UNIQUE STUDY POINT

IMPORTANT QUESTIONS PPT

CLASS X

ARITHMATIC PROGRESSION

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The n^{th} term of the AP $a, 3a, 5a, \ldots$ is

(a) na

(b) $(2n-1)a$

(c) $(2n+1)a$

(d) $2na$

Given AP is a, $3a$, $5a$, ...

First term is a and $d = 3a - a = 2a$

n^{th} term

$$\begin{aligned} a_n &= a + (n-1)d \\ &= a + (n-1)2a \\ &= a + 2na - 2a \\ &= 2na - a = (2n-1)a \end{aligned}$$

Thus (b) is correct option.

The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is

(a) 1

(b) $\frac{1}{p}$

(c) -1

(d) $-\frac{1}{p}$

The value of x for which $2x, (x+10)$ and $(3x+2)$ are the three consecutive terms of an AP, is

(a) 6

(b) -6

(c) 18

(d) -18

Since $2x$, $(x+10)$ and $(3x+2)$ are in AP we obtain,

$$\begin{aligned}(x+10)-2x &= (3x+2)-(x+10)\\ -x+10 &= 2x-8\\ -x-2x &= -8-10\\ -3x &= -18 \Rightarrow x = 6\end{aligned}$$

Thus (a) is correct option.

The first term of AP is p and the common difference is q, then its 10th term is

(a) $q+9p$

(b) $p-9q$

(c) $p+9q$

(d) $2p+9q$

We have $a = p$ and $d = q$

$$a_{10} = a + (10 - 1)\,d$$

$$= p + 9q$$

Thus (c) is correct option.

In an AP, if $d = -4$, $n = 7$ and $a_n = 4$, then a is equal to

(a) 6

(b) 7

(c) 20

(d) 28

In an AP,

$$\begin{aligned} a_n &= a + (n-1)d \\ 4 &= a + (7-1)(-4) \\ 4 &= a + 6(-4) \\ 4 + 24 &= a \Rightarrow a = 28 \end{aligned}$$

Thus (d) is correct option.

The 11th term of an AP $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots\ldots,$ is

(a) -20

(b) 20

(c) -30

(d) 30

Here, $a = -5,\ d = \frac{-5}{2} - (-5) = \frac{5}{2}$

nth term, $a_n = a + (n-1)\,d$

$$a_{11} = -5 + (11-1) \times \left(\frac{5}{2}\right)$$

$$a_{11} = -5 + 25 = 20$$

Thus (b) is correct option.

If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(a) 5
(b) 20
(c) 25
(d) 30

Given, the common difference of AP i.e, $d = 5$

Using, $a_n = a + (n-1)\,d$

We have, $a_{18} = a + (18-1)\,d$

and $a_{13} = a + (13-1)\,d$

Now, $a_{18} - a_{13} = a + (18-1)\,d - [a + (13-1)\,d]$

$= a + 17 \times 5 - a - 12 \times 5$

$= 85 - 60 = 25$

Thus (c) is correct option.

The 4th term from the end of an AP $-11, -8, -5, \ldots, 49$ is

(a) 37 (b) 40

(c) 43 (d) 58

Common difference,

$$d = -8 - (-11) = -8 + 11 = 3$$

Last term, $l = 49$

nth term of an AP from the end is

$$a_n = l - (n-1)d$$

, $a_4 = 49 - (4-1) \times 3$

$$= 49 - 9 = 40$$

Thus (b) is correct option.

The sum of first 16 terms of the AP 10, 6, 2, ... is

(a) -320

(b) 320

(c) -352

(d) -400

Given, AP, is 10, 6, 2 .....

We have $a = 10$ and $d = (6 - 10) = -4$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$S_{16} = \frac{16}{2}[2a + (16-1)d]$$

$$= 8[2 \times 10 + 15(-4)]$$

$$= 8(20 - 60)$$

$$= 8(-40) = -320$$

Thus (a) is correct option.

In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(a) 19

(b) 21

(c) 38

(d) 42

We have $a = 1,\ a_n = 20$ and $S_n = 399$

Now, $S_n = \frac{n}{2}(a + a_n)$

$$399 = \frac{n}{2}(1 + 20)$$

$$n = \frac{399 \times 2}{21} = 38.$$

If the sum of the series $2 + 5 + 8 + 11$ .... is 60100, then the number of terms are

(a) 100 (b) 200

(c) 150 (d) 250

We have $a = 2$. $d = 5 - 2 = 3$ and $S_n = 60100$

$$\frac{n}{2}[2a + (n-1)d] = S_n$$

$$\frac{n}{2}[4 + (n-1)3] = 60100$$

$$n(3n+1) = 120200$$

$$3n^2 + n - 120200 = 0$$

$$(n-200)(3n+601) = 0 \Rightarrow n = 200, \frac{601}{3}$$

Thus $n = 200$ because n can not be fraction.

Thus (b) is correct option.

If the nth term of an AP is given by $a_n = 5n - 3$, then the sum of first 10 terms if

(a) 225

(b) 245

(c) 255

(d) 270

We have $a_n = 5n - 3$

Substituting $n = 1$ and 10 we have

$$a = 2$$

$$a_{10} = 47$$

Thus $S_n = \frac{n}{2}(a + a_n)$

$$S_{10} = \frac{10}{2}(2 + 47)$$

$$= 5 \times 49 = 245$$

Thus (b) is correct option.

Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is

(a) -1 (b) -8

(c) 7 (d) -9

4th term of first AP,

$$a_4 = -1 + (4-1)\,d = -1 + 3d$$

and 4th term of second AP,

$$a'_4 = -8 + (4-1)\,d = -8 + 3d$$

Now, the difference between their 4th terms,

$$a'_4 - a_4 = (-1 + 3d) - (-8 + 3d)$$

$$= -1 + 3d + 8 - 3d = 7$$

Hence, the required difference is 7.

Thus (c) is correct option.

If a, b, c, d, e, f are in AP, then $e - c$ is equal to

(a) $2(c - a)$

(b) $2(d - c)$

(c) $2(f - d)$

(d) $(d - c)$

Let x be the common difference of the AP a, b, c, d, e, f.

For an AP,

$$\begin{aligned} a_n &= a+(n-1)d \\ e &= a+(5-1)x \\ e &= a+4x \qquad \text{...(1)} \end{aligned}$$

and

$$\begin{aligned} c &= a+(3-1)x \\ c &= a+2x \qquad \text{...(2)} \end{aligned}$$

Using equation (1) and (2), we get

$$\begin{aligned} e-c &= a+4x-a-2x \\ &= 2x = 2(d-c) \end{aligned}$$

Thus (b) is correct option.

If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its term will be

(a) 7

(b) 11

(c) 18

(d) 0

In an AP, $a_n = a + (n-1)d$

Now, according to the question,

$$7a_7 = 11a_{11}$$

$$7[a + (7-1)d] = 11[a + (11-1)d]$$

$$7(a + 6d) = 11(a + 10d)$$

$$7a + 42d = 11a + 110d$$

$$4a + 68d = 0$$

$$4(a + 17d) = 0$$

$$a + 17d = 0 \quad \text{...(1)}$$

18th term of an AP,

$$a_{18} = a + (18-1)d = a + 17d$$

But from equation (1) this is zero.

If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(a) 30 (b) 33

(c) 37 (d) 38

We have $a_2 = 13$, and $a_5 = 25$

In an AP, $a_n = a + (n-1)d$

$$a_2 = a + (2-1)d = 13$$

$$a + d = 13 \quad \text{...(1)}$$

and $a_5 = a + (5-1)d = 25$

$$a + 4d = 25 \quad \text{...(2)}$$

Subtracting equation (1) from equation (2), we get

$$3d = 25 - 13 = 12 \Rightarrow d = 4$$

From equation (1), $a = 13 - 4 = 9$

Now, 7th term, $a_7 = a + (7-1)d$

$$= 9 + 6 \times 4 = 33$$

Thus (b) is correct option.

Find the number of natural numbers between 102 and 998 which are divisible by 2 and 5 both.

$$a_n = a + (n-1)d$$
$$990 = 110 + (n-1)10$$
$$880 = 10(n-1)$$
$$88 = n-1$$
$$n = 88 + 1 = 89$$

If the sum of n terms of an AP is $2n^2 + 5n$, then find the 4^{th} term.

$$
\begin{aligned}
a_n &= S_n - S_{n-1} \\
a_n &= (2n^2 + 5n) - [2(n-1)^2 + 5(n-1)] \\
&= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5] \\
&= 2n^2 + 5n - 2n^2 - n + 3 \\
&= 4n + 3
\end{aligned}
$$

Thus 4^{th} term $\quad a_4 = 4 \times 4 + 3 = 19$

If the sum of first m terms of an AP is the same as the sum of its first n terms, show that the sum of its first $(m+n)$ terms is zero.

$$S_m = S_n$$

$$\frac{m}{2}\{2a+(m-1)d\} = \frac{n}{2}\{2a+(n-1)d\}$$

$$2a(m-n)+\{m(m-1)-n(n-1)d\}=0$$

$$2a(m-n)+[(m^2-n^2)-(m-n)d]=0$$

$$(m-n)[2a+(m+n-1)d]=0$$

$$2a+(m+n-1)d=0$$

Now, $S_{m+n} = \frac{m+n}{2}\{2a+(m+n-1)d\}$

$$= \frac{m+n}{2}\times 0 = 0$$

Which term of the AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

Given AP is 3, 15, 27, 39......

Here, first term, $a = 3$ and common difference, $d = 12$

Now, 21^{st} term of AP is

$$a_n = a + (n-1)d$$

$$a_{21} = 3 + (21-1) \times 12$$

$$= 3 + 20 \times 12 = 243$$

Therefore, 21^{st} term is 243.

Now we need to calculate term which is 120 more than 21^{st} term i.e it should be $243 + 120 = 363$

Therefore,

$$a_n = a + (n-1)d$$

$$363 = 3 + (n-1)12$$

$$360 = 12(n-1)$$

$$n - 1 = 30 \Rightarrow n = 31$$

So, 31^{st} term is 120 more than 21^{st} term.

Find the middle term of the AP 213, 205, 197, ... 37.

Let the first term of an AP be a, common difference be d and number of terms be m.

Here, $a = 213, d = 205 - 213 = -8, a_m = 37$

$$a_m = a + (m-1)d$$

$$37 = 213 + (m-1)(-8)$$

$$37 - 213 = -8(m-1)$$

$$m - 1 = \frac{-176}{-8} = 22$$

$$m = 22 + 1 = 23$$

The middle term will be $= \dfrac{23+1}{2} = 12^{th}$

$$a_{12} = a + (12-1)d = 213 + (12-1)(-8)$$

$$= 213 - 88 = 125$$

Middle term will be 125.

Find the values of a, b and c such that the numbers a, 10, b, c, 31 are in AP.

Let the first term be a and common difference be d.
Since $a, 10, b, c, 31$ are in AP, then

$$a + d = 10 \tag{1}$$
$$a + 4d = a_5$$
$$a + 4d = 31 \tag{2}$$

Solving (1) and (2) we have

$$d = 7 \text{ and } a = 3$$

Now $a = 3, b = 3 + 14 = 17, c = 3 + 21 = 24$

Thus $a = 3, b = 17, c = 24$.

Find the sum of all two digit natural numbers which are divisible by 4.

First two digit multiple of 4 is 12 and last is 96

So, $a = 12, d = 4$. Let n^{th} term be last term $a_n = 96$

Now

$$\begin{aligned} a + (n-1)d &= a_n \\ 12 + (n-1)4 &= 96 \\ (n-1)4 &= 96 - 12 = 84 \\ n - 1 &= 21 \\ n &= 21 + 1 = 22 \end{aligned}$$

Now,

$$\begin{aligned} S_{22} &= \frac{22}{2}[12 + 96] \\ &= 11 \times 108 \\ &= 1188 \end{aligned}$$

How many terms of the Arithmetic Progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

$$
\begin{aligned}
S_n &= \frac{n}{2}[2a + (n-1)d] \\
180 &= \frac{n}{2}[2 \times 45 + (n-1)(-6)] \\
360 &= n(90 - 6n + 6) \\
360 &= n(96 - 6n) \\
60 &= n(16 - n) \\
n^2 - 16n + 60 &= 0 \\
n^2 - 6n - 10n + 60 &= 0 \\
n(n-6) - 10(n-6) &= 0 \\
(n-10)(n-6) &= 0 \\
n &= 10 \text{ or } n = 6
\end{aligned}
$$

Find $\left(4-\frac{1}{n}\right)+\left(7-\frac{2}{n}\right)+\left(10-\frac{3}{n}\right)+$ ..... upto n terms.

$$= (4 + 7 + 10 + ... + n \text{ terms}) - \left(\frac{1}{n} + \frac{2}{n} + \frac{3}{n} ..... + 1\right)$$

$$= (4 + 7 + 10 + ... + n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + ...n)$$

$$= \frac{n}{2}[2 \times 4 + (n - 1)(3)] - \frac{1}{n} \times \frac{n}{2}[2 \times 1 + (n - 1)(1)]$$

$$= \frac{n}{2}[8 + 3n - 3] - \frac{1}{2}[2 + n - 1]$$

$$= \frac{n}{2}(3n + 5) - \frac{1}{2}(n + 1)$$

$$= \frac{3n^2 + 5n - n - 1}{2}$$

$$= \frac{3n^2 + 4n - 1}{2}$$