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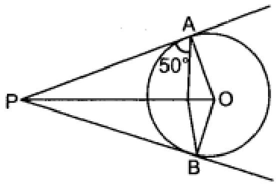
UNIQUE STUDY POINT
IMP QUESTIONS : CLASS X
CIRCLES

1

From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.

Solution:



Given, $\angle PAB = 50^\circ$

$\angle PAB + \angle OAB = 90^\circ$

[∵ angle between radius OA and tangent PA is 90°]

$\Rightarrow 50^\circ + \angle OAB = 90^\circ$

$\Rightarrow \angle OAB = 90^\circ - 50^\circ = 40^\circ$

Now, $PA = PB$ [∴ tangents from an external point are same]

$\Rightarrow \angle PBA = \angle PAB$

$\Rightarrow \angle PBA = 50^\circ$

$\angle PBA + \angle OBA = 90^\circ$ [∴ angle between radius OB and tangent PB is 180°]

$\Rightarrow 50^\circ + \angle OBA = 90^\circ$

$\Rightarrow \angle OBA = 90^\circ - 50^\circ = 40^\circ$

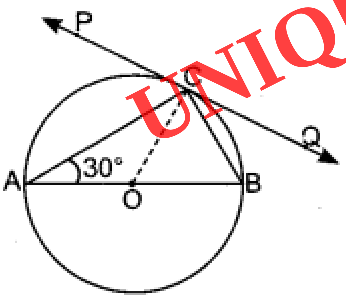
Now in ΔAOB we have

$\angle AOB + \angle ABO + \angle BAO = 180^\circ$ [∵ sum of angles in triangle is 180°]

$\Rightarrow \angle AOB + 40^\circ + 40^\circ = 180^\circ \Rightarrow \angle AOB = 180^\circ - 80^\circ = 100^\circ$

2

In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA



Solution:

Construction: Join AO.

Given: PQ is tangent. AB is diameter ∠CAB = 30°.

To Find: ∠PCA

Solution: In ΔAOC, $AO = CO$ (∵ Equal radii)

$\angle CAO = \angle OCA$ (∵ Angles opposite to equal sides are equal)

or $\angle CAB = \angle OCA$

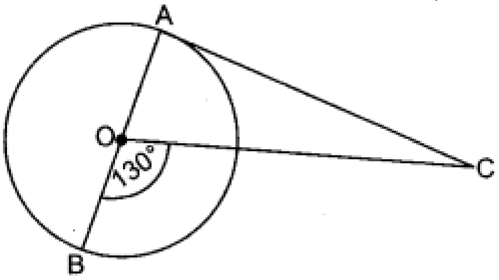
But, $\angle CAB = 30^\circ$ So, $\angle OCA = 30^\circ$ (*i*)

Since, $OC \perp PQ$ (∵ Tangent is perpendicular to radius at point of contact)

$\Rightarrow \angle PCO = 90^\circ \Rightarrow \angle OCA + \angle PCA = 90^\circ \Rightarrow 30^\circ + \angle PCA = 90^\circ$

$\therefore \angle PCA = 60^\circ$

3 In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠BOC = 130°, then find ∠ACO.



Solution:

$\angle AOC + \angle BOC = 180°$ [∵ Linear Pair Axiom]

$\angle AOC + 130° = 180°$

$\angle AOC = 180° - 130°$

$\angle AOC = 50°$

Now, $\angle OAC = 90°$ [angle between radius OA and tangent AC is 90°]

Now, in ΔAOC,

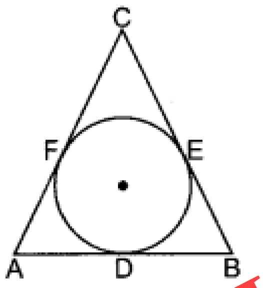
$\angle OAC + \angle AOC + \angle ACO = 180°$ [∵ sum of angles in triangle is 180°]

$90° + 50° + \angle ACO = 180°$

$\angle ACO = 180° - 140°$

$\angle ACO = 40°$

4 In given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF



Solution:

Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $AD = AF = x$ [∵ Tangent drawn from external point to circle are equal]

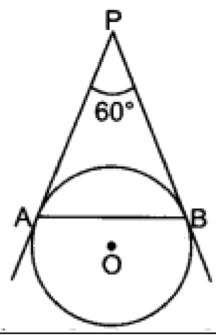
∴ $DB = BE = 12 - x$ and $CF = CE = 10 - x$

$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$

$\Rightarrow x = 7$

∴ AD = 7 cm, BE = 5 cm and CF = 3 cm

5 If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and ∠APB = 60°. Find the length of chord AB.



Solution:

In ΔAPB we have $\quad AP = BP$

$\Rightarrow \quad \angle PAB = \angle PBA$

[$\because$ Tangents from an external point are equally inclined to segment joining centre to point]

Let $\quad \angle PAB = x,$

then in ΔAPB, $\quad x + x + 60° = 180°$

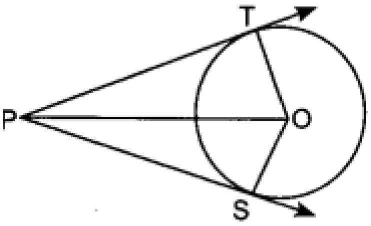
$2x = 180° - 60° = 120°$

$x = 60°$

As all three angles of ΔAPB are 60°. So ΔAPB is an equilateral triangle.

Hence AP = BP = AB = 5 cm

6 In given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that ∠SPT = 120°, Prove that OP = 2PS



Solution:

Let PT = x = PS $\quad$ [$\because$ Tangent drawn from external point to circle are equal]

$\angle SPT = 120°$

In ΔOTP and ΔOSP, $\quad \angle OTP = \angle OSP$

[$\because$ each equal to 90°, since tangent perpendicular r radius]

$OT = OS \quad$ [$\because$ Equal radii]

$OP = OP \quad$ [common]

$\Rightarrow \quad \Delta OSP \cong \Delta OTP \quad$ [$\because$ By SAS congruence rule]

$\therefore \quad \angle TPO = \angle SPO \quad$ [$\because$ By CPCT]

$\Rightarrow \quad \angle TPO = \frac{1}{2}\angle SPT = \frac{1}{2} \times 120 = 60°$

In ΔOTP, $\quad \frac{OP}{x} = \text{Sec } 60°$

$\Rightarrow \quad \frac{OP}{x} = 2 \Rightarrow OP = 2x \Rightarrow OP = 2PS$

Hence proved.

7 n given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8cm, find the length of BP

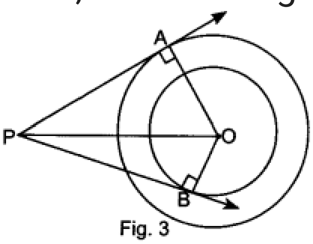


Fig. 3

Solution:

$OA = 6$ cm [$\because$ Given radius]

$OB = 4$ cm [$\because$ Given radius]

$AP = 8$ cm

In ΔOAP, $OP^2 = OA^2 + AP^2 = 36 + 64 = 100$ [$\because$ Pythagoras theorem]

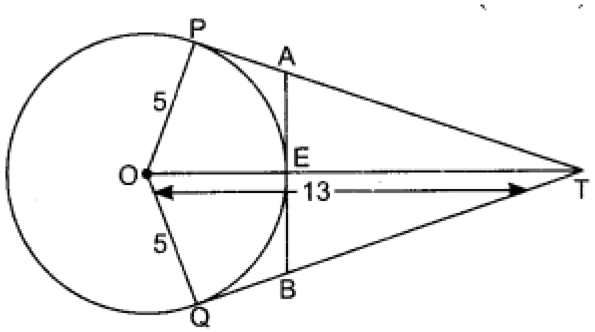
$\Rightarrow$ $OP = 10$ cm

In ΔOBP, $BP^2 = OP^2 - OB^2 = 100 - 16 = 84$ [$\because$ Pythagoras theorem]

$BP = 2\sqrt{21}$ cm

8

In given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



Solution:

In ΔOPT, $OP^2 + PT^2 = OT^2$ [$\because$ Pythagoras theorem]

$PT = \sqrt{OT^2 - OP^2}$

$= \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE = 13 - 5 = 8$ cm

Let $PA = AE = x$ [tangent from outer point A]

In ΔTEA, $TE^2 + EA^2 = TA^2$ [$\because$ Pythagoras theorem]

$(8)^2 + (x)^2 = (12 - x)^2$

$64 + x^2 = (12 - x)^2$

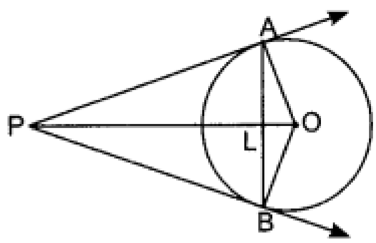
$\Rightarrow$ $64 + x^2 = 144 + x^2 - 24x$

$\Rightarrow$ $80 = 24x \Rightarrow x = 3.3$ cm

Thus $AB = 2 \times 3.3$ cm $= 6.6$ cm [$\because$ AE = EB, as AB is tangent to circle at E]

9

n given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA



Solution:

Let $PL = x$

As OP is perpendicular bisector of AB. Then

$AL = BL = 8$ cm

In ΔALO, $OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 36 \Rightarrow OL = 6$ cm

$AP^2 = OP^2 - OA^2$ [$\because$ Pythagoras theorem]

In ΔOAP, $AP^2 = (x+6)^2 - 10^2$

$AP^2 = AL^2 + PL^2$ [$\because$ Pythagoras theorem]

In ΔALP, $AP^2 = x^2 + 64$

Now, $(x+6)^2 - 10^2 = x^2 + 64$

$x^2 + 12x + 36 - 100 = x^2 + 64$

$\Rightarrow 12x = 128$

$\Rightarrow x = \dfrac{128}{12}$

$= \dfrac{32}{3}$ cm

From ΔALP, $AP^2 = \left(\dfrac{32}{3}\right)^2 + 64$

$= \dfrac{1024}{9} + 64$

$= \dfrac{1024 + 576}{9}$ cm

$AP^2 = \dfrac{1600}{9}$ cm

$AP = \dfrac{40}{3}$ cm $= 13.3$ cm

10 In figure, PA and PB are tangents to the circle with centre O such that $\angle APB = 50°$. Write the measure of $\angle OAB$

Solution:

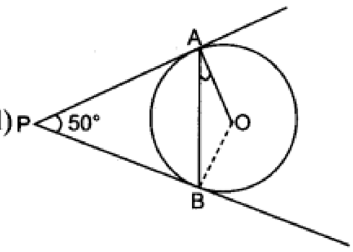
Join OB.

$\because$ PA and PB are tangents to the circle drawn from an external point P. We know that, tangent is perpendicular to radius.

$\angle OAP = \angle OBP = 90°$

Then, $\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

(ASP of quadrilateral)



$\therefore \angle APB + \angle AOB = 180°$

$\Rightarrow 50° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 130°$

In ΔOAB, $OA = OB$ ($\because$ equal radii)

$\Rightarrow \angle A = \angle B = x$ (say)($\because$ angles opposite to equal sides are equal)

$\angle A + \angle B + \angle AOB = 180°$ ($\because$ ASP of triangles)

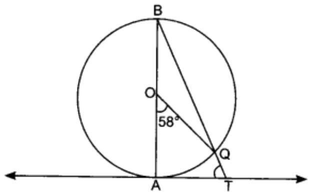
$\Rightarrow x + x + 130° = 180°$

$\Rightarrow 2x = 50°$

$\Rightarrow x = 25°$

$\therefore \angle OAB = 25°$

11 n figure, AB is the diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58°, find ∠ATQ



Solution:

∵ AT is a tangent and BA is a diameter.

So, $OA \perp AT$

[radius is perpendicular to the tangent at point of contact]

$\Rightarrow \angle OAT = 90^\circ$ or $\angle BAT = 90^\circ$

Arc AQ subtends an angle of 58° at the circle.

$\angle AOQ = 2\angle ABQ$

So, $\angle ABQ = 29^\circ$ [angle subtended by the arc at the centre is double the angle subtended by the same arc on the circle]

In ΔABT,

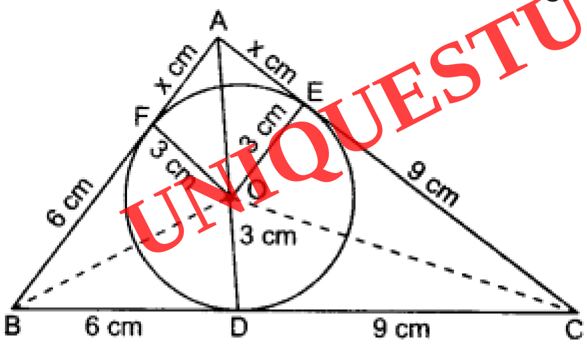
$\angle A + \angle ABT + \angle ATB = 180^\circ$

$\Rightarrow 90^\circ + 29^\circ + \angle ATB = 180^\circ$

$\Rightarrow \angle ATB = 61^\circ$

Hence, ∠ATQ = 61°

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC



Solution:

Let AF = x cm, BC = (6 + 9) = 15 cm

∵ AF = AE

[tangents drawn from an external point are equal]

∴ AE = x cm

Also BD = BF = 6 cm

and CD = CE = 9 cm

∴ AB = $(x + 6)$ cm

In ΔABC, AC = $(x + 9)$ cm

Area ΔABC = Area ΔBOC + Area ΔCOA + Area ΔAOB

$\Rightarrow 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$

$\Rightarrow 54 \times 2 = 15 \times 3 + (9 + x) \times 3 + (6 + x) \times 3$

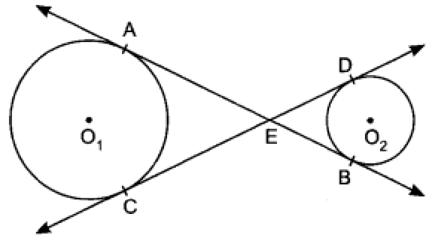
$108 = 45 + 18 + 3x + 27 + 3x$

$6x = 18 \Rightarrow x = 3$

$\Rightarrow AB = 6 + x = 6 + 3 = 9$ cm

$AC = 9 + x = 9 + 3 = 12$ cm

12 n figure, common tangents AB and CD to the two circles with Centres O1 and O2 intersect at E. Prove that AB = CD.



Solution:

In the given figure, AB and CD are common tangents to the two given circles with centres O_1 and O_2 respectively.

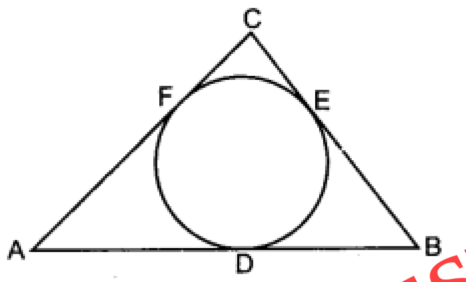
We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$\therefore$ AE = EC and EB = ED

$\Rightarrow$ AE + EB = CE + ED

$\Rightarrow$ AB = CD.

13 In the given figure, a circle inscribed in ΔABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF



Solution:

Given, AB = 12 cm, BC = 8 cm, AC = 10 cm

Let AD = x cm

$\therefore$ BD = AB − AD = $(12 - x)$ cm

$\because$ AD = AF [tangents from point A]

$\therefore$ AF = x cm

Now, CF = AC − AF = $(10 - x)$ cm

Also, CE = CF $\Rightarrow$ CE = $(10 - x)$ cm

And BD = BE [$\because$ tangents from B]

$\Rightarrow$ BE = $(12 - x)$ cm [From (*i*)]

Now, BC = CE + BE

$\Rightarrow$ $8 = (10 - x) + (12 - x)$

$\Rightarrow$ $8 = 22 - 2x \Rightarrow 2x = 14$

$\Rightarrow$ $x = 7$ cm

$\Rightarrow$ AD = 7 cm

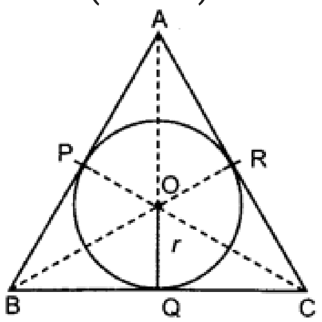
BE = $12 - x = 12 - 7 = 5$ cm

and CF = $10 - x = 10 - 7 = 3$ cm

14 n the given figure, the sides AB, BC and CA of ΔABC touch a circle with centre O and radius r at P, Q and R respectively.

Prove that:

1. AB + CQ = AC + BQ
2. Area (ΔABC) = 1/2 (perimeter of ΔABC) X r



Solution:

(i) We have, $AP = AR$ [Tangents from A] ...(i)

Similarly, $BP = BQ$ [Tangents from B] ...(ii)

$CR = CQ$ [Tangents from C] ...(iii)

Now, we have

$\because \quad AP = AR$

$\Rightarrow \quad (AB - BP) = (AC - CR)$

$\Rightarrow \quad AB + CR = AC + BP$

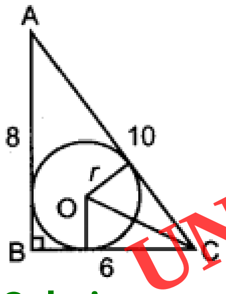
$\Rightarrow \quad AB + CQ = AC + BQ$ [Using eq. (ii) and (iii)]

(ii) $\text{Area}(\Delta ABC) = \text{Area}(\Delta ABO + \Delta OBC + OAC)$

$= \frac{1}{2}(AB + BC + AC) \times r$ [$\therefore$ Area $(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}$]

$= \frac{1}{2}(\text{perimeter of } \Delta ABC) \times r$

15 In figure, a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, find the value of r.



Solution:

$\because$ ABC is right angle Δ, right $\angle d$ at B.

So, By Pythagoras theorem

$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 100$

$AC = 10$ cm

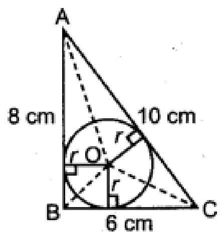
So, $\text{ar}(\Delta ABC) = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

Also, $\text{ar}(\Delta ABC) = \text{ar}(\Delta OBC) + \text{ar}(\Delta OAC) + \text{ar}(\Delta OAB)$

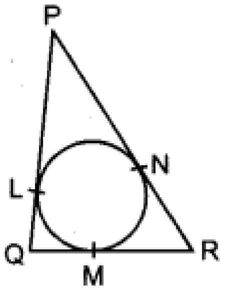
$\Rightarrow \quad 24 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$

$\Rightarrow \quad 24 = 3r + 5r + 4r \quad \Rightarrow \quad 12r = 24$

$\Rightarrow \quad r = 2$ cm



16 In figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths QM, RN and PL.



Solution:

We know that the tangents drawn from an external point to a circle are equal.

Therefore

Let $QM = x = QL$

$MR = y = RN$

and $PL = z = PN$

Now $PQ = 10$ cm, $QR = 8$ cm, $PR = 12$ cm

$\Rightarrow \quad x + y = 8, y + z = 12, z + x = 10$

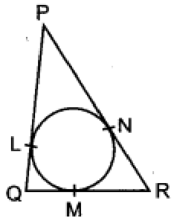
$\Rightarrow \quad 2x + 2y + 2z = 8 + 12 + 10 = 30$

$\Rightarrow \quad x + y + z = 15 \Rightarrow 8 + z = 15 \Rightarrow z = 7$

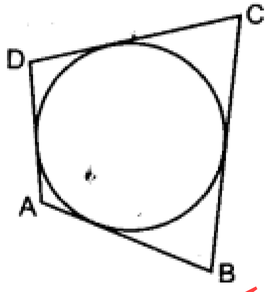
$\Rightarrow \quad x + 12 = 15 \Rightarrow x = 3$

$\Rightarrow \quad y + 10 = 15 \Rightarrow y = 5$

Hence, QM = 3 cm, RN = 5 cm and PL = 7 cm.



17 In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.

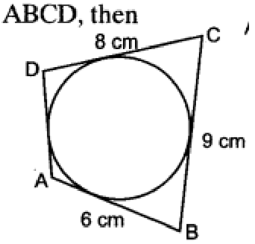


Solution:

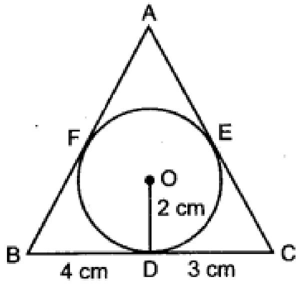
If a circle touches all the four sides of quadrilateral ABCD, then we know that $AD + BC = AB + CD$

$\therefore \quad AD + 9 = 6 + 8$

$\Rightarrow \quad AD = 5$ cm



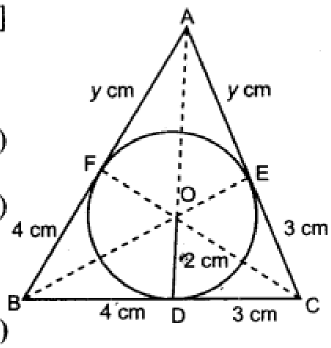
18 **In figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are the lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21\ cm^2$, then find the lengths of sides AB and AC.**



Solution:

Let AE = AF = y (say)

[Tangents drawn from an external point are equal]



$$\text{ar } \Delta BOC = \frac{1}{2} \times 7 \times 2 = 7 \text{ cm}^2 = b \text{ (say)}$$

$$\text{ar } \Delta AOB = \frac{1}{2} \times (4+y) \times 2 = (4+y) \text{ cm}^2 = a \text{ (say)}$$

$$\text{ar } \Delta AOC = \frac{1}{2} \times (3+y) \times 2 = (3+y) \text{ cm}^2 = c \text{ (say)}$$

Now, $\text{ar } \Delta ABC = \text{ar } \Delta AOB + \text{ar } \Delta BOC + \text{ar } \Delta AOC$

$= 4 + y + 7 + 3 + y$

$\text{ar } \Delta ABC = 14 + 2y$ ...(*i*)

For ΔABC,

Semi-perimeter, $s = \dfrac{a+b+c}{2} = \dfrac{4+y+7+3+y}{2} = \dfrac{14+2y}{2} = 7 + y$

$\therefore$ $\text{ar } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ [$\because$ By Heron's formula]

$= \sqrt{(7+y)(7+y-4-y)[7+y-7)(7+y-3-y)}$

$= \sqrt{(7+y) \times 3 \times y \times 4}$

$\text{ar } \Delta ABC = 2\sqrt{3y(7+y)}$ ...(*ii*)

From (*i*) and (*ii*)

$\Rightarrow$ $2\sqrt{3y(7+y)} = 14 + 2y$

$\Rightarrow$ $\sqrt{3y(7+y)} = 7 + y$

Squaring both sides, we get

$\Rightarrow$ $3y(7+y) = (7+y)^2 \Rightarrow 21y + 3y^2 = 49 + y^2 + 14y$

$\Rightarrow$ $2y^2 + 7y - 49 = 0 \Rightarrow 2y^2 + 14y - 7y - 49 = 0$

$\Rightarrow$ $2y(y+7) - 7(y+7) = 0 \Rightarrow (2y-7)(y+7) = 0$

$\Rightarrow$ $y = \frac{7}{2}, y = -7$ [Rejected]

Hence, length of side AB = 4 + 3.5 = 7.5 cm and AC = 3 + 3.5 = 6.5 cm.