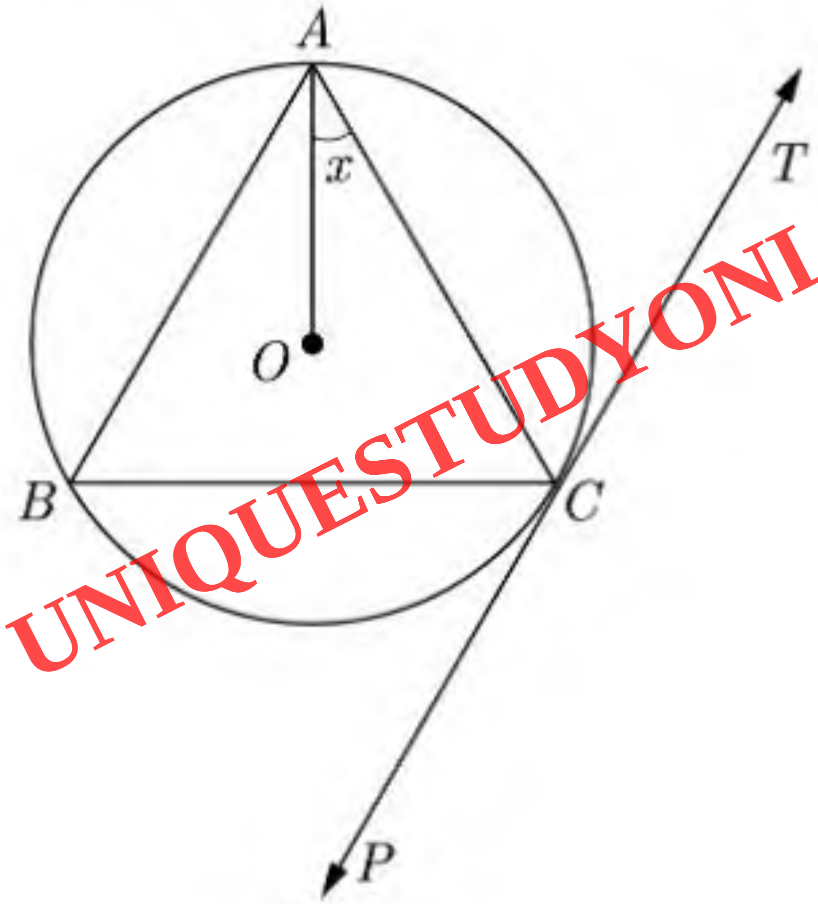
But we have $PT = 3.8$ cm

Thus $$PR = PT = 3.8 \text{ cm}$$

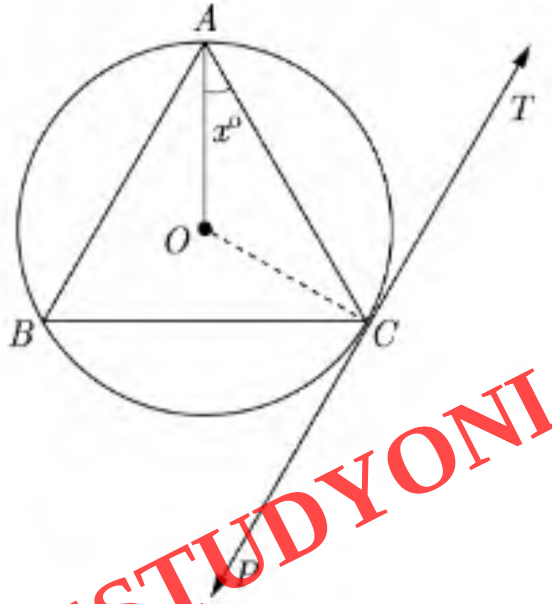
Now $$QR = QP + PR$$

$$= 3.8 + 3.8 = 7.6 \text{ cm.}$$

In the adjoining figure, PT is a tangent at point C of the circle. O is the circumference of ΔABC. If $\angle ACP = 118^\circ$, then find the angle $\angle x$?



We join OC as shown in the below figure. Here OC is the radius and PT is the tangent to circle at point C.



Thus $\quad OC \perp PT$

$$\angle OCP = 90^\circ$$

Given, $\quad \angle ACP = 118^\circ$

$$\angle ACO = \angle ACP - \angle OCP$$

$$= 118^\circ - 90^\circ = 28^\circ$$

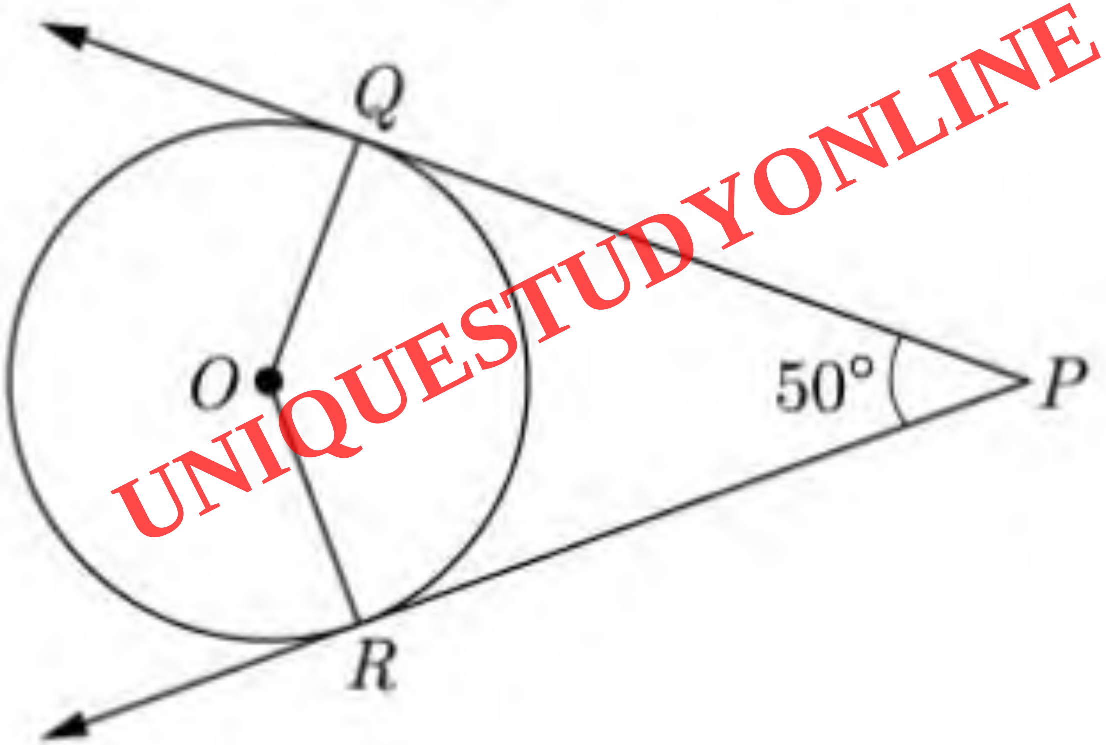
$$\angle ACO = 28^\circ$$

Since O is the circumcentre, thus $OA = OC$ (radius)

$$\angle OAC = \angle ACO$$

$$x = 28^\circ$$

In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR = 50°$, Then find $\angle OQR$.



We have $\angle QPR = \angle 50^\circ$ (Given)

Since $\angle QOR$ and $\angle QPR$ are supplementary angles

$$\angle QOR + \angle QPR = 180^\circ$$

$$\angle QOR = 180^\circ - \angle QPR$$

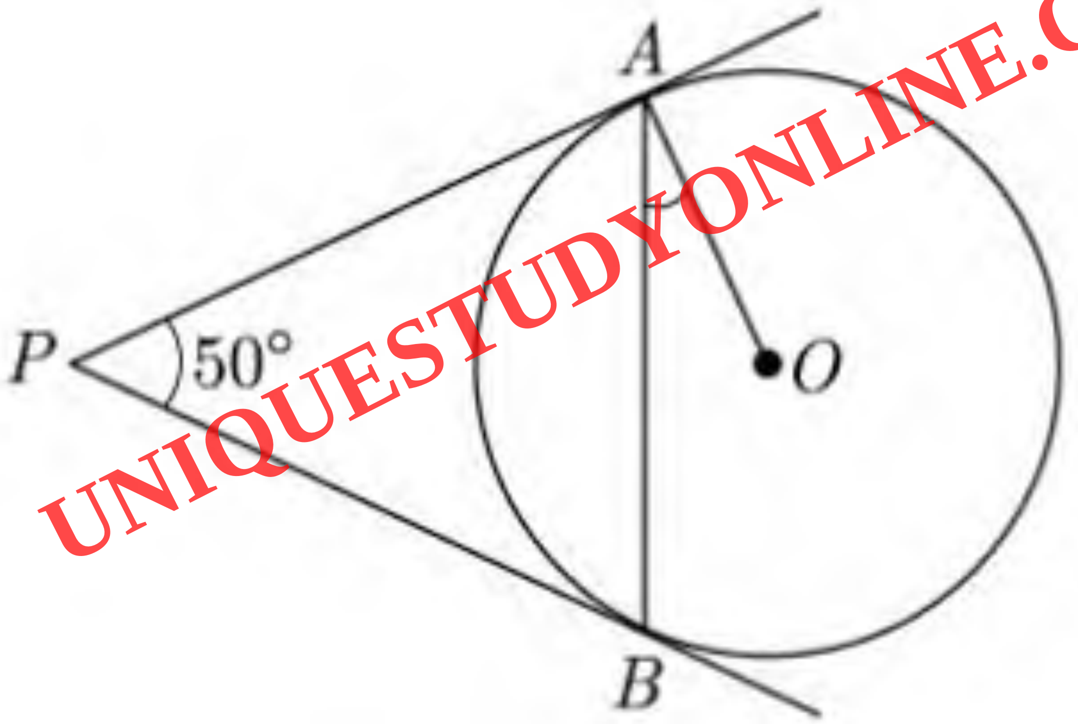
$$= 180^\circ - 50^\circ = 130^\circ$$

From $\Delta\ OQR$ we have

$$\angle OQR = \angle ORQ = \frac{180^\circ - 130^\circ}{2}$$

$$= \frac{50^\circ}{2} = 25^\circ$$

In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$.



We have $$\angle APB = 50^\circ$$

$$\angle PAB = \angle PBA = \frac{180^\circ - 50^\circ}{2} = 65^\circ$$

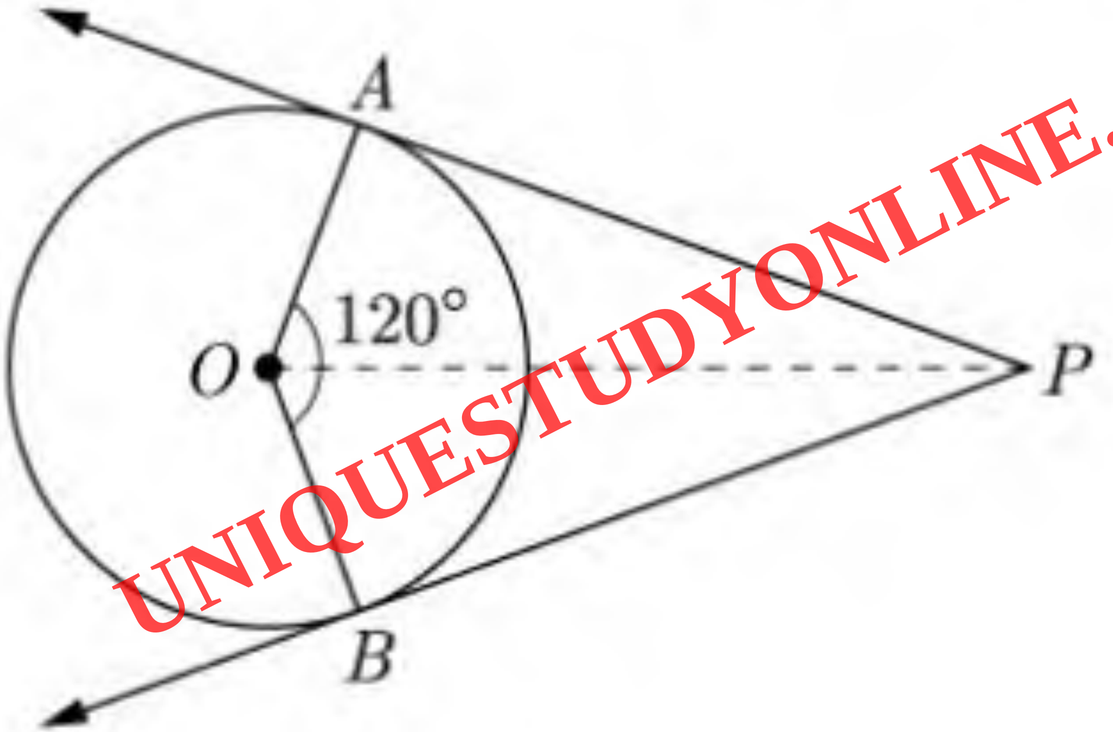
Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^\circ$$

Now $$\angle OAB = \angle OAP - \angle PAB$$

$$= 90^\circ - 65^\circ = 25^\circ$$

In the figure, PA and PB are tangents to a circle with centre O. If $\angle AOB = 120^\circ$, then find $\angle OPA$.



Here OA is radius and AP is tangent at A, since radius is always perpendicular to tangent at point of contact, we have

$$\angle OAP = 90^\circ$$

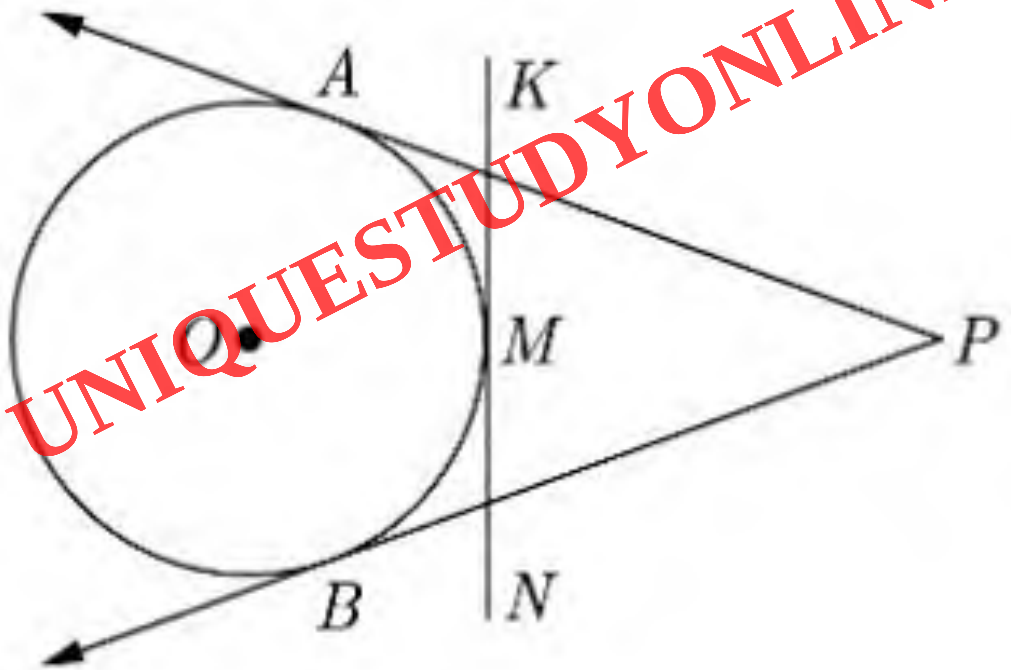
Due to symmetry we have

$$\angle AOP = \frac{\angle AOB}{2} = \frac{120^\circ}{2} = 60^\circ$$

Now in right ΔAOP we have

$$\angle APO + \angle OAP + \angle AOP = 180^\circ$$
$$\angle APO + 90^\circ + 60^\circ = 180^\circ$$
$$\angle APO = 180^\circ - 150^\circ = 30^\circ.$$

PA and PB are tangents from point P to the circle with centre O as shown in figure. At point M, a tangent is drawn cutting PA at K and PB at N. Prove that $KN = AK + BN$



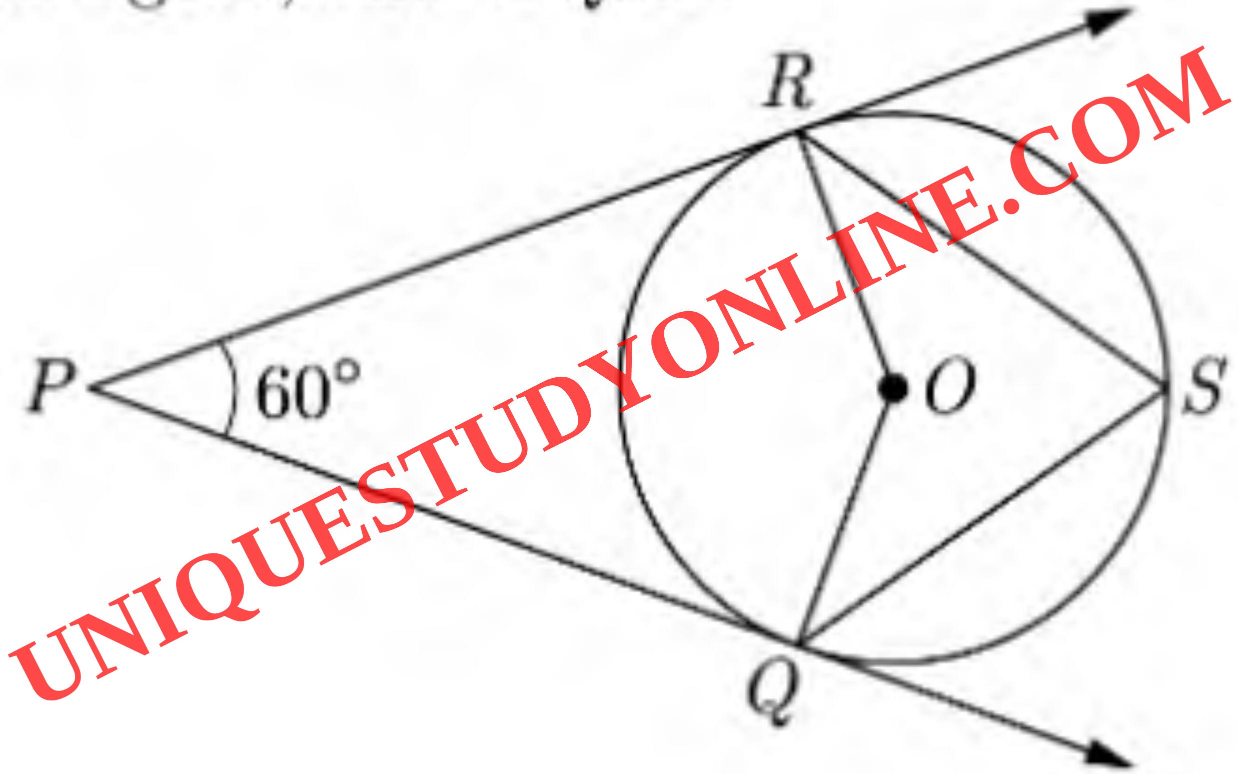
Since length of tangents from an external point to a circle are equal,

$$PA = PB, KA = KM, NB = NM,$$

$$KA + NB = KM + NM$$

$$AK + BN = KN.$$ Hence Proved

In the given figure, find $\angle QSR$.



Sum of the angles between radii and between intersection point of tangent is always 180°.

Thus $$\angle ROQ + \angle RPQ = 180^\circ$$

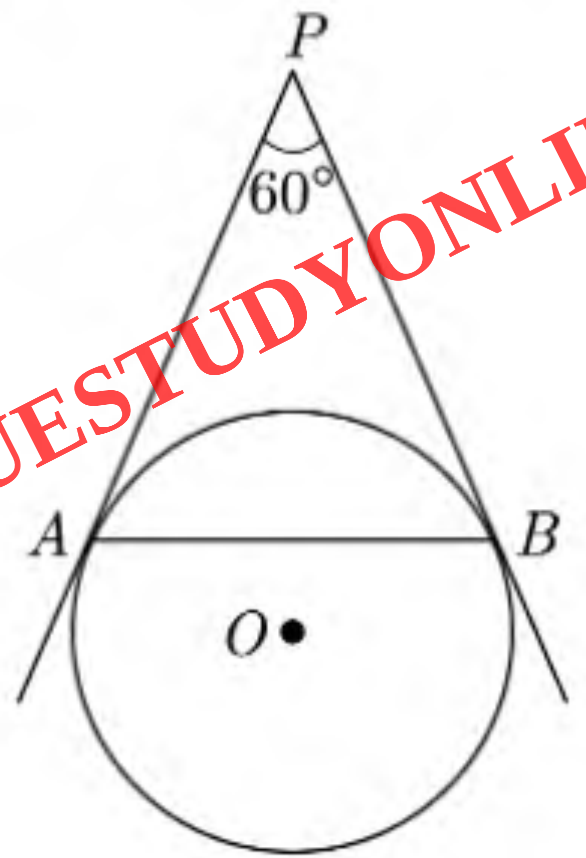
$$\angle ROQ = 180^\circ - 60^\circ = 120^\circ$$

We know that angle subtended on the centre of a circle is twice of the angle subtended on circumference of circle

Thus $$\angle QSR = \frac{1}{2}\angle ROQ = \frac{1}{2} \times 120^\circ$$

$$= 60^\circ$$

In figure, AP and BP are tangents to a circle with centre O, such that $AP = 5$ cm and $\angle APB = 60^\circ$. Find the length of chord AB.



Since length of 2 tangents drawn from an external point to a circle are equal, we have

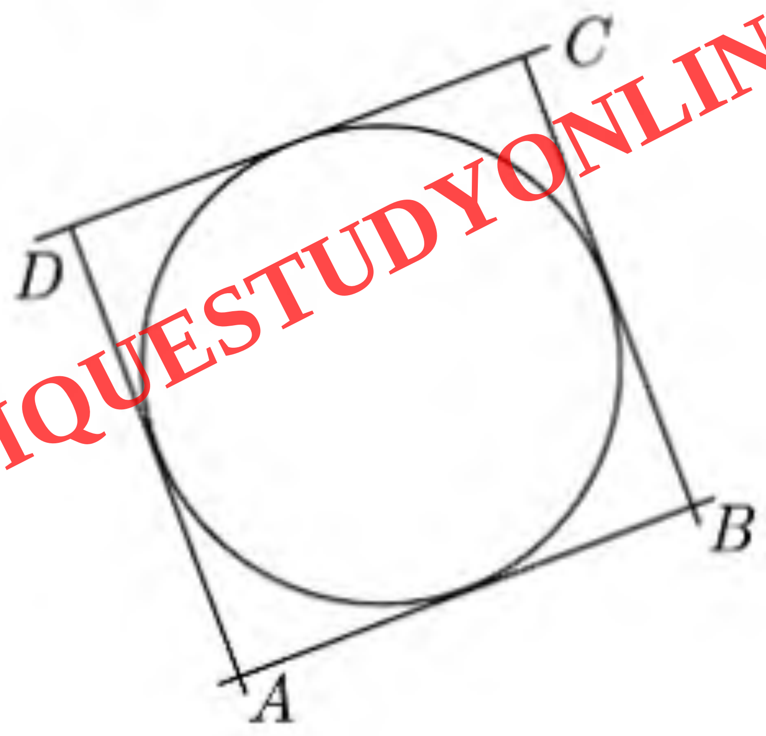
$$PA = PB$$

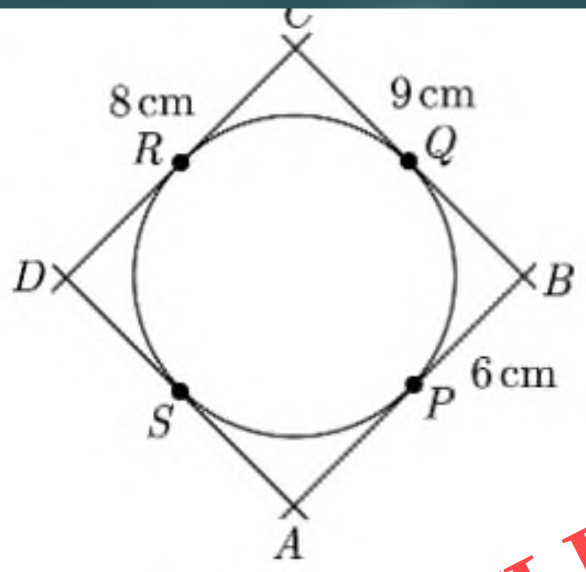
Thus $\angle PAB = \angle PBA = 60^\circ$

Hence ΔPAB is an equilateral triangle.

Therefore $AB = PA = 5$ cm.

In figure, a circle touches all the four sides of a quadrilateral $ABCD$. If $AB = 6$ cm, $BC = 9$ cm and $CD = 8$ cm, then find the length of AD.





Tangents drawn from an external point to a circle are equal in length.

Thus $AP = AS$ and let it be x.

Similarly, $BP = BQ$, $CQ = CR$ and $RD = DS$

Now $\quad BP = AB - AP = 6 - x$

$$BP = BQ = 6 - x$$

$$CQ = BC - BQ = 9 - (6 - x) = 3 + x$$

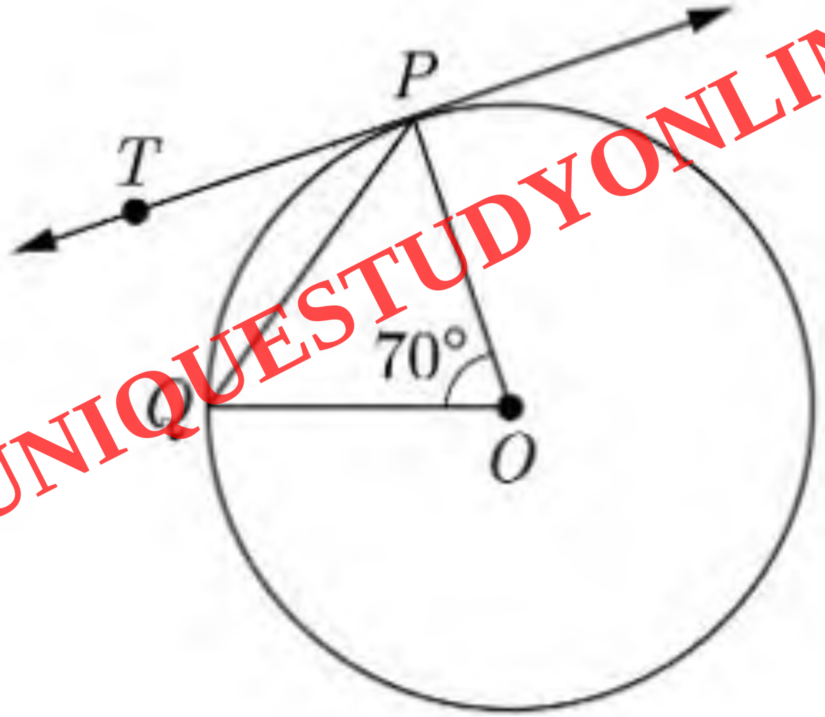
Now, $\quad CQ = CR = 3 + x$

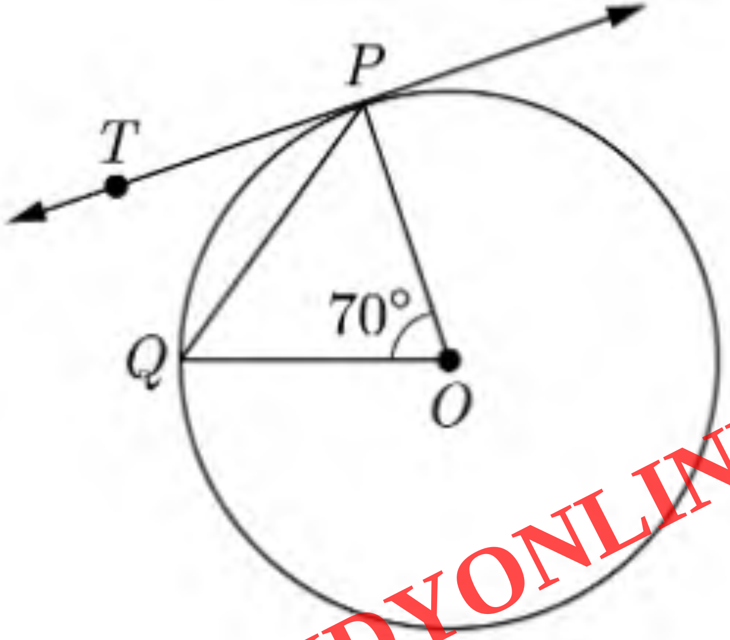
$$RD = CD - CR = 8 - (3 + x) = 5 - x$$

Now, $\quad RD = DS = 5 - x$

$$AD = AS + SD = x + 5 - x = 5$$

In figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. Find $\angle OPQ$ and $\angle TPQ$





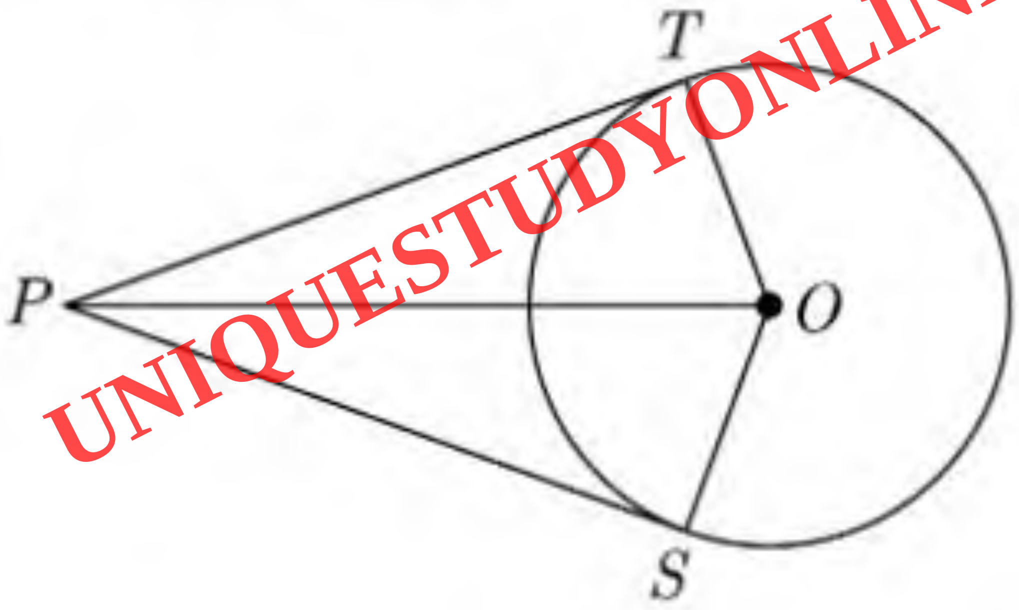
Sol :

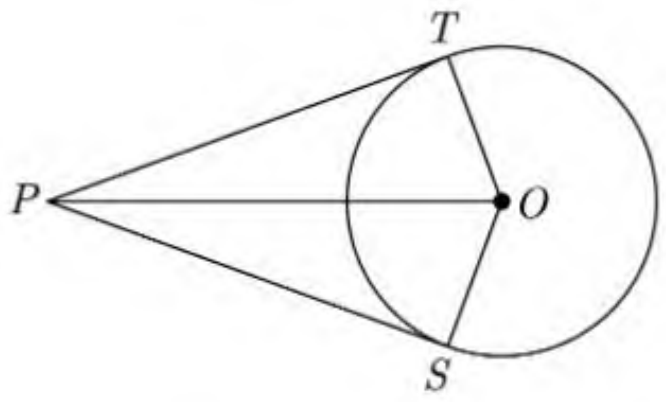
We have $\angle OPQ = \angle OQP$

$$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$$

Thus $\angle TPQ = 90^\circ - 55^\circ = 35^\circ$

In the given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that $\angle SPT = 120^\circ$, Prove that $OP = 2PS$.





Sol :

We have $\angle SPT = 120^\circ$

As OP bisects $\angle SPT$,

$$\angle OPS = \frac{120^\circ}{2} = 60^\circ$$

Since radius is always perpendicular to tangent,

$$\angle PTO = 90^\circ$$

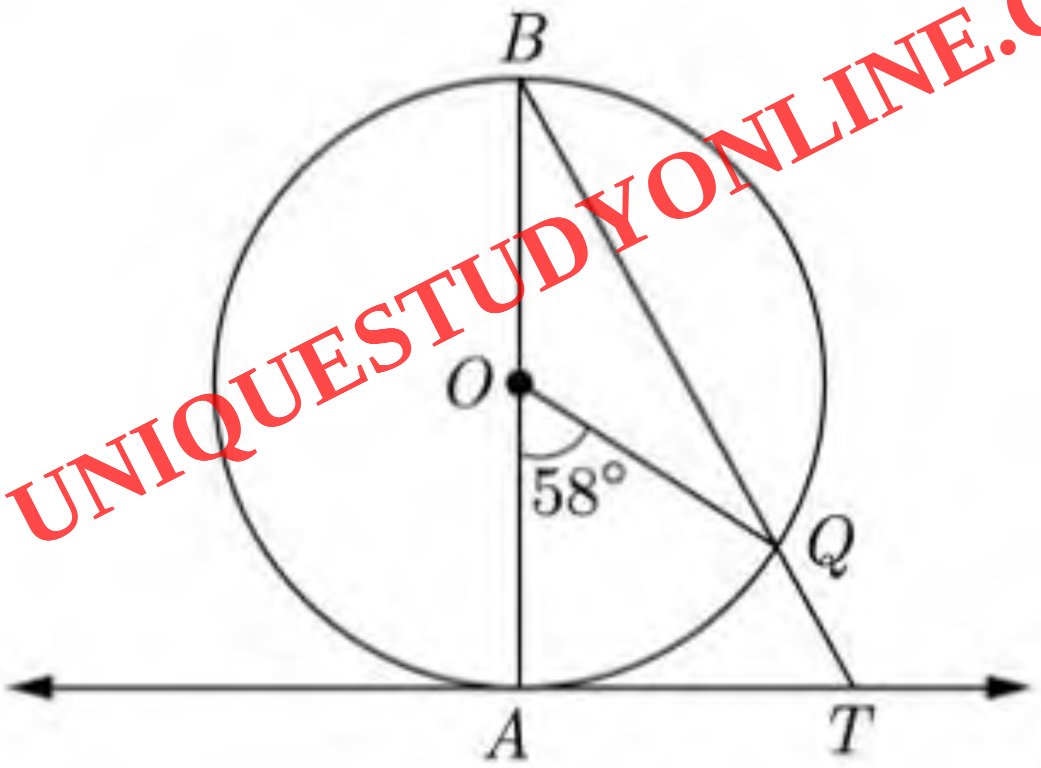
Now in right triangle POS, we have

$$\cos 60^\circ = \frac{PS}{OP}$$

$$\frac{1}{2} = \frac{PS}{OP}$$

$$OP = 2PS$$ Hence proved.

In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 58^\circ$, find $\angle ATQ$.



We have $\angle AOQ = 58^\circ$

Since angle $\angle ABQ$ and $\angle AOQ$ are the angle on the circumference of the circle by the same arc,

$$\angle ABQ = \frac{1}{2}\angle AOQ$$

$$= \frac{1}{2} \times 58^\circ = 29^\circ$$

Here OA is perpendicular to TA because OA is radius and TA is tangent at A.

Thus $\angle BAT = 90^\circ$

$$\angle ABQ = \angle ABT$$

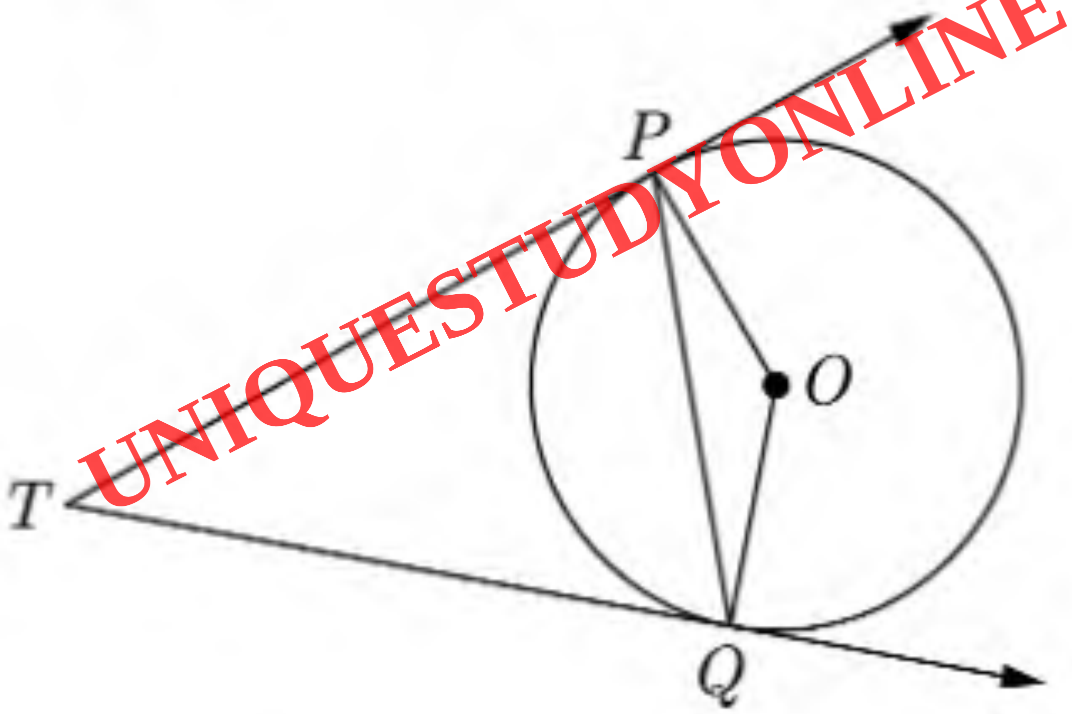
Now in ΔBAT,

$$\angle ATB = 90^\circ - \angle ABT$$

$$= 90^\circ - 29^\circ = 61^\circ$$

Thus $\angle ATQ = \angle ATB = 61^\circ$

In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.



Sol :

We have $PQ = 6$ cm, $OP = OQ = 6$ cm

Since $PQ = OP = OQ$, triangle ΔPQO is an equilateral triangle.

Thus $\angle POQ = 60^\circ$

Now we know that $\angle POQ$ and $\angle PTQ$ are supplementary angle,

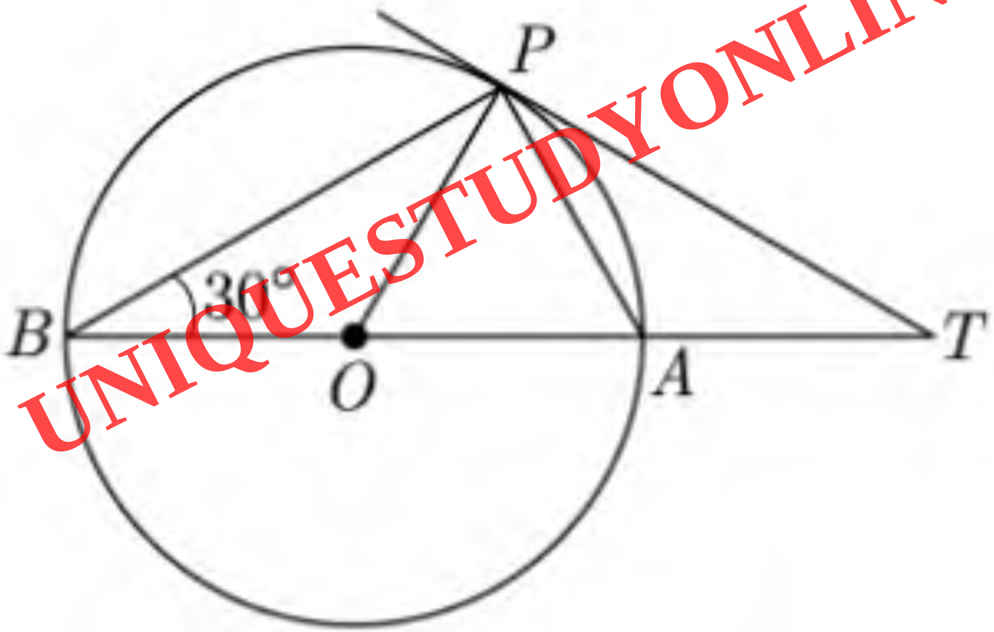
$$\angle POQ + \angle PTQ = 180^\circ$$

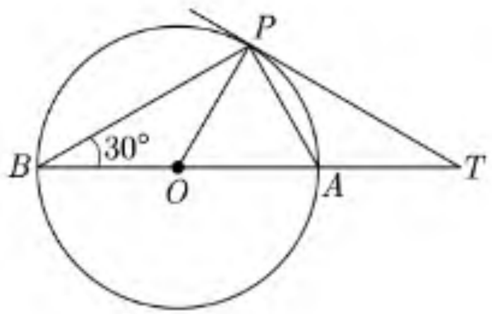
$$\angle PTQ = 180^\circ - \angle POQ$$

$$= 180^\circ - 60^\circ = 120^\circ$$

Thus $\angle PTQ = 120^\circ$

In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^0$, what is the measure of $\angle PTA$?





Sol :

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^{\circ}$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^{\circ}$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^{\circ} + 30^{\circ} = 60^{\circ}$$

Thus $\angle POT = \angle POA = 60^{\circ}$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^{\circ}$$

Now in right angle ΔOPT,

$$\angle PTO = 180^{\circ} - (\angle OPT + \angle POT)$$

Substituting $\angle OPT = 90^{\circ}$ and $\angle POT = 60^{\circ}$ we have

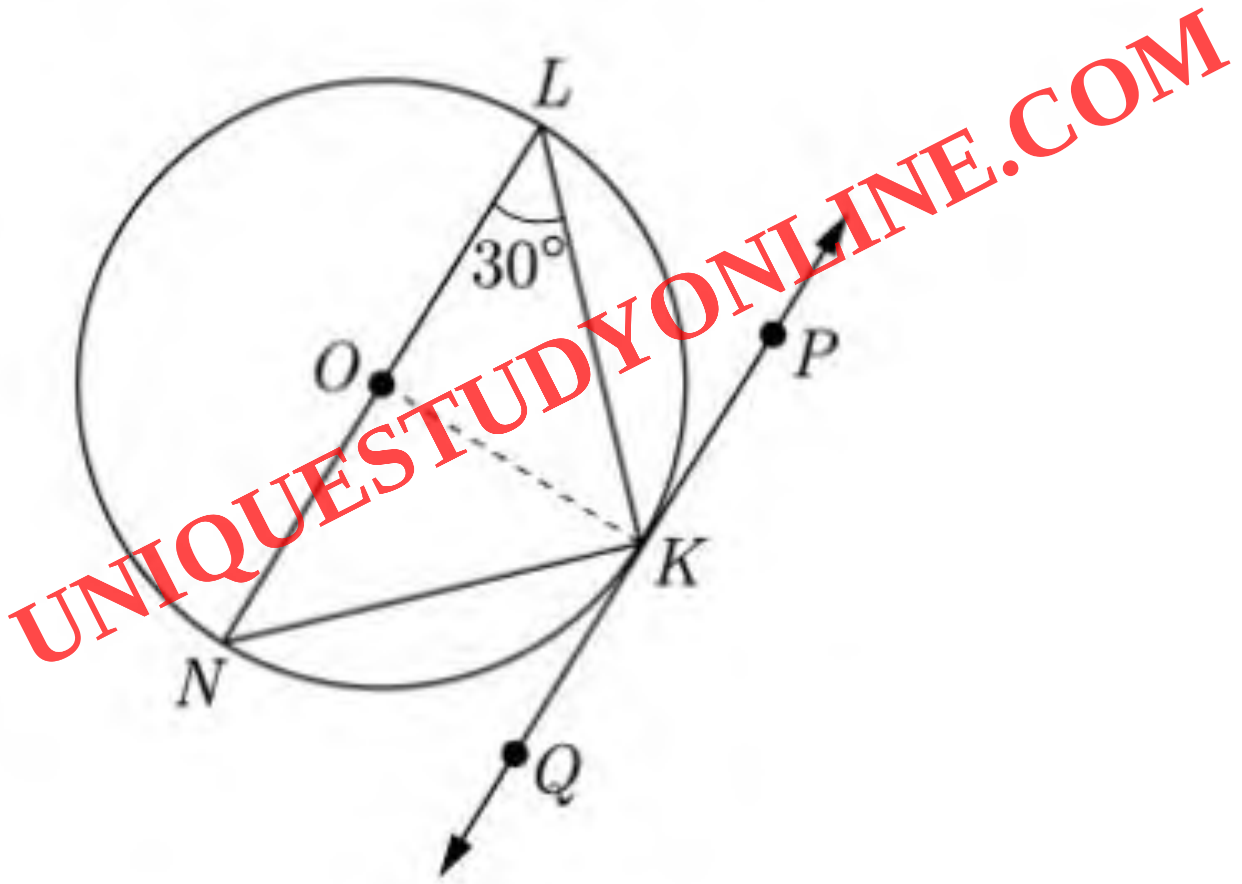
$$\angle PTO = 180^{\circ} - (90^{\circ} + 60^{\circ})$$

$$= 180^{\circ} - 150^{\circ}$$

$$= 30^{\circ}$$

Thus $\angle PTA = \angle PTO = 30^{\circ}$

In figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at K and $\angle KLN = 30^\circ$, find $\angle PKL$.



Sol :

Since OK and OL are radius of circle, thus

$$OK = OL$$

Angles opposite to equal sides are equal,

$$\angle OKL = \angle OLK = 30^\circ$$

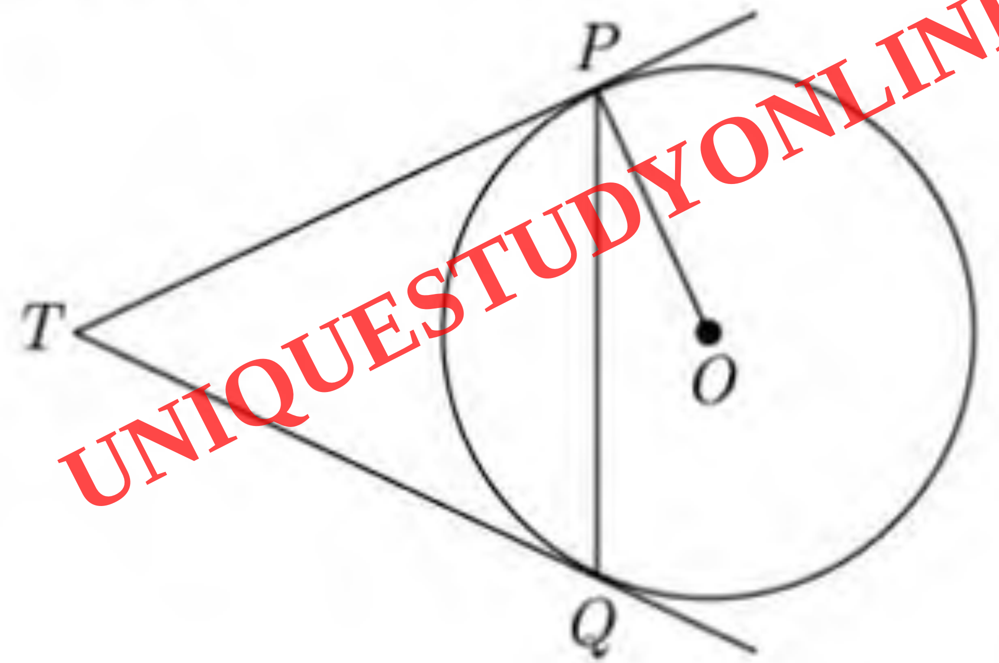
Tangent is perpendicular to the end point of radius,

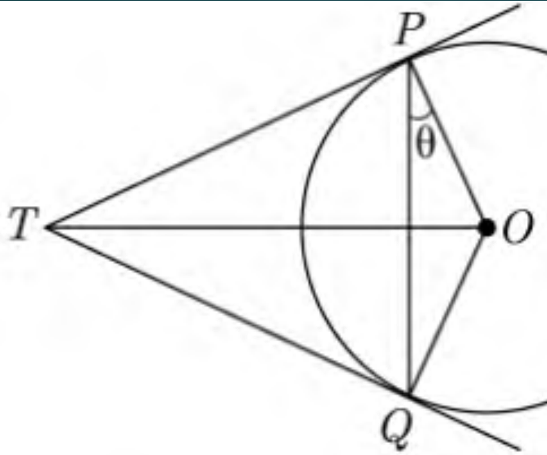
$$\angle OKP = 90^\circ \qquad \text{(Tangent)}$$

Now $$\angle PKL = \angle OKP - \angle OKL$$

$$= 90^\circ - 30^\circ = 60^\circ$$

In figure, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.





Let $\angle OPQ$ be θ, then

$$\angle TPQ = 90^\circ - \theta$$

Since, $TP = TQ$, due to opposite angles of equal sides we have

$$\angle TQP = 90^\circ - \theta$$

From angle sum property of a triangle we can write,

$$\angle TPQ + \angle TQP + \angle PTQ = 180^\circ$$

$$90^\circ - \theta + 90^\circ - \theta + \angle PTQ = 180^\circ$$

$$\angle PTQ = 180^\circ - 180^\circ + 2\theta$$

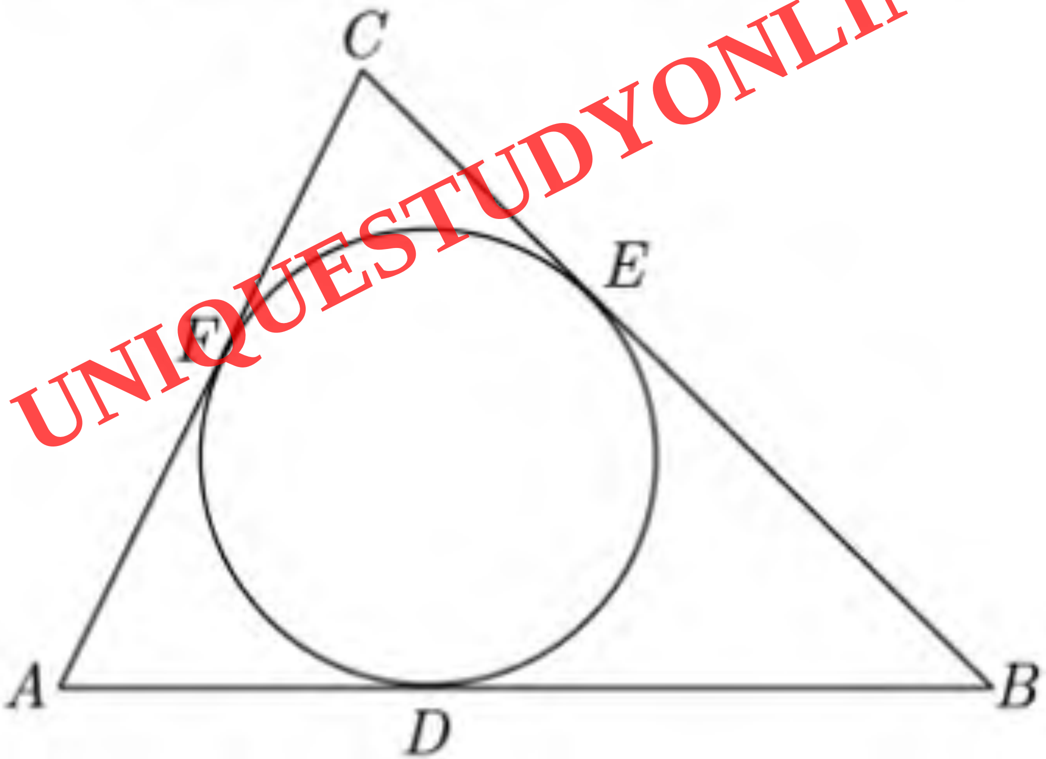
$$\angle PTQ = 2\theta$$

Hence, $$\angle PTQ = 2\angle OPQ$$

A circle is inscribed in a ΔABC, with sides AC, AB and BC as 8 cm, 10 cm and 12 cm respectively. Find the length of AD, BE and CF.

Sol :

As per question we draw figure shown below.



We have $AC = 8$ cm

$AB = 10$ cm

and $BC = 12$ cm

Let AF be x. Since length of tangents from an external point to a circle are equal,

At A, $AF = AD = x$ (1)

At B $BE = BD = AB - AD = 10 - x$ (2)

At C $CE = CF = AC - AF = 8 - x$ (3)

Now $BC = BE + EC$

$12 = 10 - x + 8 - x$

$2x = 18 - 12 = 6$

or $x = 3$

Now $AD = 3$ cm,

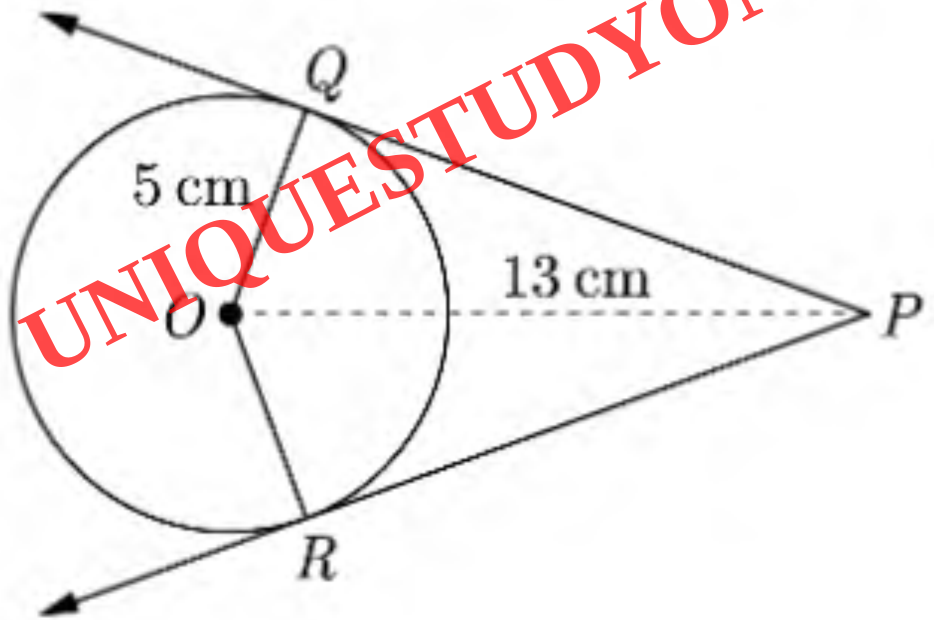
$BE = 10 - 3 = 7$ cm

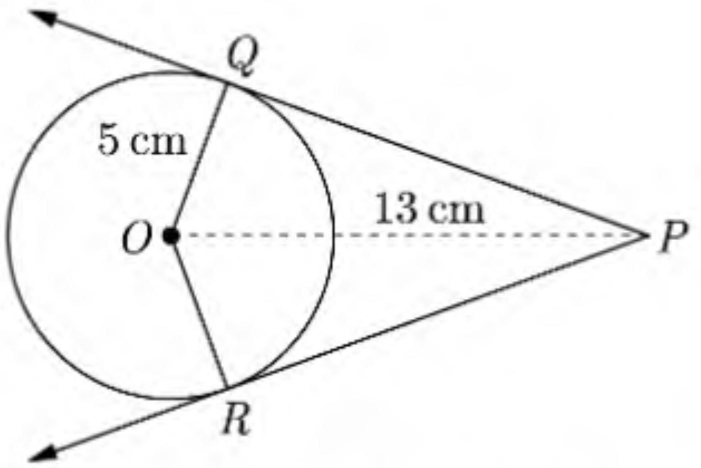
and $CF = 8 - 3 = 5$

From a point P, which is at a distant of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR are drawn to the circle, then the area of the quadrilateral $PQOR$ (in cm^2).

Sol :

As per the given question we draw the figure as below.





Here OQ is radius and QP is tangent at Q, since radius is always perpendicular to tangent at point of contact, $\Delta\, OQP$ is right angle triangle.

Now $$PQ = \sqrt{OP^2 - OB^2}$$
$$= \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$$
$$= \sqrt{144} = 12 \text{ cm}$$

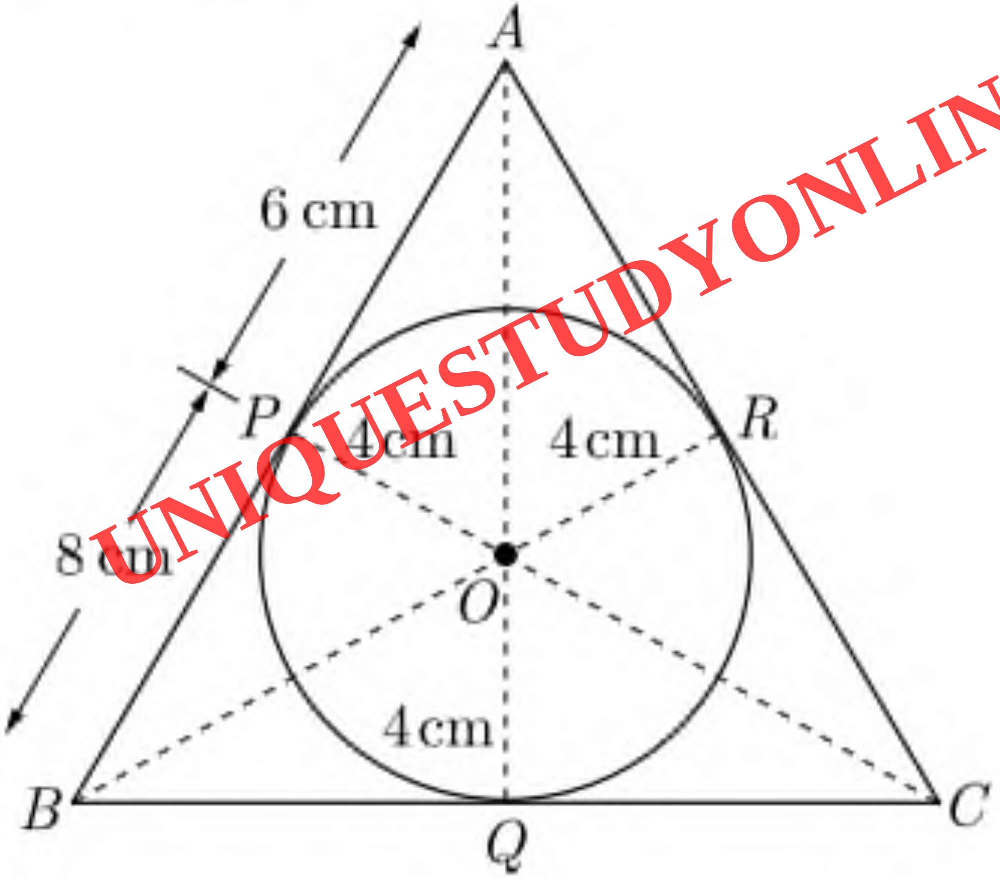
Area of triangle $\Delta\, OQP$,

$$\Delta = \tfrac{1}{2}(OQ)(QP) = \tfrac{1}{2} \times 12 \times 5 = 30$$

Area of quadrilateral $PQOR$,

$$2 \times \Delta\, POQ = 2 \times 30 = 60 \text{ cm}^2$$

In Figure the radius of incircle of ΔABC of area 84 cm^2 and the lengths of the segments AP and BP into which side AB is divided by the point of contact are 6 cm and 8 cm Find the lengths of the sides AC and BC.



Since length of tangents from an external point to a circle are equal,

At A, $\quad AP = AR = 6$ cm $\quad (1)$

At B, $\quad ,BP = BQ = 8$ cm $\quad (2)$

At C, $\quad CR = CQ = x \quad (3)$

Perimeter of ΔABC,

$$p = AP + PB + BQ + QC + CR + RA$$
$$= 6 + 8 + 8 + x + x + 6 = 28 + 2x$$

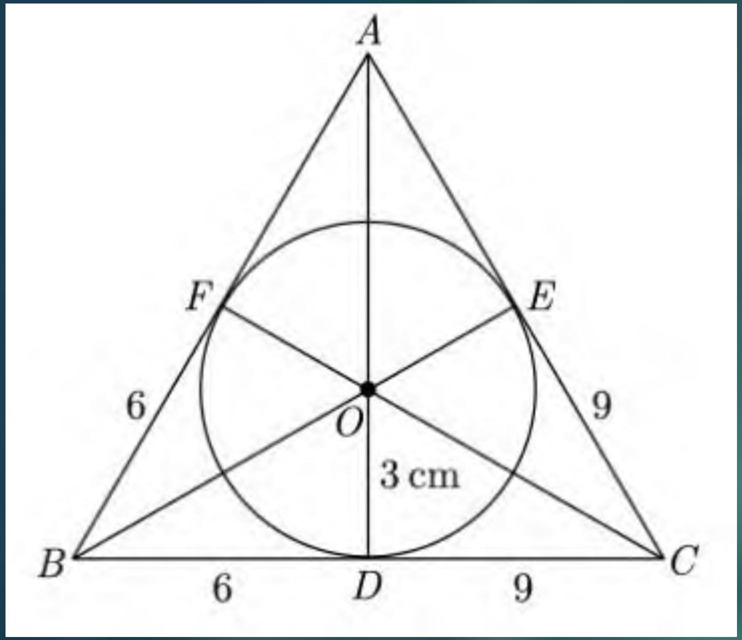
Now area $\quad \Delta ABC = \frac{1}{2} rp$

Here $r = 4$ is the radius of circle. Substituting all values we have

$$84 = \frac{1}{2} \times 4 \times (28 + 2x)$$
$$84 = 56 + 4x$$
$$21 = 14 + x \Rightarrow x = 7$$

Thus $\quad AC = AR + RC = 6 + 7 = 13$ cm

$\quad BC = BQ + QC = 8 + 7 = 15$ cm



Since tangents from an external point to a circle are equal,

$$AF = AE$$
$$BF = BD = 6 \text{ cm}$$
$$CE = CD = 9 \text{ cm}$$

Let $AF = AE = x$

Now $AB = AF + FB = 6 + x$

$$AC = AE + EC = x + 9$$
$$BC = 6 + 9 = 15 \text{ cm}$$

Perimeter of ΔABC,

$$p = 15 + 6 + x + 9 + x$$
$$= 30 + 2x$$

Now area, $\Delta ABC = \frac{1}{2} rp$

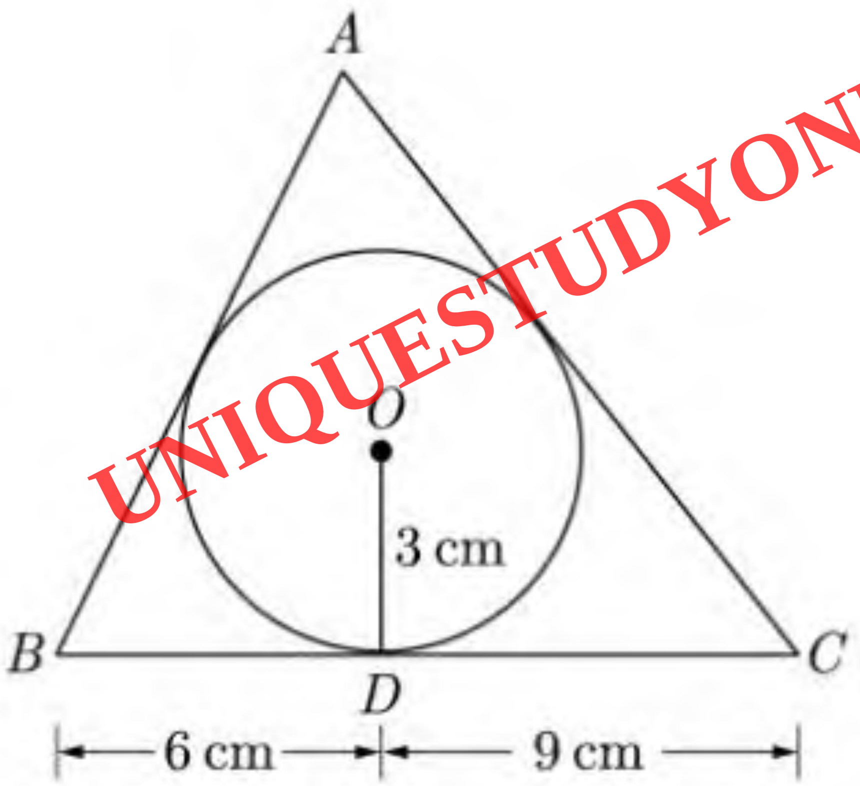
Here $r = 3$ is the radius of circle. Substituting all values we have

$$54 = \frac{1}{2} \times 3 \times (30 + 2x)$$
$$54 = 45 + 3x$$

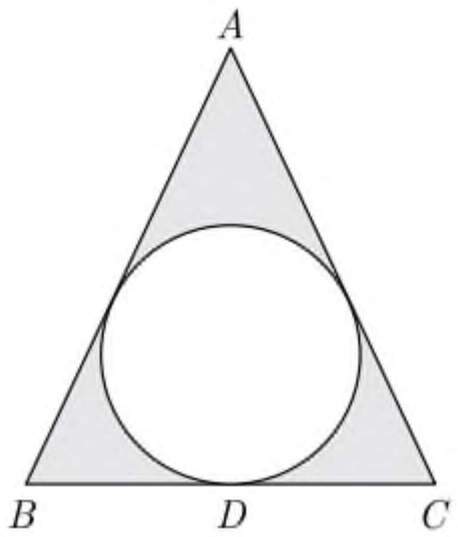
or $x = 3$

Thus $AB = 9$ cm, $AC = 12$ cm and $BC = 15$ cm.

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC.



A children's park is in the triangular shape as shown in Figure below. In the middle of the park, there is a circular region for younger children to play. It is fenced with three layers of wire. The radius of the circular region is 3 m.



Based on the above, answer the following questions:

(i) Find the perimeter (or circumference) of the circular region.

(ii) What is the total length of wire used?

(iii) What is the area of the circular region?

(iv) If $BD = 6\text{ m}$, $DC = 9\text{ m}$ and $ar(\Delta ABC) = 54\text{ m}^2$, then find the length of sides AB and AC, respectively.

(v) Find the perimeter of ΔABC.

(i) Perimeter of circular region $= 2\pi r$

$$= 2\pi \times 3$$

$$= 6\pi \text{ m}$$

(ii) Total length of wire $= 3 \times$ Circumference of circle

$$= 3 \times 6\pi$$

$$= 18\pi \text{ m}$$

(iii) Area of circular region, $A = \pi r^2$

$$= \pi (3)^2$$

$$= 9\pi \text{ m}^2$$

Since tangents drawn from external points are of equal length. So,

$$BF = BD = 6 \text{ m}$$

$$CE = CD = 9 \text{ m}$$

Let $AF = AE = x$ m

Now,

$$\text{ar}(\Delta ABC) = \text{ar}(\Delta AOB) + \text{ar}(\Delta BOC) + \text{ar}(\Delta AOC)$$

$$54 = \frac{1}{2} AB \times OF + \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE$$

$$54 = \frac{1}{2}(6 + x) \times 3 + \frac{1}{2}(6 + 9) \times 3 + \frac{1}{2}(9 + x) \times 3$$

$$54 = \frac{3}{2}[6 + x + 15 + 9 + x]$$

$$36 = 2x + 30 \rightarrow x = 3$$

So, $AB = x + 6 = 9 \text{ m}$

and $AC = x + 9 = 12 \text{ m}$

(v) Perimeter of ΔABC $s = AB + BC + AC$

$$= 9 + 15 + 12$$

$$= 36 \text{ m}$$