QUADRATIC EQUATION PART 1

If the equation $(m^2+n^2)x^2-2(mp+nq)x+p^2+q^2=0$ has equal roots, then

(a) $mp=nq$

(b) $mq=np$

(c) $mn=pq$

(d) $mq=\sqrt{np}$

For equal roots,

$$\begin{aligned} b^2 &= 4ac \\ 4(mp+nq)^2 &= 4(m^2+n^2)(p^2+q^2) \\ m^2q^2+n^2p^2-2mnpq &= 0 \\ (mq-np)^2 &= 0 \\ mq-np &= 0 \\ mq &= np \end{aligned}$$

Thus (b) is correct option.

The quadratic equation $2x^2 - 3\sqrt{2}\,x + \frac{9}{4} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

The quadratic equation $x^2 + 4x - 3\sqrt{2} = 0$ has

(a) two distinct real roots

(b) two equal real roots

(c) no real roots

(d) more than 2 real roots

Assertion : $4x^2 - 12x + 9 = 0$ has repeated roots.

Reason : The quadratic equation $ax^2 + bx + c = 0$ have repeated roots if discriminant $D > 0$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Assertion : The equation $x^2 + 3x + 1 = (x - 2)^2$ is a quadratic equation.

Reason : Any equation of the form $ax^2 + bx + c = 0$ where $a \neq 0$, is called a quadratic equation.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Solve for x : $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

We have

$$\begin{aligned} x^2 - (\sqrt{3} + 1)x + \sqrt{3} &= 0 \\ x^2 - \sqrt{3}\,x - 1x + \sqrt{3} &= 0 \\ x(x - \sqrt{3}) - 1(x - \sqrt{3}) &= 0 \\ (x - \sqrt{3})(x - 1) &= 0 \end{aligned}$$

Thus $x = \sqrt{3}, x = 1$

Solve the following quadratic equation for x :

$$9x^2 - 6b^2x - (a^4 - b^4) = 0$$

We have $9x^2 - 6b^2x - (a^4 - b^4) = 0$

Comparing with $Ax^2 + Bx + C = 0$ we have

$$A = 9, B = -6b^2, C = -(a^4 - b^4)$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x = \frac{6b^2 \pm \sqrt{(-6b^2)^2 - 4 \times 9 \times \{(a^4 - b^4)\}}}{2 \times 9}$$

$$= \frac{6b^2 \pm \sqrt{36b^4 + 36a^4 - 36b^4}}{18}$$

$$= \frac{6b^2 \pm \sqrt{36a^4}}{18} = \frac{6b^2 \pm 6a^2}{18}$$

Thus $x = \frac{a^2 + b^2}{3}, \frac{b^2 - a^2}{3}$

If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find k.

We have $3x^2 + px - 8 = 0$

Since 2 is a root of above equation, it must satisfy it.

Substituting $x = 2$ in $3x^2 + px - 8 = 0$ we have

$$12 + 2p - 8 = 0$$

$$p = -2$$

Since $4x^2 - 2px + k = 0$ has equal roots,

or $4x^2 + 4x + k = 0$ has equal roots,

$$D = b^2 - 4ac = 0$$

$$4^2 - 4(4)(k) = 0$$

$$16 - 16k = 0$$

$$16k = 16$$

Thus $k = 1$

Solve for $x : \frac{1}{x} + \frac{2}{2x-3} = \frac{1}{x-2}, x \neq 0, \frac{2}{3}, 2.$

We have

$$\begin{aligned}\frac{1}{x}+\frac{2}{2x-3} &= \frac{1}{x-2}\\ \frac{2x-3+2x}{x(2x-3)} &= \frac{1}{x-2}\\ \frac{4x-3}{x(2x-3)} &= \frac{1}{x-2}\\ (x-2)(4x-3) &= 2x^2-3x\\ 4x^2-11x+6 &= 2x^2-3x\\ 2x^2-8x+6 &= 0\\ x^2-4x+3 &= 0\\ (x-1)(x-3) &= 0\end{aligned}$$

Thus $x = 1,3$

Find for x : $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}$; $x \neq 0, 1, 2$

We have

$$\frac{1}{x-2}+\frac{2}{x-1} = \frac{6}{x}$$

$$\frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$$

$$3x^2-5x = 6x^2-18x+12$$

$$3x^2-13x+12 = 0$$

$$3x^2-4x-9x+12 = 0$$

$$x(3x-4)-3(3x-4) = 0$$

$$(3x-4)(x-3) = 0$$

$$x = \frac{4}{3} \text{ and } 3$$

Hence, $x = 3, \frac{4}{3}$

Sum of the areas of two squares is 468 m^2. If the difference of their perimeter is 24 m, find the sides of the squares.

$$4x - 4y = 24$$

$$x - y = 6 \quad (1)$$

According to the question we get

$$x^2 + y^2 = 468 \quad (2)$$

Substituting $x = y + 6$ from equation (1) in (2) we have

$$(y+6)^2 + y^2 = 468$$

$$2y^2 + 12y + 36 = 468$$

$$2y^2 + 12y - 432 = 0$$

$$y^2 + 6y - 216 = 0$$

$$(y+18)(y-12) = 0$$

Thus $\quad y = -18, 12$