Find the 9th term from the end (towards the first term) of the A.P. 5, 9,13,185.

Solution:

Reversing the given A.P., we get

185, 181, 174, ..., 9, 5

Now, first term $(a) = 185$

Common difference, $(d) = 181 - 185 = -4$

We know that nth term of an A.P. is given by $a + (n-1)d$

$$\text{Ninth term } a_9 = a + (9-1)d$$

$$= 185 + 8 \times (-4) = 185 - 32 = 153$$

For what value of k will k + 9,2k -1 and 2k + 7 are the consecutive terms of an A.P.?

Solution:

Given that $k + 9, 2k - 1$ and $2k + 7$ are in A.P.

Then $(2k - 1) - (k + 9) = (2k + 7) - (2k - 1)$

$\Rightarrow k - 10 = 8 \Rightarrow k = 18$

How many terms of the A.P. 18,16,14,... be taken so that their sum is zero?

Solution:

Let the number of terms taken for sum to be zero be n.

Then, sum of n terms $(S_n) = 0$

First term $(a) = 18$

Common difference $(d) = -2$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)] \quad \Rightarrow \quad 0 = 38 - 2n$

$\Rightarrow \quad n = 19$

$\therefore$ Hence, sum of 19 terms is 0.

How many terms of the A.P. 27,24,21,… should be taken so that their sum is zero?

Solution:

In the given A.P.,

Here, first term $(a) = 27$

Common difference $(d) = -3$

Sum of n terms $(S_n) = 0$

Therefore, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$0 = \frac{n}{2}[2 \times 27 + (n-1)(-3)]$$

$\Rightarrow$ $54 - 3n + 3 = 0$

$\Rightarrow$ $3n = 57 \Rightarrow n = 19$

Thus, the sum of 19 terms of given A.P. is zero.

The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term.

Solution:

Let a be first term and d be the common difference of the A.P. Then

$$a_n = a + (n-1)d$$

$$a_4 = a + (4-1)d$$

$$0 = a + 3d \Rightarrow a = -3d \qquad [\because \text{ Given, } a_4 = 0]$$

Now $$a_{25} = a + (25-1)d$$

$$= a + 24d = -3d + 24d = 21d = 3 \times 7d$$

Hence, $$a_{25} = 3 \times a_{11}$$

$$[\because \text{ Since } a_{11} = a + (11-1)d = -3d + 10d = 7d]$$

The sum of three numbers in A.P. is 12 and sum of their cubes is 288, Find the numbers.

Solution:

Let the three numbers in A.P. are $a - d, a, a + d$

Then $a - d + a + a + d = 12$ [$\because$ Given that, $S_3 = 12$]

$\Rightarrow \quad 3a = 12 \Rightarrow a = 4$

$\Rightarrow \quad (4 - d)^3 + (4)^3 + (4 + d)^3 = 288$ ...eir cubes = 288]

$\Rightarrow \quad 64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

$\Rightarrow \quad 24d^2 + 192 = 288 \Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

For $d = 2$, the numbers will be 2, 4, 6. For $d = -2$, numbers will be 6, 4, 2.

Hence, required numbers are 2, 4, 6.

The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find its common difference

Solution:

Let 1st term of the AP $= a$

Common Difference $= d$

Now, $a_4 = 11$ [Given]

$\Rightarrow a + 3d = 11 \Rightarrow a = 11 - 3d$ ...(*i*)

Also, $a_5 + a_7 = 34$ [Given]

$a + 4d + a + 6d = 34$

$2a + 10d = 34 \Rightarrow a = 17 - 5d$ ...(*ii*)

From (*i*) and (*ii*) $11 - 3d = 17 - 5d$

$\Rightarrow 2d = 6 \Rightarrow d = 3$

The 14th term of an AP is twice its 8th term. If its 6th term is -8, then find the sum of its first 20 terms.

Solution:

Let 1st term of AP = a and common difference = d

A.T.Q. $a_{14} = 2a_8$

$\Rightarrow$ $a + 13d = 2(a + 7d) \Rightarrow a = -d$

Also, given $a_6 = -8 \Rightarrow a + 5d = -8$

$\Rightarrow$ $-d + 5d = -8 \Rightarrow d = -2$

$\Rightarrow$ $a = 2$

$\therefore$ Sum of first 20 terms, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times -2) = 10 \times (-34) = -340$

$\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

The 13th term of an AP is four times its 3rd term. If its fifth term is 16, then find the sum of its first ten terms.

Solution:

Let 1st term of the AP = a and Common difference = d

A.T.Q., $a_{13} = 4 \times a_3$ [Given]

$$a + 12d = 4(a + 2d)$$

$$a + 12d = 4a + 8d \Rightarrow 3a = 4d$$

$$a = \frac{4}{3}d \quad \ldots(i)$$

Also $a_5 = 16 \Rightarrow a + 4d = 16$

$\Rightarrow$ $\frac{4}{3}d + 4d = 16$ [Using (i)]

$\Rightarrow$ $16d = 48 \Rightarrow d = 3$

When $d = 3$, (i) becomes $a = \frac{4}{3} \times 3 = 4$

$\Rightarrow$ $a = 4$

Now, sum of first 10 terms, $S_{10} = \frac{10}{2}(2a + 9d)$ $\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$$= 5(2 \times 4 + 9 \times 3) = 5 \times 35 = 175.$$

An arithmetic progression 5,12,19,… has 50 terms. Find its last term. Hence find the sum of its last 15

Solution:

Given, A.P. is 5, 12, 19, ...

Now, first term $a = 5, d = 7, n = 50$

Now, $a_{50} = a + 49d = 5 + 49 \times 7 = 348$

$\therefore \quad S_{50} = \frac{50}{2}(a + a_{50}) = 25(5 + 348) = 8825$

$\left\{\because S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$\therefore$ Sum of last 15 terms $= S_{50} - S_{35}$, where

$S_{35} = \frac{35}{2}(2 \times 5 + 34 \times 7) = 4340$

$\therefore$ Sum of last fifteen terms $= 8825 - 4340 = 4485.$

The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

Solution:

Numbers between 101 and 999 which are divisible by both 2 and 5 (*i.e.* by 10) are 110, 120, 130, ... 990.

An A.P. is formed with $a = 110$, $d = 10$ and $a_n = 990$

Now, we know that $a_n = a + (n - 1)d$

$\Rightarrow \quad 990 = 110 + (n - 1)10$

$\Rightarrow \quad 880 = (n - 1)10$

$\Rightarrow \quad 88 = n - 1$

$\Rightarrow \quad n = 89$

$\therefore$ Natural numbers which are divisible by 2 and 5 both are 89.

Find the number of all three-digit natural numbers which are divisible by 9.

Solution:

3-digit numbers which are divisible by 9 are 108, 117, 126 .... 999.

Here, $a = 108, a_n = 999, d = 9.$

$\therefore$ Then, $a_n = a + (n-1)d$

$\Rightarrow 999 = 108 + (n-1)9$

$999 - 108 = (n-1)9 \Rightarrow 891 + 9 = 9n$

$\Rightarrow \frac{900}{9} = n \Rightarrow n = 100$

$\therefore$ There are 100 three-digit natural numbers which are divisible by 9.

The 8th term of an AP is equal to three times its 3rd term. If its 6th term is 22, find the AP.

Solution:

$$\text{Consider 1st term} = a$$

$$\text{Common difference} = d$$

Given that $a_8 = 3a_3$, where $a_n = a + (n-1)d$

$$a + 7d = 3(a + 2d)$$

$$a + 7d = 3a + 6d$$

$$7d - 6d = 3a - a$$

$$d = 2a \quad \text{...(i)}$$

Now, $a_6 = a + 5d$

$$22 = a + 5d \quad [\because \text{ Given that } a_6 = 22]$$

$$22 = a + 5(2a) \quad [\text{Using (i)}]$$

$$22 = 11a$$

$$a = 2$$

$\therefore$ $d = 2a = 2(2) = 4$

Hence, required AP 2, 6, 10, 14 .....