UNIQUE STUDY POINT

IMPORTANT QUESTIONS PPT

CLASS X

TRIGONOMETRY

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If ΔABC is right angled at C, then the value of $\cos(A+B)$ is

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{\sqrt{3}}{2}$

$$\angle A + \angle B + \angle C = 180^\circ$$

But right angled at C i.e., $\angle C = 90^\circ$, thus

$$\angle A + \angle B + 90^\circ = 180^\circ$$

$$A + B = 90^\circ$$

$$\cos(A + B) = \cos 90^\circ = 0$$

Thus (a) is correct option.

Given that $\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\beta = 0$, then the value of $\beta - \alpha$ is

(a) 0° (b) 90°

(c) 60° (d) 30°

We have $\sin\alpha = \frac{\sqrt{3}}{2}$

$$\sin\alpha = \sin 60^\circ \Rightarrow \alpha = 60^\circ \quad ...(1)$$

and $\cos\beta = 0$

$$\cos\beta = \cos 90^\circ \Rightarrow \beta = 90^\circ \quad ...(2)$$

Now, $\beta - \alpha = 90^\circ - 60^\circ = 30^\circ$

Thus (d) is correct option.

If $4\tan\theta = 3$, then $\left(\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\dfrac{2}{3}$

(b) $\dfrac{1}{3}$

(c) $\dfrac{1}{2}$

(d) $\dfrac{3}{4}$

Given,

$$4\tan\theta = 3$$

$$\tan\theta = \frac{3}{4} \qquad \ldots(i)$$

$$\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\frac{\sin\theta}{\cos\theta} - 1}{4\frac{\sin\theta}{\cos\theta} + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1}$$

$$= \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$$

Thus (c) is correct option.

If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, than $x^2 + y^2$ is equal to

(a) 0 (b) 1/2

(c) 1 (d) 3/2

$$\begin{aligned}
\text{We have,} \qquad x\sin^3\theta + y\cos^3\theta &= \sin\theta\cos\theta \\
(x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta &= \sin\theta\cos\theta \\
x\sin\theta(\sin^2\theta) + (x\sin\theta)\cos^2\theta &= \sin\theta\cos\theta \\
x\sin\theta(\sin^2\theta + \cos^2\theta) &= \sin\theta\cos\theta \\
x\sin\theta &= \sin\theta\cos\theta \Rightarrow x = \cos\theta \\
\text{Now,} \qquad x\sin\theta &= y\cos\theta \\
\cos\theta\sin\theta &= y\cos\theta \\
y &= \sin\theta \\
\text{Hence,} \qquad x^2 + y^2 &= \cos^2\theta + \sin^2\theta = 1
\end{aligned}$$

Thus (c) is correct option.

Assertion : The value of $\sin\theta = \frac{4}{3}$ is not possible.

Reason : Hypotenuse is the largest side in any right angled triangle.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

$$\sin\theta = \frac{P}{H} = \frac{4}{3}$$

Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

Thus (a) is correct option.

If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$, $A>B$, then the value of A is ....

We have $\tan(A+B) = \sqrt{3}$

$$
\begin{aligned}
\tan(A+B) &= \sqrt{3} \\
&= \tan 60^\circ
\end{aligned}
$$

Hence, $A + B = 60^\circ \quad ...(1)$

Again,

$$
\begin{aligned}
\tan(A-B) &= \frac{1}{\sqrt{3}} \\
&= \tan 30^\circ \\
A - B &= 30^\circ \quad ...(2)
\end{aligned}
$$

Adding equation (1) and (2) we get

$$2A = 90^\circ \Rightarrow A = 45^\circ$$

If $x = 3\sin\theta + 4\cos\theta$ and $y = 3\cos\theta - 4\sin\theta$ then prove that $x^2 + y^2 = 25$.

We have $x = 3\sin\theta + 4\cos\theta$

and $y = 3\cos\theta - 4\sin\theta$

$$\begin{aligned} x^2 + y^2 &= (3\sin\theta + 4\cos\theta)^2 + (3\cos\theta - 4\sin\theta)^2 \\ &= (9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta) + \\ &\quad + (9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta) \\ &= 9(\sin^2\theta + \cos^2\theta) + 16(\sin^2\theta + \cos^2\theta) \\ &= 9 + 16 = 25 \end{aligned}$$

If $\sin\theta + \sin^2\theta = 1$ then prove that $\cos^2\theta + \cos^4\theta = 1$.

We have

$$\begin{aligned}
\sin\theta + \sin^2\theta &= 1 \\
\sin\theta + (1 - \cos^2\theta) &= 1 \\
\sin\theta - \cos^2\theta &= 0 \\
\sin\theta &= \cos^2\theta
\end{aligned}$$

Squaring both sides, we get

$$\begin{aligned}
\sin^2\theta &= \cos^4\theta \\
1 - \cos^2\theta &= \cos^4\theta \\
\cos^4\theta + \cos^2\theta &= 1
\end{aligned}$$

Hence Proved

Prove that $1 + \frac{\cot^2 \alpha}{1 + \operatorname{cosec} \alpha} = \operatorname{cosec} \alpha$

$$1 + \frac{\cot^2\alpha}{1 + \text{cosse}\ \alpha} = 1 + \frac{\text{cosec}^2\alpha - 1}{1 + \text{cosse}\ \alpha}$$

$$= 1 + \frac{(1 + \text{cosec}\ \alpha)(\text{cosec}\ \alpha - 1)}{1 + \text{cosec}\ \alpha}$$

$$= 1 + \text{cosec}\ \alpha - 1$$

$$= \text{cosec}\ \alpha$$

Hence Proved

Prove that $\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1} = 2\sec^2\theta.$

$$\text{LHS} = \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta - 1} + \frac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta + 1}$$

$$= \operatorname{cosec}\theta\left[\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1}\right]$$

$$= \operatorname{cosec}\theta\left[\frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}\right]$$

$$= \operatorname{cosec}\theta\left(\frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}\right)$$

$$= \frac{2\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta - 1} = \frac{2\operatorname{cosec}^2\theta}{\cot^2\theta}$$

$$= \frac{2 \times \frac{1}{\sin^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}} = \frac{2}{\cos^2\theta}$$

$$= 2\sec^2\theta = \text{RHS}$$

Hence Proved

Evaluate :

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$\frac{3\tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$= \frac{3\times\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\sqrt{3}\right)^2 + 2 - 1}{(1)^2}$$

$$= \frac{3\times\frac{1}{3} + 3 + 2 - 1}{1}$$

$$= 1 + 3 + 2 - 1 = 5$$

Find the value of θ, if,

$$\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta}=4;\theta\le 90^\circ$$

We have

$$\frac{\cos\theta}{1-\sin\theta}+\frac{\cos\theta}{1+\sin\theta} = 4$$

$$\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = 4$$

$$\frac{\cos\theta[1+\sin\theta+1-\sin\theta]}{1-\sin^2\theta} = 4$$

$$\frac{\cos\theta(2)}{\cos^2\theta} = 4$$

$$\frac{1}{\cos\theta} = 2$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = \cos 60^\circ$$

Prove that $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

$$
\begin{aligned}
\text{LHS} &= (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\
&= \left(1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}\right) \\
&= \left(\frac{\sin A + \cos A - 1}{\sin A}\right)\left(\frac{\cos A + \sin A + 1}{\cos A}\right) \\
&= \frac{(\sin A + \cos A - 1)(\cos A + \sin A + 1)}{\sin A \cos A} \\
&= \frac{(\sin A + \cos A)^2 - (1)^2}{\sin A \cos A} \\
&= \frac{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}{\sin A \cos A} \\
&= \frac{1 + 2\sin A \cos A - 1}{\sin A \cos A} \\
&= 2 = \text{RHS}
\end{aligned}
$$

Prove that $\dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \dfrac{1}{1 - 2\cos^2 A}$

$$\text{LHS} = \frac{\tan^2 A}{\tan^2 A - 1} + \frac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A} - 1} + \frac{\frac{1}{\sin^2 A}}{\frac{1}{\cos^2 A} - \frac{1}{\sin^2 A}}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A}} + \frac{\frac{1}{\sin^2 A}}{\frac{\sin^2 A - \cos^2 A}{\cos^2 A \sin^2 A}}$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A}$$

$$= \frac{1}{1 - \cos^2 A - \cos^2 A}$$

$$= \frac{1}{1 - 2\cos^2 A}$$

$$= \text{RHS}$$

In ΔABC, $\angle B = 90°$, $BC = 5$ cm, $AC - AB = 1$,

Evaluate : $\dfrac{1 + \sin C}{1 + \cos C}$.

We have $AC - AB = 1$

Let $AB = x$, then we have

$$AC = x + 1$$

Now
$$AC^2 = AB^2 + BC^2$$
$$(x+1)^2 = x^2 + 5^2$$
$$x^2 + 2x + 1 = x^2 + 25$$
$$2x = 24$$
$$x = \frac{24}{2} = 12 \text{ cm}$$

Hence, $AB = 12$ cm and $AC = 13$ cm

Now
$$\sin C = \frac{AB}{AC} = \frac{12}{13}$$
$$\cos C = \frac{BC}{AC} = \frac{5}{13}$$

Now
$$\frac{1+\sin C}{1+\cos C} = \frac{1+\frac{12}{13}}{1+\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{18}{13}} = \frac{25}{18}$$

Prove that : $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\tan\theta(\cos^2\theta - \sin^2\theta)}{(\cos^2\theta - \sin^2\theta)}$$

$$= \tan\theta$$

Prove that : $(\cot\theta - \operatorname{cosec}\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

$$\cot\theta - \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$$

$$(\cot\theta - \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$$

$$= \left(\frac{\cos\theta - 1}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} \quad [[\sin^2\theta + \cos^2\theta = 1]]$$

$$= \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta} \qquad \text{Hence Proved.}$$

Prove that: $\dfrac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \dfrac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$

$$\text{LHS} = \frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta}$$

$$= \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta + 1}$$

$$= \frac{1 - \cos^2\theta}{\cos\theta + 1} = \frac{(1-\cos\theta)(1+\cos\theta)}{\cos\theta + 1}$$

$$= 1 - \cos\theta \qquad \ldots(1)$$

Now, $$\text{RHS} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$$

$$= 2 + \frac{\sin\theta}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}} = 2 + \frac{\sin^2\theta}{\cos\theta - 1}$$

$$= 2 + \frac{1 - \cos^2\theta}{\cos\theta - 1} = 2 - \frac{(\cos^2\theta - 1)}{(\cos\theta - 1)}$$

$$= 2 - \frac{(\cos\theta - 1)(\cos\theta + 1)}{\cos\theta - 1}$$

$$= 2 - (\cos\theta + 1) = 1 - \cos\theta$$

Prove that: $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta.$

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}\times\frac{(1+\sin\theta)}{(1+\sin\theta)}}+\sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}\times\frac{(1-\sin\theta)}{(1-\sin\theta)}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}}+\sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}+\sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$=\frac{1+\sin\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}=\frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$=\frac{2}{\cos\theta}=2\sec\theta$$

Hence Proved