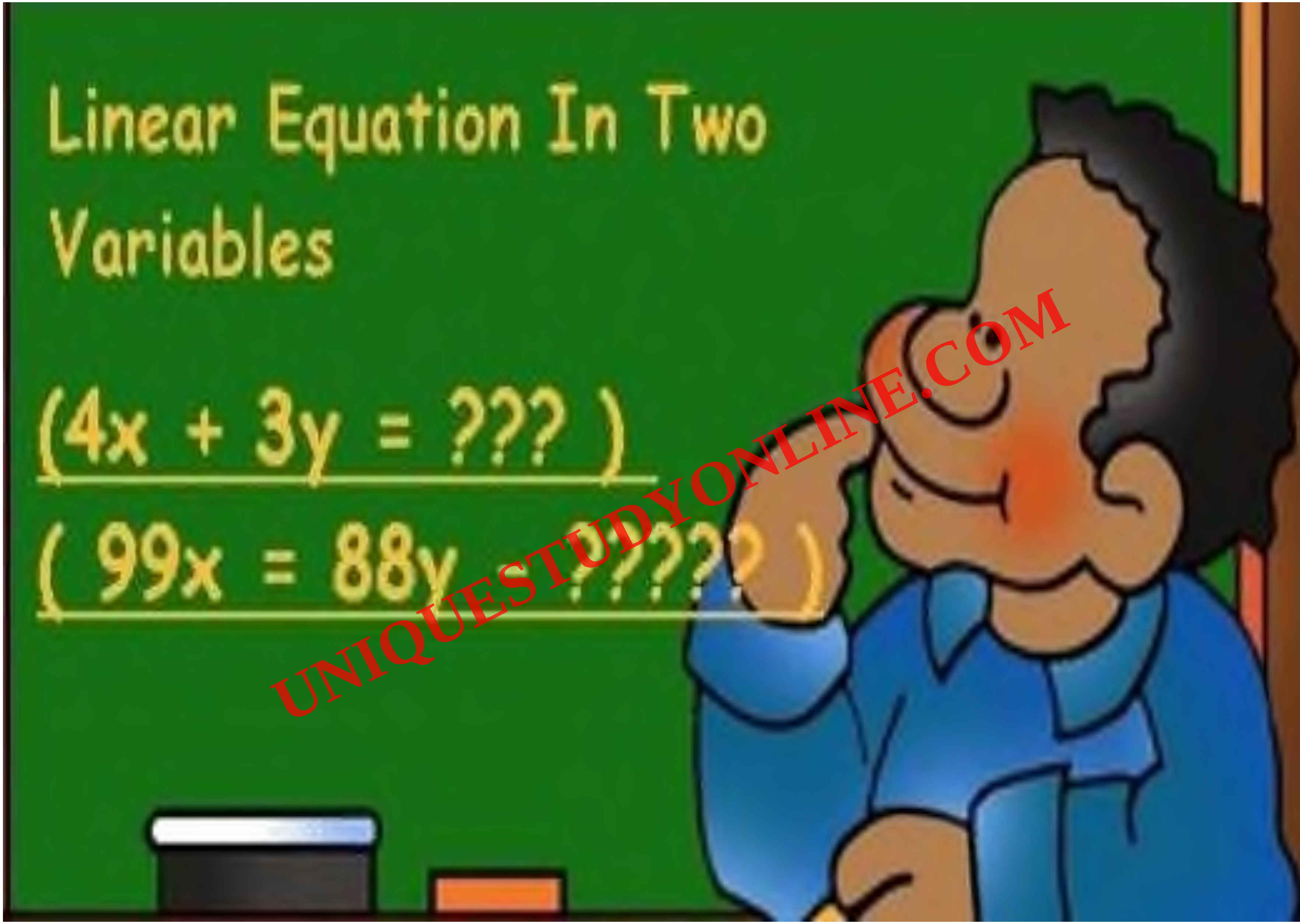

Linear Equation In Two Variables
(4x + 3y = ???)
(99x = 88y ?????)

LINEAR EQUATION: A Linear Equation is an algebraic equation in which terms are a constants or the product of a constants and variables. Linear Equations can have one or more variables.Ex:2x-3=5(linear equation in one variable)

Ex:2x+3y=7(linear equation in two variables)

System of equations or Simultaneous equations

A pair of linear equations in two variables is said to form a system of simultaneous (together) linear equations.

For Example, $2x - 3y + 4 = 0$

$$x + 7y - 1 = 0$$

Form a system of two linear equations in variables x and y.

The general form of a linear equation in two variables x and y is $ax + by + c = 0$ $a \neq 0, b \neq 0$,

where

a, b and c being real numbers.

A solution of such an equation is a pair of values, one for x and the other for y, which makes two sides (LHS AND RHS) of the equation equal.

Every linear equation in two variables has infinitely many solutions which can be represented on a certain line.

A pair of linear equations in two variables can be solved by the:

(i) Graphically method

(ii) Algebraic method

GRAPHICAL SOLUTIONS OF A LINEAR EQUATION

- Let us consider the following system of two simultaneous linear equations in two variable.

$2x - y = -1$

$3x + 2y = 9$

Here we assign any value to one of the two variables and then determine the value of the other variable from the given equation.

For the equation

$2x - y = -1$ ---(1)

Solve for y

ADD 1 and add y to both sides

$2x + 1 = y$

Switching sides

$Y = 2x + 1$

X	0	2
Y	1	5

$3x + 2y = 9$ --- (2)

Solve for y

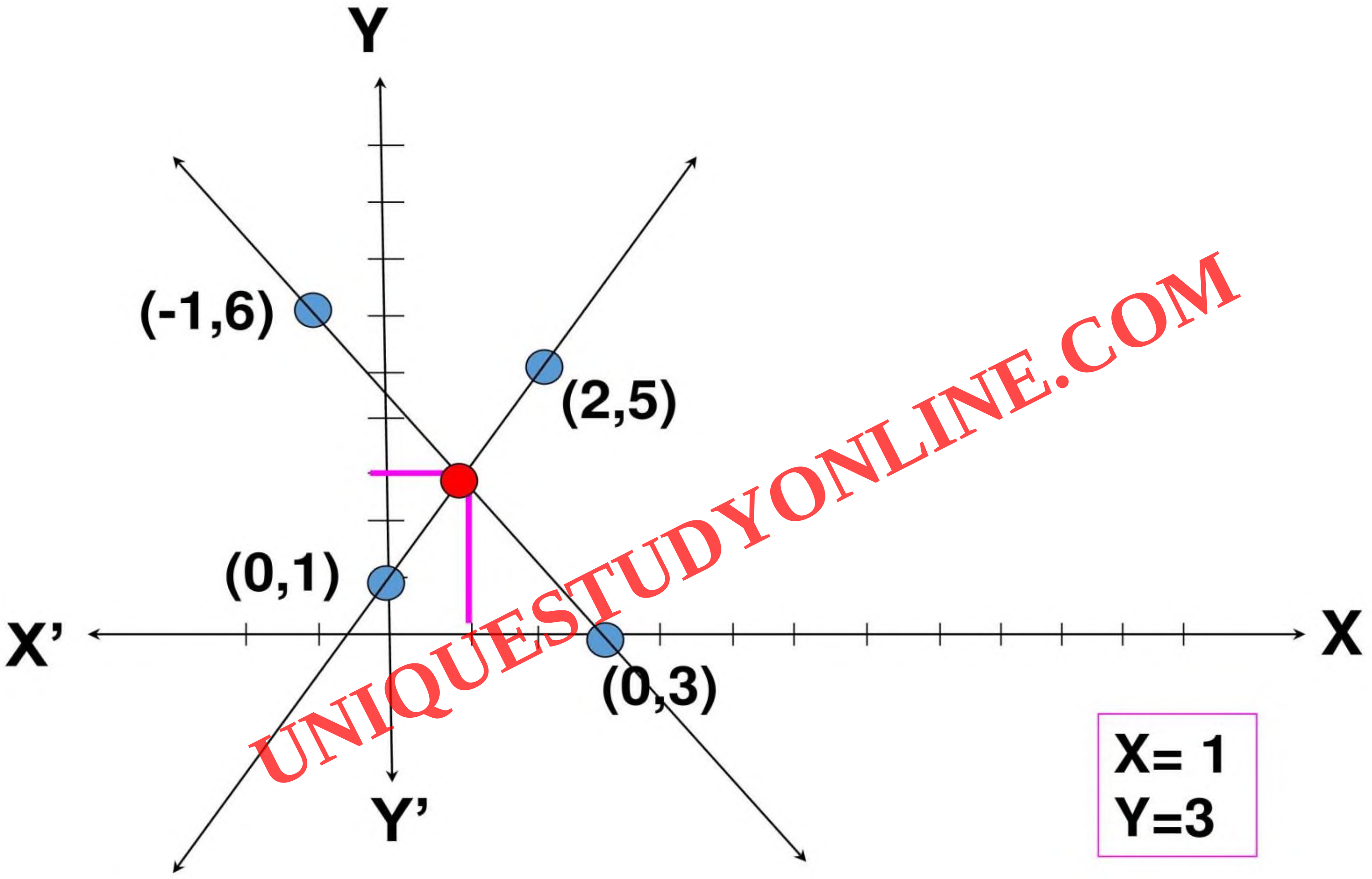
Subtract 3x on both sides

$2y = 9 - 3x$

Divide by 2 on both sides

$y=(9-3x)/(2)$

X	3	-1
Y	0	6


Y
(-1,6)
(2,5)
(0,1)
X'
X
(0,3)
Y'
X= 1
Y=3

Types of Solutions of Systems of Equations

- One solution – the lines cross at one point

- No solution – the lines do not cross

- Infinitely many solutions – the lines coincide

(i) $x - 2y = 0$ and $3x + 4y - 20 = 0$ (The lines intersect)

(ii) $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ (The lines coincide)

(iii) $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ (The lines are parallel)

Let us now write down, and compare, the values of $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ in all the three examples. Here, a_1, b_1, c_1 and a_2, b_2, c_2 denote the coefficents of equations given in the general form in Section 3.2.

Table 3.4

Sl No.	Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
1.	$x - 2y = 0$ $3x + 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
2.	$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3.	$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

To solve a pair of linear equations in two variables algebraically, we have following methods:

(i) Substitution method

(ii) Elimination method

(iii) Cross-multiplication method

SUBSTITUTION

Let the equations be

$a_1x + b_1y + c_1 = 0$ ----------- (i)

$a_2x + b_2y + c_2 = 0$ ----------- (ii)

STEPS

1. Choose either of the two equations, say (i) and find the value of one variable , say 'y' in terms of x
2. Substitute the value of y, obtained in the previous step in equation (ii) to get an equation in x
3. Solve the equation obtained in the previous step to get the value of x.
4. Substitute the value of x and get the value of y.

ELIMINATION BY SUBSTITUTION

Let us take an example

$x + 2y = -1$ (i)

$2x - 3y = 12$ (ii)

1. Choose either of the two equations, say (i) and find the value of one variable , say 'x' in terms of y

$x + 2y = -1$

Subtract 2y on both sides

$x = -2y - 1$ ------- (iii)

2. Substituting the value of x in equation $2x – 3y = 12$ (ii), we get

$2(-2y – 1) – 3y = 12$ (solving brackets)

$-4y – 2 – 3y = 12$ (rearranging)

$-7y = 14$ (divide by -7)

$y = -2$

Putting the value of y in eq (iii), we get

$x = -2y - 1$

$x = -2 \times (-2) - 1$

$= 4 - 1$

$= 3$

Hence the solution of the equation is

$(x,y) = (3, -2)$

ELIMINATION METHOD

- We eliminate one of the two variables to obtain an equation in one variable which can easily be solved. Putting the value of this variable in any of the given equations, the value of the other variable can be obtained.
- For example: we want to solve,

$$3x + 2y = 11$$

$$2x + 3y = 4$$

STEP 1

Let $3x + 2y = 11$ --------- (i) x 3

$2x + 3y = 4$ ---------(ii) x 2

1. Multiply 3 in equation (i)
2. Multiply 2 in equation (ii)
3. Subtracting eq iv from iii, we get

$9x + 6y = 33$ -------(iii)

$4x + 6y = 8$ ------- (iv)

$\underline{-\quad\quad -\quad\quad -}$

$5x + 0 = 25$

$x = 5$

Putting the value of y in equation (ii) we get,

$2x + 3y = 4$ (put $X=5$)

$2 \times 5 + 3y = 4$

$10 + 3y = 4$ (subtract by 10 both sides)

$3y = 4 - 10$ (simplify)

$3y = -6$ (divide by 3)

$y = -2$

Hence, $x = 5$ and $y = -2$

$(x,y) = (5, -2)$

$$a_1x + b_1y + c_1 = 0 \qquad a_2x + b_2y + c_2 = 0$$

To solve this pair of equations for x and y using cross-multiplication, we'll arrange the variables and their coefficients

a_1, a_2 and b_1, b_2 and the constants c_1 and c_2

CROSS MULTIPLICATION METHOD

Let us now see how this method works for any pair of linear equations in two variables of the form

$$a_1x + b_1y + c_1 = 0 \quad (1)$$

and

$$a_2x + b_2y + c_2 = 0 \quad (2)$$

To obtain the values of x and y as shown above, we follow the following steps:

Step 1 : Multiply Equation (1) by b_2 and Equation (2) by b_1, to get

$$b_2a_1x + b_2b_1y + b_2c_1 = 0 \quad (3)$$

$$b_1a_2x + b_1b_2y + b_1c_2 = 0 \quad (4)$$

Step 2 : Subtracting Equation (4) from (3), we get:

$$(b_2a_1 - b_1a_2)\,x + (b_2b_1 - b_1b_2)\,y + (b_2c_1 - b_1c_2) = 0$$

i.e.,

$$(b_2a_1 - b_1a_2)\,x = b_1c_2 - b_2c_1$$

So,

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \text{ provided } a_1b_2 - a_2b_1 \neq 0 \qquad (5)$$

Step 3 : Substituting this value of x in (1) or (2), we get

$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \qquad (6)$$

Now, two cases arise :

Case 1 : $a_1b_2 - a_2b_1 \neq 0$. In this case $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Then the pair of linear equations has a unique solution.

Case 2 : $a_1b_2 - a_2b_1 = 0$. If we write $\frac{a_1}{a_2} = \frac{b_1}{b_2} = k$, then $a_1 = k\,a_2$, $b_1 = k\,b_2$.

Substituting the values of a_1 and b_1 in the Equation (1), we get

$$k\,(a_2x + b_2y) + c_1 = 0. \qquad (7)$$

It can be observed that the Equations (7) and (2) can both be satisfied only if $c_1 = k\,c_2$, i.e., $\frac{c_1}{c_2} = k$.

If $c_1 = k\,c_2$, any solution of Equation (2) will satisfy the Equation (1), and vice versa. So, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k$, then there are infinitely many solutions to the pair of linear equations given by (1) and (2).

If $c_1 \neq k\,c_2$, then any solution of Equation (1) will not satisfy Equation (2) and vice versa. Therefore the pair has no solution.

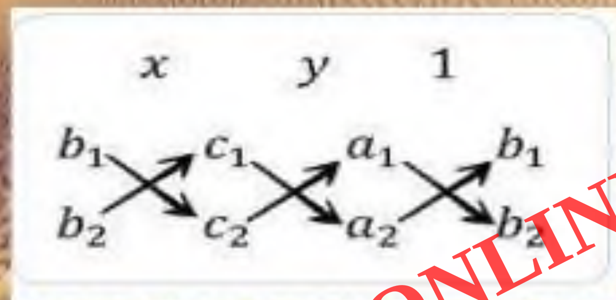
We can summarise the discussion above for the pair of linear equations given by (1) and (2) as follows:

(i) When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, we get a unique solution.

(ii) When $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, there are infinitely many solutions.

(iii) When $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, there is no solution.

CROSS MULTIPLICATION METHOD



$$\Rightarrow x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

EXAMPLE

1) $11x + 15y + 23 = 0$

$7x - 2y - 20 = 0$

X Y 1

15 23 11 15

-2 -20 7 -2

$$\frac{X}{(15)(-20)-(-2)(23)} = \frac{Y}{(23)(7)-(120)(11)}$$

$$\frac{1}{(11)(-2)-(7)(15)}$$

$=x/(-300+46) = y/(161+220) = 1/(-22-105)$

$=x/(-254) = y/(381) = 1/(-127)$

$x=-254/-127=2$

$y=381/-127=3$

$x=2$
$y=3$

THANK YOU