Problem 1 : A merchant has 120 liters and 180 liters of two kinds of oil. He wants to sell the oil by filling the two kinds in tins of equal volumes. Find the greatest volume of such a tin.

Solution : The given two quantities 120 and 180 can be divided by 10, 20,... exactly. That is, both the kinds of oils can be sold in tins of equal volume of 10, 20,... liters.

But, the target of the question is, the volume of oil filled in tins must be greatest.

So, we have to find the largest number which exactly divides 120 and 180. That is the highest common factor (HCF) of (120, 180).

HCF (120, 180) = 60 liters

The 1^{st} kind 120 liters is sold in 2 tins of of volume 60 liters in each tin.

The 2^{nd} kind 180 liters is sold in 3 tins of volume 60 liters in each tin.

Hence, the greatest volume of the tin is 60 liters.

Problem 2 : Find the least number of square tiles by which the floor of a room of dimensions 16.58 m and 8.32 m can be covered completely.

Solution : We require the least number of square tiles. So, each tile must be of maximum dimension.

To get the maximum dimension of the tile, we have to find the largest number which exactly divides 16.58 and 8.32. That is the highest common factor (HCF) of (16.58, 8.32).

To convert meters into centimeters, we have to multiply by 100.

16.58 · 100 = 1658 cm

8.32 · 100 = 832 cm

HCF (1658, 832) = 2 cm

Hence the side of the square tile is 2 cm.

Required no. of tiles :

= (Area of the floor)/(Area of a square tile)

= (1658 · 832)/(2 · 2)

= 344,864

Hence, the least number of square tiles required is 344,864.

Problem 3 : A Juice seller had three types of juice . 403 liters of 1st kind, 434 liters of 2nd kind and 465 liters of 3rd kind. Find the least possible number of casks of equal size in which different types of juice can be filled without mixing.

Solution : For the least possible number of casks of equal size, the size of each cask must be of the greatest volume.

To get the greatest volume of each cask, we have to find the largest number which exactly divides 403, 434 and 465. That is the highest common factor (HCF) of (403, 434, 465).

HCF (403, 434, 465) = 31 liters

Each cask must be of the volume 31 liters.

Required number casks :

= 403/31 + 434/31 + 465/31

= 13 + 14 + 15

= 42

Hence, the least possible number of casks of equal size required is 42.

Problem 4 :Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ? (excluding the one at start)

Solution :

For example, let the two bells toll after every 3 and 4 seconds respectively.

Then the first bell tolls after every 3, 6, 9, 12 seconds...

Like this, the second bell tolls after every 4, 8, 12 seconds...

So, if the two bell toll together now, again they will toll together after 12 seconds. This 12 is the least common multiple (LCM) of 3 and 4.

The same thing happened in our problem. To find the time, when they will all toll together, we have to find the LCM of (2, 4, 8, 6, 10, 12).

LCM (2, 4, 8, 6, 10, 12) is 120

That is, 120 seconds or 2 minutes.

So, after every two minutes, all the bell will toll together.

For example, in 10 minutes, they toll together :

10/2 = 5 times

That is, after 2, 4, 6, 8, 10 minutes. It does not include the one at the start.

Similarly, in 30 minutes, they toll together :

= 30/2

= 15 times

(excluding one at the start).

Problem 5 :The traffic lights at three different road crossings change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8:20:00 hours, when will they again change simultaneously ?

Solution :For example, let the two signals change after every 3 secs and 4 secs respectively.

Then the first signal changes after 3, 6, 9, 12 seconds...

Like this, the second signal changes after 4, 8, 12 seconds...

So, if the two signals change simultaneously now, again they will change simultaneously after 12 seconds. This 12 is the least common multiple (LCM) of 3 and 4.

The same thing happened in our problem. To find the time, when they will all change simultaneously, we have to find the LCM of (48, 72, 108).

LCM (48, 72, 108) = 432 seconds or 7 min 12 sec

So, after every 7 min 12 sec, all the signals will change simultaneously.

At 8:20:00 hours, if all the three signals change simultaneously, again they will change simultaneously after 7 min 12 sec. That is at 8:27:12 hours.

Hence, three signals will change simultaneously at 8:27:12 seconds.

Problem 6 :Find the least number of soldiers in a regiment such that they stand in rows of 15, 20, 25 and form a perfect square.

Solution :

To answer this question, we have to find the least number which is exactly divisible by the given numbers 15, 20 and 25. That is the least common multiple of (15, 20, 25).

LCM (15, 20, 25) = 300

So, we need 300 soldiers such that they stand in rows of 15, 20 , 25.
But, it has to form a perfect square (as per the question).
To form a perfect square, we have to multiply 300 by some number such that it has to be a perfect square.
To make 300 as perfect square, we have to multiply 300 by 3.
Then, it is 900 which is a perfect square.
Hence, the least number of soldiers required is 900.