Quadratic Equations

The polynomial p(x)of degree two is called quadratic polynomial and equation corresponding to p(x)=0, is a quadratic equation in variable x.

Standard form: p(x) $=ax^2+bx+c=0$, where a≠0 and a, b, c ∈ R	
Complete Quadratic Equation	$ax^2+bx+c=0$, where , a, b, c ∈ R ,a≠0 ,b≠0, c ≠0,
Pure Quadratic Equation	$ax^2+bx+c=0$, where , a ∈ R ,a≠0 ,b=0, c =0

Relation b/w Roots and Coefficients	
Sum of roots	$(\alpha+\beta)=\frac{-b}{a}=\frac{-coefficient\ of\ x}{coefficient\ of\ x^2}$
Product of roots	$\alpha\beta=\frac{c}{a}=\frac{constant\ term}{coefficient\ of\ x^2}$

- The value of x for which the polynomial becomes zero is called zero of a polynomial or root of the quadratic equation p(x) =0.
- Quadratic equation has 2 roots α and β

$D=b^2-4ac$;discriminant , $\alpha=\frac{-b+\sqrt{D}}{2a}$, $\beta=\frac{-b-\sqrt{D}}{2a}$

Formation of the Quadratic Equations when the two roots are given:

If α and β are given as two roots of the quadratic equation, then the required quadratic equation can be formed as

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$=x^2-$ (sum of roots) x + product of roots =0, by assuming coefficient of x^2 as 1

Nature of D	Nature of the roots	Inference
D=0	$\alpha=\beta=\frac{-b}{2a}$	Roots are Real and Equal
D > 0, but not a perfect square	$\alpha=\frac{-b+irrational}{2a}$, $\beta=\frac{-b-irrational}{2a}$	Root are real but irrational
D = 0 and a perfect square	$\alpha=\frac{-b+rational}{2a}$ $\beta=\frac{-b-rational}{2a}$	Roots are real and rational
D < 0	α , β both are imaginary as $\sqrt{D}=\sqrt{-ve\ number}$	Roots are virtual or imaginary ; they do not belong to real numbers

Methods of finding the Roots	
Factorization	factorizing the quadratic expression into two linear factors with the help of identities or splitting middle term method and equating each factor to zero.
Completing the square method	1. reduce the coefficient of x^2 to 1 or a perfect square 2. Add and subtract the square of half the coefficient of x so as to get an expresion of the form $(x-p)^2=q^2$ 3. Now x =p ± q, will give the roots
Use of Quadratic formula	Roots are given by the Quadratic formula $=\alpha,\beta=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Method of Solving Word problems:

- Translating the word problem into Mathematics form (symbolic form) according to the given condition(s)
- Form the word problem into Quadratic equations and solve them