QUADRATIC EQUATION PART 2

In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir. The Product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation.

Let the number of wickets is taken by Zahir be x, then number of wickets taken by Harbhajan will be $2x - 3$.

According to question, we have

$$x(2x - 3) = 20$$
$$2x^2 - 3x = 20$$

Thus required quadratic equation,

$$2x^2 - 3x - 20 = 0$$

The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction.

Let numerator be x then fraction will be $\dfrac{x}{2x+1}$

As per the question we have

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$$

$$21\left[x^2 + (2x+1)^2\right] = 58(2x^2 + x)$$

or, $\qquad 11x^2 - 26x - 21 = 0$

$$11x^2 - 33x + 7x - 20 = 0$$

$$x = 3, -\frac{7}{11} \text{ (rejected)}$$

We reject $x = -\frac{7}{3}$, thus $x = 3$ and fraction will be

$$\frac{3}{6+1} = \frac{3}{7}.$$

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A person on tour has ₹ 4200 for his expenses. If he extends his tour for 3 days, he has to cut down his daily expenses by ₹ 70. Find the original duration of the tour.

$$\frac{4200}{x} - \frac{4200}{x+3} = 70$$

$$4200\left(\frac{1}{x} - \frac{1}{x+3}\right) = 70$$

$$60\left[\frac{x+3-x}{x(x+3)}\right] = 1$$

$$60\left[\frac{3}{x(x+3)}\right] = 1$$

$$180 = x^2 + 3x$$

$$x^2 + 3x - 180 = 0$$

$$x^2 + 15x - 12x - 180 = 0$$

$$x(x+15) - 12(x+15) = 0$$

$$(x+15)(x-12) = 0$$

Thus $x = -15$ and $x = 12$

A student scored a total of 32 marks in class tests in mathematics ans science. Had he scored 2 marks less in science and 4 more in mathematics, the product of his marks would have been 253. Find his marks in two subjects.

According to question,

$$(32 - x - 2)(x + 4) = 253$$

$$(30 - x)(x + 4) = 253$$

$$26x - x^2 + 120 = 253$$

$$x^2 - 26x + 133 = 0$$

$$x^2 - 19x - 7x + 133 = 0$$

$$x(x - 19) - 7(x - 19) = 0$$

$$x = 7 \text{ or } x = 19$$

If $x = 7$ then marks in mathematics $= 7$, and marks in science $= 25$

If $x = 19$, then marks in mathematics $= 19$ and marks in science $= 13$.

The sum of ages (in years) of a son and his father is 35 years and product of their ages is 150 years, find their ages.

Let the age of father be x years and age of son be y years

Now $$x + y = 35 \qquad (1)$$

and $$xy = 150 \qquad (2)$$

Substituting the value of y, in (1) we have

$$x(35 - x) = 150$$

$$x^2 - 35x + 150 = 0$$

$$(x - 30)(x - 5) = 0$$

$$x = 30, x = 5 \text{ (Rejected)}$$

Age of father can't be 5 years, so we reject $x = 5$ and take $x = 30$.

One fourth of a herd of camels was seen in forest. Twice of square root of the herd had gone to mountain and remaining 15 camels were seen on the bank of a river, find the total number of camels.

Let the total number of camels be x.
According to the condition given we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x$$

$$3x - 8\sqrt{x} - 60 = 0$$

Let $\sqrt{x} = y$, then we have

$$3y^2 - 8y - 60 = 0$$

$$3y^2 - 18y + 10y - 60 = 0$$

$$3y(y - 6) + 10(y - 6) = 0$$

$$(3y + 10)(y - 6) = 0$$

$$y = 6 \text{ or } y = -\frac{10}{3}$$

A two digit number is four times the sum of the digits. It also equal to 3 times the product of digits. Find the number.

$$10y + x = 4(y + x)$$
$$10y + x = 4y + 4x$$
$$10y - 4y = 4x - x$$
$$6y = 3x \Rightarrow 2y = x$$

Also, $$10y + x = 3xy$$

Now substituting $x = 2y$ we have

$$10y + 2y = 3(2y)y$$
$$12y = 6y^2$$
$$6y^2 - 12y = 0$$
$$6y(y - 2) = 0$$
$$y = 0 \text{ or } y = 2$$

Three consecutive positive integers are such that the sum of the square of the first and product of the other two is 46. Find the integers.

$$x^2 + (x+1)(x+2) = 46$$

$$x^2 + x^2 + 3x + 2 = 46$$

$$2x^2 + 3x - 44 = 0$$

$$2x^2 + 11x - 8x - 44 = 0$$

$$x(2x+11) - 4(2x+11) = 0$$

$$(2x+11)(x-4) = 0$$

$$2x+11 = 0 \text{ and } x - 4 = 0$$

$$x = -\frac{11}{2} \text{ and } x = 4$$

A train covers a distance of 360 km at a uniform speed. Had the speed been 5 km/h more, it would have taken 48 min less for the journey. Find the original speed of the train.

$$\frac{360}{x} - \frac{360}{(x+5)} = \frac{48}{60}$$

$$\frac{360}{x} - \frac{360}{x+5} = \frac{4}{5}$$

$$360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{4}{5}$$

$$90\left[\frac{5}{x(x+5)}\right] = \frac{1}{5}$$

$$450 \times 5 = x^2 + 5x$$

$$x^2 + 5x - 2250 = 0$$

$$x^2 + 50x - 45x - 2250 = 0$$

$$x(x+50) - 45(x+50) = 0$$

$$(x+50)(x-45) = 0 \Rightarrow x = 45 \qquad [x \neq -50]$$

Hence, original speed of the train is 45 km/h.

If the price of a book is reduced by Rs. 5, a person can buy 5 more book for Rs. 300. Find the original list price of the book.

According to questions, we have

$$\frac{300}{x-5} - \frac{300}{x} = 5$$

$$x^2 - 5x - 300 = 0$$

$$(x - 20)(x + 15) = 0$$

Since price cannot be negative, $x = 20$

Thus original list price is 20 rs.

The sum of ages (in years) of a son and his father is 35 years and product of their ages is 150 years, find their ages.

Now $$x + y = 35 \qquad (1)$$

and $$xy = 150 \qquad (2)$$

Substituting the value of y, in (1) we have

$$x(35 - x) = 150$$
$$x^2 - 35x + 150 = 0$$
$$(x - 30)(x - 5) = 0$$
$$x = 30, x = 5 \text{ (Rejected)}$$

Age of father can't be 5 years, so we reject $x = 5$ and take $x = 30$.

Now $$y = 5$$

Hence the age of father is 30 years and the age of son is 5 years.

Speed of a boat in still water is 15 km/hour. It goes 30 km up stream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2}$$

$$\frac{30(15+x) + 30(15-x)}{15^2 - x^2} = \frac{9}{2}$$

$$900 \times 2 = 9(15^2 - x^2)$$

$$9x^2 = 2025 - 1800 = 225$$

$$x^2 = 25$$

$$x = \pm 5$$

the speed of the stream = 5 km/hr